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Tito M. Tonietti

And Yet It Is Heard

Musical, Multilingual and Multicultural History of the Mathematical Sciences — Volume 2

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Contents for Volume II

7 Introduction to Volume II 1

Part II In the World of the Scientific Revolution

8 Not Only in Latin, but also in Dutch, Chinese, Italian and German 5
8.1 Aristoxenus with Numbers, or Simon Stevin and Zhu Zaiyu 5
8.2 Reaping What Has Been Sown. Galileo Galilei, the Jesuits and the Chinese 18
8.3 Johannes Kepler: The Importance of Harmony 37

9 Beyond Latin, French, English and German: The Invention of Symbolism 107
9.1 From Marin Mersenne to Blaise Pascal 107
9.2 René Descartes, Isaac Beeckman and John Wallis 158
9.3 Constantijn and Christiaan Huygens 192

10 Between Latin, French, English and German: The Language of Transcendence 227
10.1 Gottfried Wilhelm Leibniz 227
10.2 Sir Isaac Newton and Mr. Robert Hooke 265
10.3 Symbolism and Transcendence 291

11 Between Latin and French 327
11.1 Jean-Philippe Rameau, the Bernoullis and Leonhard Euler 327
11.2 Jean le Rond d'Alembert, Jean-Jacques Rousseau and Denis Diderot 368
11.3 Counting, Singing and Listening: From Rameau to Mozart 412

12 From French to German 431
12.1 From Music-Making to Acoustics: Luigi Giuseppe Lagrange e Joseph Jean-Baptiste Fourier 431

12.2 Too Much Noise, from Harmony to Harmonics: Bernhard Riemann and Hermann von Helmholtz 439
12.3 Ludwig Boltzmann and Max Planck 466
12.4 Arnold Schönberg and Albert Einstein 483

Part III It Is Not Even Heard

13 In the Language of the Venusians 511
13.1 Black Languages 511
13.2 Stones, Pieces of String and Songs 513
13.3 Dancing, Singing and Navigating 514

14 Come on, Apophis 527
14.1 Gott mit uns 527

Bibliography 535

Index 575

Contents for Volume I

1 Introduction 1

Part I In the Ancient World

2 Above All with the Greek Alphabet 9
2.1 The Most Ancient of All the Quantitative Physical Laws 9
2.2 The Pythagoreans 11
2.3 Plato 19
2.4 Euclid 25
2.5 Aristoxenus 35
2.6 Claudius Ptolemaeus 41
2.7 Archimedes and a Few Others 54
2.8 The Latin Lucretius 64
2.9 Texts and Contexts 82

3 In Chinese Characters 97
3.1 Music in China, Yuejing, Confucius 97
3.2 Tuning Reed-Pipes 100
3.3 The Figure of the String 111
3.4 Calculating in Nine Ways 117
3.5 The Qi 124
3.6 Rules, Relationships and Movements 140
3.6.1 Characters and Literary Discourse 141
3.6.2 A Living Organism on Earth 143
3.6.3 Rules, Models in Movement and Values 147
3.6.4 The Geometry of the Continuum in Language 151
3.7 Between Tao and Logos 153

4 In the Sanskrit of the Sacred Indian Texts 169
4.1 Roots in the Sacred Books 169
4.2 Rules and Proofs 172
4.3 Numbers and Symbols 179

4.4 Looking Down from on High 187
4.5 Did a Mathematical Theory of Music Exist in India, or Not? 194
4.6 Between Indians and Arabs 203

5 Not Only in Arabic 209
5.1 Between the West and the East 209
5.2 The Theory of Music in Ibn Sina 210
5.3 Other Theories of Music 219
5.4 Beyond the Greek Tradition 225
5.5 Did the Arabs Use Their Fractions and Roots for the Theory of Music, or Not? 232
5.6 An Experimental Model Between Mathematical Theory and Practice 238
5.7 Some Reasons Why 244

6 With the Latin Alphabet, Above All 257
6.1 Reliable Proofs of Transmission 257
6.2 Theory and Practice of Music: Severinus Boethius and Guido D'Arezzo 259
6.3 Facing the Indians and the Arabs: Leonardo da Pisa 271
6.4 Constructing, Drawing, Calculating: Leon Battista Alberti, Piero della Francesca, Luca Pacioli, Leonardo da Vinci 276
6.5 The Quadrivium Still Resisted: Francesco Maurolico, the Jesuits and Girolamo Cardano 291
6.6 Variants of Pythagorean Orthodoxy: Gioseffo Zarlino, Giovan Battista Benedetti 303
6.7 The Rebirth of Aristoxenus, or Vincenzio Galilei 309

A *[From the]* **Suanfa tongzong** *[Compendium of Rules for Calculating] by Cheng Dawei* 329

B **Al-qawl 'ala ajnas alladhi bi-al-arba'a** *[Discussion on the Genera Contained in a Fourth] by Umar al-Khayyam* 335

C **Musica** *[Music] by Francesco Maurolico* 341
C.1 Rules to Compose Consonant Music 348
C.2 Rule of Unification 353
C.3 Rule of Taking Away 354
C.4 The Calculation of Boethius for the Comparison of Intervals 359
C.5 Comment on the Calculation of Boethius 360
C.6 Guido's Icosichord 362
C.7 MUSIC 374

D **The Chinese Characters** 379

Bibliography 395

Chapter 7
Introduction to Volume II

> We are forced, for the sake of quickness and sharpness, to file down each word to its narrowest edge of meaning. Nature would seem to have become less like a paradise and more and more like a factory.
>
> Ernest Fenollosa & Ezra Pound

In Part II, we find ourselves confronted by the world of the generally so-called scientific revolution. I must therefore warn the reader not to fall into the trap of overused commonplaces, which are unfortunately particularly widespread in many environments. When natural philosophers left the highroad of calculations with ratios as a means to solve their problems, to break out of the rigid framework of the classical *quadrivium*, did they forget music, or not? Certainly not! I will show, rather, that even the most famous characters continued to deal with the art of sounds. Indeed, they did so to such an extent, at times with an attention comparable to that reserved for the questions of mathematical sciences, that music revealed, or at least made more evident, their ideas and notions sustained with other means. In certain cases, we may even go so far as to say that through investigations into the division of the octave, instrument-tuning procedures, or how to justify consonances, creating synergies or otherwise, the evolution of sciences was influenced, particularly at critical moments.

I terminated Part I by showing that it was a context of musical problems that prompted the extension of the notion of number to include irrationals, abandoning the Pythagorean *quadrivium*. However, I must underline that those natural philosophers were not yet ready to follow their speculations through, even if they realised what was at stake. However, musicians like Nicola Vicentino and Vincenzio Galilei, spurred on by the new harmonies that they desired, had already stepped out into the new world. Still on subject to understand the extent of the influence of music on the mathematical sciences, among others, we shall take another look at the pages left to us by Galileo Galilei, Kepler, Descartes, Huygens, Newton, d'Alembert, Euler, Riemann and Planck.

The references to this topic are numerous, and come above all from Europe, though not exclusively. The reader might therefore receive the wrong impression

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that we are abandoning the other scientific cultures, to concentrate our history on the new results achieved by Italians, Germans, Frenchmen, Dutchmen and Englishmen, as if the so-called scientific revolution were exclusively a thing of their own. But this would denote a serious misunderstanding of my historical argument. It is clear that the majority of the texts that we are about to examine had been written in a European national language, or else still in Latin. However, all the traces of other cultures, regardless of their dimensions, will be followed up and duly pointed out, in contrast with what usually happens. The different qualities of these will be the most important aspects to be noted, without being overcome by the quantity of other better-known and more frequently studied books and manuscripts.

If, as a result of my laziness, or ease of consultation, I do not succeed in avoiding the temptation to quote too many texts of old Europe, above all regarding music, I hope this will not appear to be a value judgement on the other cultures. Here, simply, we have kept a greater number of documents, which are available. On the contrary, may it be seen as the right and proper expression of the limitations to my competence and my patience, on such a wide-ranging subject, which cannot be dealt with by a single pen in only two volumes. Rather, may it represent an invitation to scholars of mother tongues different from my own to start exploring the innumerable archives outside Europe, which are full of manuscripts that have been waiting for centuries to be brought to the attention of the world. Colleagues of other cultures have not always attended to this job: the time has come for them to take their places in the ranks.

Compared with their endeavours, whoever they may be, this attempt of mine will act simply as a modest proposal, a beginning that will need additions, corrections, clarifications, and maybe another radical revision, made necessary by the great multicultural age in which we are living, which will see many further developments.

I would like to thank the anonymous referees for helping me to explain myself better, and encouraging me to introduce further improvements in Part II, in the musical and extra-European aspects.

Part II
In the World of the Scientific Revolution

(The trial of harmony and invention)

Youwu xiangsheng
[Being and non-being are born from one another.]
Daodejing, II.

To be, or not to be, that is the question.
Shakespeare, *Hamlet*, III, 1.

Everything in life is so diverse, so opposed, so obscure,
that we cannot be assured of any truth.
Erasmus from Rotterdam, *In praise of folly*, XLV.

Then Aristotle, observing Bacon advance in furious mien, drew his
bow to the head, and let fly his arrow, which missed the valiant
modern and went whizzing over his head; but Descartes it hit;
the steel point quickly found a defect in his headpiece; it pierced
the leather and the pasteboard, and went in at his right eye.
The torture of the pain whirled the valiant bow-man round, till death,
like a star of superior influence, drew him into his own vortex.
Jonathan Swift, *The battle of the books*.

Chapter 8
Not Only in Latin, but also in Dutch, Chinese, Italian and German

But they most despise the low, grovelling populace when they bring out their mathematics of triangles, squares, circles, and other similar geometric figures, which then they lay the one on the other, and mix up as in a labyrinth: at last they astonish the idiots with several letters, arranged like an army in order of battle, and subdivided in different companies.
But at last the preacher brought it thus far, that he could demonstrate the whole Trinity to be represented by these first rudiments of grammar, as clearly and plainly as it was barely possible for a mathematician to draw a proof in the sand.
Erasmus from Rotterdam, *In praise of folly*, 204, 239.

Mark how one string, sweet husband to another, Strikes each in each by mutual ordering; Resembling sire and child and happy mother, Who, all in one, one pleasing note to sing.
Shakespeare, *Sonnet*, VIII.

8.1 Aristoxenus with Numbers, or Simon Stevin and Zhu Zaiyu

The rules for music presented by Maurolico were predominant in texts written during the second half of the sixteenth century, and we have to consider them to represent orthodoxy in that period. Aristoxenus[1] had sustained, on the contrary, that the tone could be divided into equal parts. We can easily imagine that in playing his stringed instrument, a musician would not find any particular difficulty in placing his finger at the point where his ear, or the requirements of music, guided him, and thus

[1] Part I, Sect. 2.5.

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Table 8.1 Stevin's equal temperament

1	Selftoon [Selftone]	Eerste [First (prime)]
$\sqrt[12]{\frac{1}{2}}$	Halftoon [Semitone]	Cleen tweede [Minor second]
$\sqrt[6]{\frac{1}{2}}$	Toon [Whole tone]	Groote tweede [Major second]
$\sqrt[4]{\frac{1}{2}}$	Onderhalftoon [One and a half tones]	Cleen derde [Minor third]
$\sqrt[3]{\frac{1}{2}}$	Tweetoon [Ditone]	Groote derde [Major third]
$\sqrt[12]{\frac{1}{32}}$	Twee en halftoon [Two and a half tones]	Vierde [Fourth]
$\sqrt[2]{\frac{1}{2}}$	Drietoon [Tritone]	Qua groote vierde [Bad major fourth] of qua cleene vijfde [or bad minor fifth]
$\sqrt[12]{\frac{1}{128}}$	Drie en halftoon [Three-tone-and-half]	Vijfde [Fifth]
$\sqrt[3]{\frac{1}{4}}$	Viertoon [Four tones]	Cleen seste [Minor sixth]
$\sqrt[4]{\frac{1}{8}}$	Vier en halftoon [Four and a half tones]	Groote seste [Major sixth]
$\sqrt[6]{\frac{1}{32}}$	Vijftoon [Five tones]	Cleen sevende [Minor seventh]
$\sqrt[12]{\frac{1}{2048}}$	Vijf en halftoon [Five and a half tones]	Groote sevende [Major seventh]
$\frac{1}{2}$	Sestoon [Six tones]	Dobbeleerste, achtste [Double-first (octave)]

also at the point corresponding to $\sqrt{72}$. Now the discussion about the division of the tone assumed the characteristics of controversies between continuous or discrete conceptions of the world, becoming its musical representation. We have followed these in their manifold variations, found in the cultures that were discussed in Part I.

As we have seen, natural philosophers confuted Aristoxenus using arguments based on Pythagorean concepts of numbers, and even invoking God, thus revealing a greater interest in the mathematical theory of music than in composing or playing actual notes. The position of Aristoxenus remained, at most, the unorthodox opinion of a heretical minority, though not totally silent. We have seen that it was also defended by Vincenzio Galilei, in contrast with the orthodox variant of Zarlino.[2]

But now that we arrive at the seventeenth century, it was Simon Stevin (1548–1620) who challenged the prevailing orthodoxy of the Pythagoreans most clearly and decidedly. And he did so on their own ground: that of numbers.

He refused the Greeks' way of dividing the octave 6:3, by means of the harmonic mean $4 = 2 \times \frac{6\times3}{6+3}$, into a fifth and a fourth. From this, he wrote, "oneindelicke ydelheden volghen" ["Endless vanities follow"].[3] He had decided that all the tones and all the semitones should be equal to one another, and consequently he calculated them as follows (Table 8.1).[4]

[2] Part I, Sect. 6.7.

[3] Stevin 1966, pp. 428–429.

[4] Stevin 1966, pp. 426–427 and 440–441. Tonietti 2003b, pp. 226–228.

Table 8.2 Stevin's calculation of the roots for the equal temperament

First 10,000
Minor second 9,438
Major second 8,908
Minor third 8,409
Major third 7,936
Good fourth 7,491
Bad fourth 7,071
Fifth 6,674
Minor sixth 6,298
Major sixth 5,944
Minor seventh 5,611
Major seventh 5,296
Double-first [octave] 5,000
Double minor second 4,719
Double major second 4,454

Stevin (1966, pp. 442–449)

He expected the objection that such a sweet sound as the fifth could not derive from a number like $\sqrt[12]{\frac{1}{128}}$, which he described as "onuijtsprekelick, onredelick, ongeschickt" ["unspeakable, irrational, and inappropriate"],[5] but he rebutted that. He had demonstrated elsewhere that also numbers like these possessed an equally natural perfection, like all the others, and that what was irrational, rather, was the lack of understanding that they had met with. In another table (Table 8.2), he calculated approximations for the above square roots, and then compared them with the "ghedwaelde" ["erroneous"][6] divisions of Pythagoras, Ptolemy, Boethius and Zarlino.

In his *Arithmétique [Arithmetic]* (1585), Stevin had sustained that numbers like $\sqrt{8}$ were not "absurds, irrationels, irreguliers, inexplicables, sourds" ["absurd, irrational, irregular, inexplicable, surd"][7]. Because incommensurability could not determine the absurdity of incommensurable terms which were numerical, if it did not cause the same with geometrical terms, like lines and surfaces. He repeated the concept in the third of his *Thèses mathématiques* [mathematical theses]: "Que racine quelconque est nombre" ["Any root is a number"]. And again in the fourth: "Qu'il n'y a aucuns nombres absurds, irrationels, irreguliers, inexplicables, ou sourds." ["No numbers are absurd, irrational, irregular, inexplicable or surd".][8]

Many of the characters that we have met in our story also knew of some procedures to extract the square root or cubic root of a number. But those who had remained prisoners of the Pythagorean tradition could not consider the result as

[5] Stevin 1966, p. 440.

[6] Stevin 1966, pp. 446–447.

[7] Stevin 1958, pp. 532–533.

[8] Stevin 1958, p. 738.

an appropriate solution to any problem. Now, however, Stevin used roots like any other number, and consistently did not separate numbers from geometry and did not consider them to be discontinuous magnitudes. He had been able to conceive of the equable temperament in 12 equal semitones because he no longer believed in the Pythagorean principles of numbers. $\sqrt[12]{\frac{1}{128}}$ was not to be considered different in quality from $\frac{2}{3}$, because they were both points in the geometrical *continuum* of numbers.

However, this Dutchman did not succeed in gaining consensus for his numerical equable temperament. His incomplete manuscript *Vande Spiegheling der Singconst [On the theory of the art of singing]* was to be discovered only during the nineteenth century in the library of the musician Constantijn Huygens, secretary to the Prince of Orange, and the father of Christiaan Huygens, who we shall meet later on in Sect. 9.3.

Stevin expounded his theory by giving first the definitions, and then the postulates and the theorems, as if he took his inspiration from Euclid. He postulated that the depth of a note depended on the part [length] of the string considered, because this did not require any demonstration, but derived from experience.[9] His second postulate stated that "Heele toonen al even groot te wesen, dat sgelijcx oock halve toonen al even groot sijn." ["All whole tones are equal and likewise all semitones are equal".][10] This he explained as it were the choice to consider equal the intervals $ut[do] - re, re - mi, fa - sol, sol - la$. The explanation was followed by the values of the roots (Table 8.1) called "*Vertooch*" ["Theorem"?] with a "*Bewijs*" ["Proof"], which however is missing.

Our Dutchman made his division of the octave into 12 equal parts depend on the term "*everedenheijt*" ["Equirationality"]. He sustained that it was now easy for him to consider all the semitones as having the same ratio because the word existed in the "*duytsche*" ["Dutch"] language. On the contrary, the Greeks could not divide it in the same way (and 6:4 [Fifth] did not have the same ratio as 4:3 [Fourth]) because they would not have had any way of writing it in Greek. "The cause of these errors is that the Greek language could not interpret this term together with all other mathematical terms as properly as the Dutch language does." Stevin went on here to discuss the idea that Greek had been "useful in order to bring to light various discoveries of the Greeks, Latin (because it happened to become the common language of the world) serves to be understood in all countries, and also to study all sciences about all subjects, which have been described in it by various kinds of people, French, Italian, Spanish, Polish, etc. It serves to carry on trade, everybody according to his situation. But DUTCH[11] serves to teach the liberal arts, to fathom the hidden secrets of nature, and to prove that miracle is not a miracle."[12]

[9] Stevin 1966, p. 425.

[10] Stevin 1966, pp. 426–427.

[11] Emphasis in the text.

[12] Stevin 1966, pp. 426–429.

Simon Stevin took this motto, that a miracle was not really a miracle, as his own, placing it on the title page of his book dedicated to static mechanics *De Beghinselen der Weeghconst [Elements for the art of weighing]* (1586). Here, among other things, he gave the rules for the inclined plane.[13] Beyond the love for his native tongue, Stevin's considerations about Greek and Latin are particularly interesting. They reveal that Latin was starting, at least in certain contexts, to lose its role of lingua franca for communication between natural philosophers; it was gradually to be eclipsed by the various national languages, as the single States strengthened their claims for prerogatives. The progressive weakening of Latin was to stimulate the search for another common language, at least for mathematical sciences. We shall see that the problem was to lead to the invention of a special symbolism for these sciences.

Just as the Dutchman made use of geometry to evaluate the relationships between figures that were not easy to recognise purely by sight, so his hearing would not be sufficient to judge relationships between the depth of sounds. The ancients knew, as was clear by listening, that the semitone was greater than the Pythagorean division of the tone, 256:243. And yet they decided to take it as correct, because they would have considered that "... the defect was in our singing (as if one should say: the sun may lie, but the clock cannot)". The Greeks had also rejected the pleasures of minor and major thirds, because they had taken the wrong numbers. Nor were the attempts of Zarlino to fare any better. And he repeated that all these errors derived from his persuasion: it was not possible in these languages to understand the nature of "... equirationality ... tool ...", which on the contrary existed in Dutch, "... the most perfect language of languages." It is important to note that in his text, Stevin wrote $\frac{3}{2}$ or $\frac{9}{8}$, in other words, *fractions*, where other theoreticians had used ratios, 3:2 or 9:8.[14]

He now took the keyboard of an organ or a harpsichord, and he explained how to tune instruments by means of the fifths, fourths, tones and semitones that he had calculated (Table 8.1). He wrote for musicians, because he knew perfectly well that they were the ones who would receive most benefit from the equable temperament.[15]

Therefore the notes were to be arranged in a scale, calculated in accordance with the proportional means of the relative numbers, which, as we have seen above, were no longer to be called irrational. To "... obtain the true perfect sounds of natural singing", he then divided the classic monochord in accordance with his own geometrical progression based on $\sqrt[12]{\frac{1}{2}}$. Even he invoked Euclid, and drew a line, but the distances on this line were calculated by extracting the necessary numerical roots, accepting only the octave as a ratio between whole numbers. And assuming this to be $\frac{10{,}000}{5{,}000}$, he obtained the following table, with suitable approximations (Table 8.2).

[13] Dijksterhuis 1971, pp. 432–440. Minnaert 1970, pp. 48–49.

[14] Stevin 1966, pp. 429–433.

[15] Stevin 1966, pp. 436–437.

In any case, the ear would not be sufficient to tune all the intervals of the fifth correctly on organs and harpsichords.[16] But an organist made the objection that [on his instrument which had not been well tempered] three major thirds did not make an octave.[17] And yet it would have been the musicians and composers that transformed his heretical position into the common modern-day practice of the equable temperament, now used everywhere. They only had to start to prefer moving freely with their modulations between one tonality and another, in a desire to play them on their fixed-tuning instruments. This happened above all with Johann Sebastian Bach (1685–1750) and Jean Philippe Rameau (1683–1764), while the composers of the sixteenth century continued to write polyphonic vocal compositions with several voices, which gave rather a sense of eternal immobility, like those of a certain Roland de Lassus (1530–1594).

Stevin had been one of the first to adopt also fractions and decimal figures in calculations. His many interests led him above all to solve the most varied kinds of practical problems: conical gears for the transmission of movement, hydraulics for floodgates, canals, drainage, windmills, navigation, astronomy and wartime fortifications. These activities had been requested to him by the historical context. The Orange dynasty, for whom he worked, was seeking its independence from the Spaniards by fair means or foul, and was expanding overseas.

His ideas of the mathematical sciences were the most appropriate for these purposes: fewer theorems and more calculations. Thus with him, the long journey undertaken by numbers, starting from the Orient, with the Chinese, the Indians and the Arabs, and passing through the abacus schools and Renaissance workshops, arrived in seventeenth century Europe. When the orthodox European tradition of the *quadrivium* had placed numbers among the discontinuous magnitudes, it was this Dutchman who now moved them among the continuous ones. "Que nombre n'est poinct quantité discontinue." ["A number is not a discontinuous quantity at all"]. "... à une continue grandeur, correspond le continue nombre qu'on lui attribue ... le nombre est quelque chose telle en grandeur, comme l'humidité en l'eau, car comme ceste ci s'estende par tout & en chasque partie de l'eau; ..." ["... a continuous magnitude corresponds to the continuous number that is attributed to it ... number in magnitude is something like humidity in water, because it extends everywhere, in every part of the water; ..."]. Instead of being the general generating principle, the number 1 was put together with the others.[18]

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[16]Stevin 1966, pp. 456–459.

[17]Stevin 1966, p. 420. Even in the twentieth century, the editor of the manuscript, Adriaan D.Fokker, displayed a far greater tendency to censure towards Stevin's avoidance of Pythagorean whole numbers, judging that he had acted "... *haughtily and aggressively* ..."; Stevin 1966, p. 419. On the contrary, the Dutchman, Rudolf Rasch has recently provided us with more interesting details about the original pages of his earlier fellow-countryman, duly pointing out the influence that he had, more often than not in a negative sense, on Beeckman, Mersenne, Descartes and the two Huygens; Rasch 2008. See below, Sects. 9.1–9.3. Cf. Bos 2004.

[18]Stevin 1958, pp. 501–503 and 738.

Following these ideas, Stevin developed the method of exhaustion used by Archimedes. He substituted the latter's proofs with the argument that if the difference between two magnitudes could be made smaller than any quantity assigned, then the two magnitudes would be equal. This was the case for the difference between the polygon with an increasing number of sides and the circle. For this reason, he should be included in the history of infinitesimal calculus, which would spoken in the same way of a 'limit'.

Our Dutchman used procedures of the same kind to calculate curves on a sphere that intersected the meridians at constant angles. This problem had become important for navigation, where Stevin determined a ship's route along a maximum circle or along a line of the above kind. For sailors, over distances that are not too great, it is much more convenient to follow this second method, to which Willebrord Snellius (1580–1626), in translating Stevin's writings into Latin, gave the Greek name of 'loxodrome' [an oblique course]. Using the compass, it is easier to keep the prow of the ship at a constant angle with respect to North (the meridian), tracing the route on Mercator's nautical maps. This famous mapmaker, whose real name was Gerhard Kremer (1512–1594), was another Dutchman who, thanks to the relative projection of the sphere on to the cylinder tangent to the equator, had drawn nautical maps where meridians and parallels intersect at right angles, and the loxodromes are straight.[19]

In a word, Stevin proposed putting an end to that millennial separation between numbers and geometry to which the Pythagoreans, Plato and Euclid had accustomed us. Furthermore, his numbers became effective to solve the problems of the world, because they had become a part of it, as humidity in water. But even in this field, he did not immediately find many followers. Borrowing from others, our Dutchman also tried to introduce a 'little' novelty into symbolic notation, writing algebraic equations. Instead of the word 'thing' for the unknown, he indicated the power to which a number is raised **1**, **2**, **3** in a small circle for the square, or the cube, where nowadays we would put x, x^2, x^3. He too, as everybody did, anyway continued to feel the need to anchor these numbers and reasonings to the solid propositions of Euclid; after all, the Arabs had done the same. And he interpreted an equation, rewriting it in the form of a ratio, even if elsewhere he regularly used fractions.[20]

We can be almost certain that Stevin had read some Arabic texts, because he quoted "Mahomet", together with Pacioli, Cardano, Tartaglia, Michael Stifel (1487–1567) and others.[21] In astronomy, he wrote a book to sustain the model of Copernicus. From this Dutchman, we can arrive directly, passing through the intermediation of only one real person, on the one hand at Christiaan Huygens, and on the other at René Descartes.[22]

[19] Struik 1958, pp. 7–9.

[20] Struik 1958, pp. 463, 471 and 474.

[21] Struik 1958, p. 474.

[22] Struik 1958, p. 8.

Stevin served as a general commissioner in the army of the newly-formed Holland, dealing with various matters: from training for soldiers to siege tactics, from fortifications to camps. He actually took part in some battles.[23]

Had it not been for all these warlike activities, so far removed from the world of Confucius, our Dutchman would have appeared to be at the same time the European natural scholar farthest, with his novelties, from his orthodox colleagues, and closest to the style that was common in China. Among the characteristics of his, which were seen before in the Country-at-the-Centre,[24] we may indicate the interest above all in practical problems, and the idea of fractional and decimal numbers to be used for measuring any magnitude, far from the Pythagorean prohibitions of irrationals, and thus seen as a *continuum*. Last but not least, was a certain indifference to justifications by means of philosophies of transcendence. This led him not to take sides with one religion or another, in a period of schisms and heresies when this was almost compulsory. Rather, he accepted the authority of political power, which was to be obeyed in any case.[25] Living in the land of the windmills, he had also invented a cart propelled by sail power over land, like boat over water, another device that was already present in China.[26]

For all these reasons, we may entertain the hypothesis that, in the crowded, industrious ports of his country, this Dutchman might have profited from contact with the Chinese culture, and might have taken some inspiration from this. However, there is no confirmation or proof of this. Thus, putting aside the model of the single origin for scientific and technical inventions, we will continue to follow the idea of their independence. And yet Joseph Needham indicated another case by means of which he tried to demonstrate the possibility of hypothesising the transmission of a not negligible contribution from China to Europe. In those same years sure, at the end of the sixteenth century, also Zhu Zaiyu, a prince of the Ming dynasty, proposed, and calculated numerically a division of the musical octave into 12 equal semitones.[27]

The *lülü* tuning system, which we have already seen above, as described in the text of Cheng Dawei,[28] was not the only one that existed in the late Ming empire of the sixteenth century. In 1584, Zhu Zaiyu (1536–1611) had already calculated and proposed a new system.

> Xinzhi lüzhun
> [Exact standard for *lü* based on a new system].[29]

[23] Minnaert 1970, p. 50.

[24] Part I Chap. 3.

[25] Minnaert 1970, pp. 47 and 50–51.

[26] Needham, Wang & Robinson 1962, p. 228.

[27] Needham & Wang & Robinson 1962, pp. 220–228. Tonietti 2003b, pp. 234–237.

[28] Part I Sect. 3.2.

[29] Zhu 1584, p. 20. Needham, Wang & Robinson 1962, p. 222. Chinese characters in Appendix D.

Table 8.3 Zhu's calculation of equal temperament

Lülü	Length (chi)	Diameter (cun)
1. Huáng Zhong	2	0.500000
2. Dà Lǚ	1.88774862	0.4857659
3. Dà Cù	1.78179743	0.4719371
4. Jiá Zhong	1.68179283	0.4585020
5. Gu Xǐ	1.58740105	0.4454493
6. Zhòng Lǚ	1.49830707	0.4327682
7. Ruí Bin	1.41421356	0.4204482
8. Lín Zhong	1.33483985	0.4084788
9. Yí Zé	1.25992104	0.3968502
10. Nán Lǚ	1.18920711	0.3855527
11. Wú Shè	1.12246204	0.3745767
12. Yìng Zhong	1.05946309	0.3639132
13. Huáng Zhong	1	0.3535533

This famous prince constructed an instrument with 12 strings which was to serve as a standard to tune the others. He fixed studs in it, to indicate where to place the fingers in order to obtain the desired note (Fig. 8.1).

He, too, used numbers to calculate the relative length of the strings, but he did this: "... for seeking harmony in the notes, and did not make the notes submit to numbers."[30]

In studying the texts of the "decreasing, increasing" tradition, he noticed differences of tuning between the *lülü* fixed by this method and the stringed instruments constructed by artisans and used by musicians. Thus he proposed a new method.

> Chuangli xinfa. Zhiyi chiwei shiyi milü chuzhifan shier biansuoqiu lülü zhenshu.
> [I give origin to a new rule. As the dividend, let one *chi* [about 33 cm] be fixed; to [obtain] the right ratio, divide it [extract the square root[31]]. In brief, find the right numbers for the twelve *lülü* from one place to another].[32]

In another book of 1596, Zhu explicitly gave his own procedure for calculating the length of the strings. These also included the octave, because he also considered the length twice that of the basic *chi*. Therefore the octave was divided into 12 equal semitones, using the ratio established by $\sqrt[12]{2}$ (Table 8.3).[33]
Thus Zhu proposed what is nowadays called the equable temperament in Europe and all over the world; in order to achieve it, he calculated the length of strings and pipes, using the appropriate square or cubic roots of two, or rather, the appropriate geometrical means.

[30] Needham, Wang & Robinson 1962, pp. 220–221.

[31] In China, where no Pythagorean prohibition of irrational numbers existed, the extraction of a root was seen as a kind of division; Tonietti 2006a; see above, Part I, Chap. 3.

[32] Zhu 1584, p. 8. Needham, Wang & Robinson 1962, p. 223. Chinese characters in Appendix D.

[33] Zhu 1596.

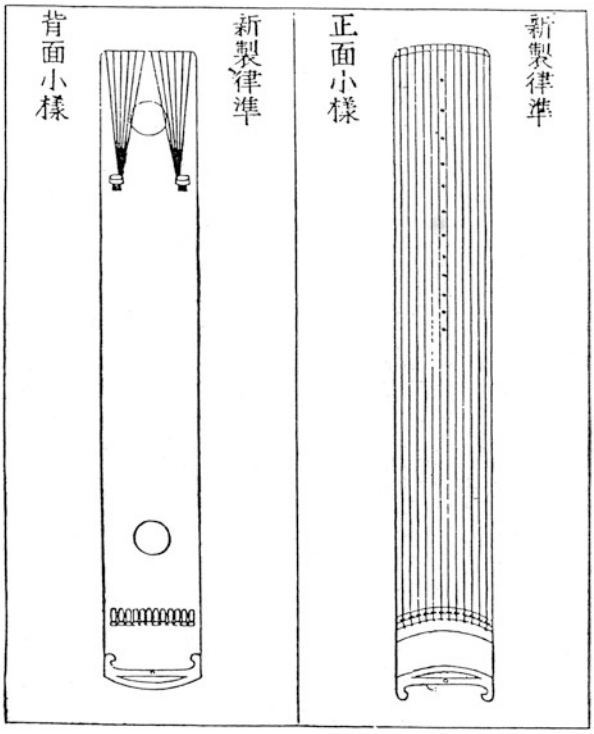

Fig. 8.1 The instrument on which Zhu Zaiyu tempered the twelve lülü in an equable manner Zhu Zaiyu ("Picture of the stringed instrument", 1584)

Our prince took a string and a Huang Zhong pipe 2 *chi* long, that is to say, tuned to the low octave of the basic note, and calculated the length of the Ying Zhong to be 1.05946 *chi*. This consequently represented a tempered semitone, seeing that it was the interval between the 12th note and the octave, or (which is the same thing, since all the semitones are now equal) between the first and the second note. He obtained the lengths of the other strings in a geometrical succession, by dividing 2 by 1.05946 and repeating the same operation serially ten more times. After the 12th operation, the length of 1 *chi* of the initial Huang Zhong is obtained, because $2 : (\sqrt[12]{2})^{12} = 1 : 1$.

Zhu equabily tempered not only strings, but also took up the ancient tradition of varying the diameter of pipes.[34] He must have been aware of the fact that the sound of pipes depends not only on their length but also on their diameter. Thus he also

[34]Part I, Sect. 3.2.

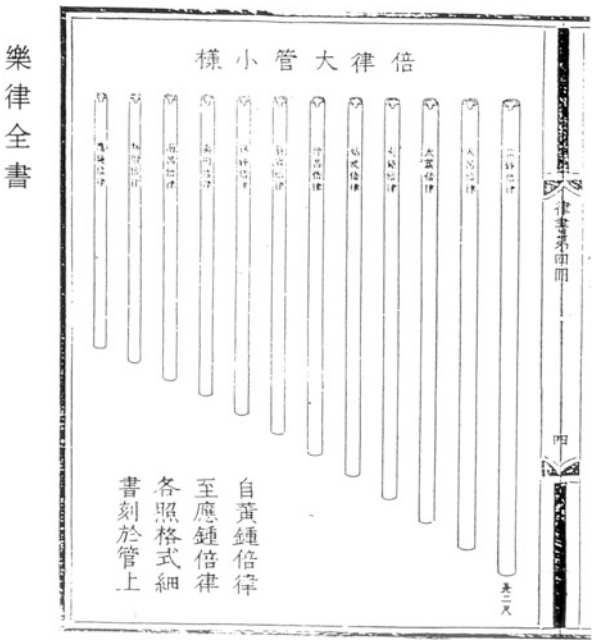

Fig. 8.2 The pipes tempered equably by Zhu Zaiyu (Zhu Zaiyu, "picture of the 12 pipes", 1584)

tempered the diameters, dividing them progressively, this time by $\sqrt[24]{2}$. Also in this respect, he deviated from Cheng Dawei.[35] (Fig. 8.2)

The Ming noble thus used numbers obtained by the extraction of roots or from proportional means (which would have been called irrational in Europe). But this was not the only novelty with respect to the other Chinese scholars, because it critically depended on considering also the double measure and the half measure among the lengths suitable for music. That is to say, among the others, Zhu also used the notes generated by these latter pipes (in Europe they would have been called the octaves). Few scholars before him had introduced, together with the Huang Zhong of 9 *cun* a Shao Gong [lesser palace] of $4\frac{1}{2}$ *cun*.[36] The tradition of the "decreasing, increasing" system had remained the orthodox one, which did not contemplate it because multiples of $\frac{4}{3}$ or of $\frac{2}{3}$ cannot be included among the multiples of 2. This can be understood from the report of Cheng Dawei, who was unaware of the octave. It is interesting to note that Cheng did not justify the inversion of the rule (two successive upward generations, multiplying twice by $\frac{4}{3}$, where we would expect a decrease of $\frac{2}{3}$ the second time), writing that he did so in order to remain inside the

[35]Robinson 1980, p. 118. But we may note that he should have divided the diameters in the same proportion $\sqrt[12]{2}$ as the length, in order to obtain a good tempering also of the pipes; Tonietti 2003b, p. 240.

[36]Needham, Wang & Robinson 1962, p. 214.

proportion of the octave. On the contrary, he invoked the increase of the *yin* at the summer solstice.[37]

In the Country-at-the-Centre, those who sought mathematical models for music could not have had the idea of the equable temperament if they had not also considered the ratio 1:2. Orthodox tradition did not provide for this, and so it was not possible to divide into equal parts what did not exist. This seems to have been the main reason why here the equable temperament remained unknown up to Zhu Zaiyu. In the Chinese culture, the roots of any number were not subject to those prohibitions seen in Europe. The procedures for their calculation, though approximate, gave results that could be used. This attitude towards numbers was supported by the idea that they, too, like sounds, which were vibrations of the *qi*, and like the whole world, were part of an all-inclusive geometrical *continuum*, where they found their justification. Thus, once the octave was accepted, it would become possible to divide it into equal parts, as Zhu did, without any further problems.

In Europe, the absence, among the orthodox mathematical sciences, of the equable temperament with numbers, stemmed from other reasons. Here, the octave ratio of 1:2 had always been considered among the fundamental ones. But up to this time, we have always been confronted with the rigid separation, made in the Pythagorean-Platonic-Euclidean tradition, between commensurable relationships, which could be represented by numbers, and incommensurable ones, for which this kind of representation was not allowed (nowadays we would speak of rational and irrational numbers). It had thus been prevented to use the instrument by means of which the octave could be divided into equal parts. The division of the Pythagorean tone into equal parts gives a slightly higher value than that of the equable temperament. But, even though it was not strictly speaking equivalent in the precise numbers, the division of the tone into equal parts would have been a good step towards the equable temperament. If the proportional mean could be used, why not calculate it in order to divide the octave? And then to divide again the interval between this note (fa♯ in Europe, Rui Bin in China) and the starting note? This is exactly what Zhu Zaiyu did.[38] The idea of the equable temperament could then have easily occurred to anyone who knew how to extract roots. Anyone who reflected that also the square root of 2 could be considered a number like the others (and therefore usable as the solution to a problem) would have been able to divide the octave into equal parts. And this was what Simon Stevin did.[39]

It was not only the theory that made the two cultural contexts different. Typical instruments of the Chinese musical tradition, like *zhong* [bells] and *qing* [chime-stones] appeared to be more expensive to construct, and more difficult to tune than any string instrument or wind instrument. Thus the desire to play a greater number of melodies on them, with a greater number of modulations between one modality and another, was bound to stimulate the search for an equable tuning. This need found

[37]Tonietti 2003b, pp. 231–232; see above, Part I, Sect. 3.2.

[38]Robinson 1980, p. 115; Chen 1999.

[39]See above in this section. Tonietti 2003b.

partial expression in that unique invention of the Chinese artisans which consisted of bells with an asymmetrical shape. When struck at different fixed points, they are able to produce two different notes.[40]

Even European instruments with a fixed tuning, like organs, not to speak of lutes, seem to be less difficult to tune than sets of bells. Anyway, a comparable stimulus towards the equable temperament was also present in Europe. But here it seemed to show its effects above all among musicians. We have found the greatest openness towards the equable temperament in Aristoxenus and Vincenzio Galilei. And other musicians, too, were only to become convinced when compositions abandoned the static nature of the Gregorian *cantus firmus* and the polyphonic constructions of the Flemish for the variety of melodies and tonal harmonies. When the fixed-tuning harpsichord and the piano progressively took over the stage, becoming the reference instruments for everybody, like today, the equable temperament became an indispensable means to enrich modulations. However, here it was not easy for these requirements to break out of the community of musicians, to make themselves heard by the natural philosophers of the scientific tradition that had long prevailed in Europe. Their ears and their brains had been better prepared to hear the abstract music of the spheres than real compositions.

In China, on the other hand, sciences seem to have evolved in a constant relationship with the numerous practical problems of this culture. Thus, also in our case, it may have been relatively easier for the needs of musicians to influence the considerations of theoreticians. This is exactly what we see Zhu Zaiyu declare, in order to justify his new science of the *lülü*. It would have been practically impossible to find a similar attitude in Europe, because it would have entailed a loss of prestige for the discipline.

It does not seem to be productive for us to level our history to a question of priority. The close proximity in time between Vincenzio Galilei, Simon Stevin and Zhu Zaiyu certainly appears to be striking. The book of the first of the three, *Dialogo della musica antica et moderna* [Dialogue between ancient and modern music], which took up the subject of the equable temperament, came out in 1581. The manuscript of the second of them, which remained unpublished, was probably written a few years after 1585. While Zhu's first text, *Lüxue xinshuo [New summary of the science of lü]*, appeared in 1584.[41] But today, we do not possess any documents even to formulate a hypothesis that one of them had borrowed from the others. It was clearly much less likely that Zhu had been inspired by the Europeans. The Jesuits would have transferred to China the orthodoxy of Maurolico, if anything, rather than the heresy of Stevin, if printing had existed. In any case, both contexts contained the elements necessary to realise the equable temperament with numbers. Consequently, it is much more likely that each of them had invented it by himself.

[40] Chen 1994.

[41] Tonietti 2003b.

8.2 Reaping What Has Been Sown. Galileo Galilei, the Jesuits and the Chinese

> Che peccato che l'artiglierie non fussero al tempo
> di Aristotele! Avrebbe ben egli con esse espugnata l'ignoranza,
> e parlato senza punto titubare delle cose del mondo.
> Galileo Galilei, *Dialogo sui massimi sistemi*, "Giornata seconda".
> [What a pity that the artillery did not exist at the time
> of Aristotle! He would have swept away ignorance with it,
> and spoken without any hesitation of the things of the world.
> *Dialogue about overall systems*, "Second day".]

Galileo Galilei (1564–1642), son of Vincenzio, must have learnt from him a great deal of music, and also wrote on the subject, performing practical experiments sometimes. One page dedicated to music is found at the end of the "First day" in the *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* [Discourses and mathematical proofs regarding two new sciences] (1638). Here, Galileo Galilei took up the orthodox tradition of the Greeks, with ratios for musical intervals formed by whole numbers, justifying them by means of the theory of strokes created by sounds in the ears; we have already seen this in Francesco Maurolico.[42]

The famous scholar from Pisa now affirmed, probably taking the idea from his father, that the tension of the musical string did not make the pitch of the note rise linearly (as the ancients believed), just as the depth increases with the length, but in accordance with a lower ratio. Thus, if we want to generate, for example, the higher octave, it is not sufficient to double the tension (with a double weight), length and other conditions remaining equal, but it needs to be multiplied by four. "… vorremmo farla montare all'ottava col tirarla di più, non basta tirarla il doppio più, ma ci bisogna il quadruplo, sì che se prima era tirata dal peso d'una libbra, converrà attaccarvene quattro per inacutirla all'ottava; …". ["… if we desire to raise it to the octave by pulling it more forcefully, it is not enough to pull it twice as hard, but it is takes four times the tension, with the result that if it was previously pulled by a weight of one pound, it will be necessary to attach four of them in order to raise it to the octave; …".]

For his father Vincenzio, this phenomenon had disproved the Pythagorean-Platonic tradition, but not for his son, Galileo. "… non è la ragion prossima ed immediata delle forme de gl'intervalli musici la lunghezza delle corde, non la tensione, non la grossezza, ma sì bene la proporzione de i numeri delle vibrazioni e percosse dell'onde dell'aria che vanno a ferire il timpano del nostro orecchio, il quale esso ancora sotto le medesime misure di tempi viene fatto tremare." ["It is not the direct, immediate reason for the forms of the musical intervals, either the length of the strings, or the tension, or the thickness, but rather the ratio of the numbers of the vibrations and strokes of the air waves that strike the tympanum of our ear, which is made to vibrate at the same rate"]. When the strokes of two notes struck

[42] Part I, Sect. 6.5.

the ear together, creating a coincidence most times, there would be consonances, and in other cases dissonances. "… e crudissime saranno le dissonanze quando i tempi delle vibrazioni fussero incommensurabili; per una delle quali sarà quella quando di due corde unisone se ne suoni una con tal parte dell'altra quale è il lato del quadrato del suo diametro: dissonanza simile al tritono o semidiapente." ["and the dissonances will be truly harsh when the times of the vibrations should be incommensurable; an example of one of these will be when one of two unisonous strings is played with that part of the other one which is to the first as the side of the square to its diagonal: a dissonance similar to the tritone, or semidiapente"]. He preferred the 3:2 ratio for the fifth, "… fa una titillazione ed un solletico tale sopra la cartilagine del timpano, che temperando la dolcezza con uno spruzzo d'acrimonia, par che insieme soavemente baci e morda." ["… it creates a tickle and a titillation over the cartilage of the tympanum, that tempers sweetness with a dash of acrimony, and seems to offer sweet kisses and to bite at the same time."] Incommensurable proportions would violate the harmony of the cosmos, and would lead to chaos. "… ordine disordinato di sregolata intrecciatura, e l'udito con noia riceve gli appulsi intemperati de i tremori dell'aria …" ["a disorderly order of irregular fabrication, and the hearing receives with irritation the untempered impulses of the air trembling"].[43]

Although they are short, these pages clearly reveal Galileo's attitude. He described experiments with water, in order to make the vibrations of glasses and resonating strings visible, arriving at an "… ondeggiamento … per l'aria …" ["… waving … in the air …"]. In giving the above rule governing the tension of strings, he also criticised current explanations as being insufficient, but he did not subvert the general theory. Using geometrical designs as well, he concluded with the orthodox theory of consonances, which prohibited incommensurable ratios in music.

Our natural philosopher, who we may note started to write in Italian, left other pages about music in his *Il saggiatore* [The tester] (1623). He affirmed that the sound of the pipes in an organ depended on their size (and not the material that they were made of), but he did not offer any numbers. However, that sound was made of "… onde, dalla frequenza delle quali nasce l'acutezza del suono e la gravità dalla rarità." ["… waves, the frequency of which produces the acuteness of a sound, while its depth stems from rarity"]. However, this natural philosopher would have like to eliminate the sensations felt in the nose, on the tongue and in the ears. "… restino bene le figure, i numeri e i moti, ma non già gli odori, né i sapori, né i suoni, li quali fuor dell'animal vivente non credo che sieno altro che nomi, …". ["Let the figures, the numbers and the movements remain, but not the smells or the tastes or the sounds, which in my opinion are nothing other than names, outside the living animal."] Thus he judged music by means of numbers, rather than with his ears. As regards sight, he preferred to keep his silence: "… senso sopra tutti gli altri eminentissimo, abbia relazione la luce, ma con quella proporzione d'eccellenza qual è tra 'l finito e l'infinito, tra 'l temporaneo e l'istantaneo, tra 'l quanto e l'indivisibile,

[43] Galileo Galilei 1996 II, pp. 671–680.

tra la luce e le tenebre." ["… a most excellent sense above all the others, in association with light, but with that proportion of excellence which exists between the finite and the infinite, between the temporary and the instantaneous, between quantity and the indivisible, between light and darkness."] The degree to which the problem of music was felt in this century, involving Galileo as well, was confirmed in 1619 by Federico Cesi (1585–1630), the founder of the renowned Accademia dei Lincei. "Quella [disputa] delle proportioni musicali et della rispondenza delle corde nelli istrumenti, ..., passammo già copiosamente col *S.r* Galilei nostro in Tivoli; et veramente sarebbe bene, fusse pienamente distesa in carta …" ["In detail with our *S.r* Galilei at Tivoli, we discussed that [dispute] about musical ratios and the correspondence of strings in instruments; and indeed it would be better to put it down in black and white thoroughly …"].[44] In spite of his father's defence, there is no trace of Aristoxenus, or of the possibility of dividing the tone into equal parts in Galileo Galilei (judging by the index of name in the national edition of *Le opere*).

Following this line demonstrating the continuing influence of music on the evolution of the sciences in the seventeenth century, at the time of their so-called revolution, the testimony of Vincenzo Viviani (1622–1703), Galileo's last pupil, is extremely interesting. "Questa del pendolo si è una delle più antiche invenzioni e scoperte in natura del Galileo, e fu circa l'anno 1580, quando era studente a Pisa, nel trovarsi egli un giorno in quel Duomo, dove si abbatté di vedere, lasciata in moto, una lampada pendente da una lunghissima corda. E, come quello che da giovanetto s'era anche esercitato nella Musica, sotto la disciplina di quel gran Vincenzio suo Padre, che si dottamente scrisse poi in *Dialogo della Musica antica e moderna*; percioché aveva impressa nell'anima l'egualità de' tempi, co' quali essa si regola, riflettendo a quel moto, gli fu facile il giudicarlo in mente sua equitemporaneo, sì nelle andate lunghe e larghe al principio del moto, come nelle strette sul fine verso la quiete. In casa poi se ne chiarì in più modi replicate esperienze esattissime, trovando, coll'aiuto de' suoi compagni, che in un determinato numero di vibrazioni d'un certo pendolo, lasciato andar sempre da una distanza medesima del perpendicolo, quante ne faceva un altro pendolo delle larghe, altrettante in ciascuno ne faceva delle strette e delle strettissime." ["This finding of the pendulum is one of Galileo's earliest inventions and discoveries in nature; it was in about 1580, when he was a student at Pisa, that one day, he was in the Cathedral, where he happened to see a lamp swinging freely at the end of a very long rope. As he had also studied Music as a boy, under the discipline of his great father, Vincenzio, who wrote so learnedly in his *Dialogue on ancient and modern Music*; thus he had impressed in his soul the equality of times by which it is regulated, and reflecting on that movement, it was easy for him to judge in his mind that the same time was taken up, both in the long, broad round going at the beginning of its movement, and in the short ones at the end, when it moved towards stillness. Later, at home, he clarified the question in various ways: repeating extremely precise experiments, and finding, with the help of his companions, that in a determined number of vibrations of a certain pendulum,

[44] Galileo Galilei 1968, VI, p. 350 and XII, p. 436.

allowed to start swinging always at the same distance from the perpendicular, the number of broad vibrations of another pendulum corresponded to the number of short and very short movements for each one."][45]

This page allows us to fill in the background, and to understand why, immediately before music in his *Discorsi* ..., Galileo spoke of the pendulum, stating: "Io ho ben mille volte posto cura alle vibrazioni, in particolare, delle lampade pendenti in alcune chiese da lunghissime corde, inavvertentemente state mosse da alcuno." ["Many a time have I focused my attention, in particular, on the vibrations of lamps hanging at the end of long ropes in churches, when they were unintentionally moved by someone."] He used the same term "vibrations" both for the oscillations of the pendulum, and for the vibration of musical strings and the air. In the end, in order to explain consonances better by means of the coincidence that he imagined between strokes in the ear, he made the pendulums oscillate together. In this way, their lengths, which simulated the octave and the fifth, would make their movements concordant. "Ma quando le vibrazioni di due o più fili siano o incommensurabili, sì che mai non ritornino a terminar concordemente determinati numeri di vibrazioni, o se pur, non essendo incommensurabili, vi ritornano dopo lungo tempo e dopo gran numero di vibrazioni, allora la vista si confonde ...." ["But when the vibrations of two or more strings are either incommensurable, such that they never return to terminate together during a determined numbers of vibrations, or even if they are not incommensurable, they return there together after a long time, and a great number of vibrations, then sight is confounded"] in the above-mentioned chaotic disorder.[46]

The tempo of music must have seemed to offer a valid alternative to the beats of his own pulse, for our renowned natural philosopher in his aim to measure the movement of bodies. Subsequently, in one of the most famous pages, the one dealing with the isochronism of the pendulum, music played a far from negligible role in the advance towards the new laws of physics. Other historians[47] have attributed the correct importance to music in Galileo. But they generally used it only as the source of his experimental method. Whereas with his refusal of Aristoxenus, who was undoubtedly known to him from his father Vincenzio, and of incommensurable ratios in music, our man from Pisa betraied that he had not listened carefully enough, or experimented sufficiently. He had not believed in, or repeated, what his father had done. Thereby, apart from the new relationship between the pitch and the tension of the string, he remained within the track of traditional Pythagoreans, who had selected those whole numbers for religious, a priori reasons, and not using their ears. Judging by Galileo's letter to Christine of Lorraine of 1615, Boethius was still the

[45] Viviani 1891, 303. Cf. Galileo Galilei 1968, XIX, p. 603. Settle 1996, pp. 30–31. Also in Huygens 1890, III, p. 473.

[46] Galileo Galilei 1996, II, pp. 670 and 680. The sight is confounded, but the hearing is not, Galileo Galilei seemed to say.

[47] Palisca 1961 or Gozza 1989, or Drake 1992, or Settle 1996.

authority in the field of music.[48] If Galileo Galilei had listened, like his father (and like all of us), without any prejudices, to a fifth tuned in the equable temperament of Aristoxenus, wouldn't he have been able to conclude that this, too, offers "kisses and bites"?

Our renowned scholar must undoubtedly have used his ears, too, to listen to the notes of music, and he must have found the exercises that he did in his father's house extremely useful. And yet, if he wanted to attain the ratios of the Pythagorean-Platonic-Euclidean schools that he followed, he too would be led to make himself independent of sensations. Otherwise, he would have heard not the heavenly sounds, but only the earthly ones, like his father's lute, mass in Pisa Cathedral, running water, the blow of wind, the waves of the sea, the rubbing of bodies, the cries of animals, thunder, cannon shots. How could he cancel all these 'noises' that covered up the melodies that floated through his well-ordered cosmos? Thus he abstracted from the environment where these phenomena took place, re-inserting them into a faraway, rarefied, ideal world. He also imagined a higher world of ideas, in which movements were simplified, and emerged from the confused fog produced by our immediate sensibility, to reveal themselves in the eternal laws.

Thinking that the movements of the stars and planets could be observed in this way, not from down here on Earth, the solar system was simplified, as if by magic. Not only did he leave aside the complicated orbits of Ptolemy, but with his simple circular figures, he went beyond Copernicus (1473–1543). Galileo Galilei cancelled Aristotle's natural upward or downward movements. He ignored friction and air resistance. With the eyes of his mind and the ears of his reason, he saw the pendulum oscillate isochronically *ad infinitum*. He abstracted from friction, and the bodies that rolled down an inclined plane (but they could not have rolled without friction!) assumed a law of perfect inertia. Our renowned scholar from Pisa imagined taking away air resistance, and the cannon-balls dropped from the Leaning Tower came down implacably, in a uniformly accelerated movement, regardless of their mass and form.

There was no experiment that could add up directly; he had to seek approximations laboriously, simulating, hypothesising, calculating and measuring. Measuring? How did he measure time? By means of the beats of his pulse! It's just as well that he exploited the help of music. In his *Sidereus nuncius [Messenger of the stars]* (1610), "… novi perspicilli beneficio …" ["with the aid of the new telescope"], he announced to scholars that the moon revealed the shadow of its mountains, and that Jupiter was accompanied by its four satellites, dedicated to the Medici family, lords of Florence.

Instead of just experimenting, Galileo Galilei was capable of representing by means of drawings those things that he considered pertinent to his theory. He

[48]Galileo Galilei 1968, V p. 325. Despite the misleading title, "Galileo and the Demise of Pythagoreanism", and his use of commonplaces taken from others, who distorted and misinterpreted Vincenzio Galilei, even Jordan 1992 ended up by presenting only a Pythagorean Galileo Galilei.

had attended the Florentine Academy of Drawing. He had been influenced by, and also himself inspired, the artists of his period. Among these, he consorted with Ludovico Cardi (1559–1613), known as "il Cigoli". Beneath the feet of the Madonna, this painter placed a moon, exactly as it was observed by our mathematician. Galileo loved to make precise distinctions, separating painting from sculpture, and the classic poems of Ludovico Ariosto (1474–1533) from the baroque cantos of Torquato Tasso (1544–1595). He preferred the former, just as he admired musicians capable of moving us without weeping or singing: "… se tacendo, col solo strumento, con crudezze et accenti patetici musicali, ciò facesse, per essere le inanimate corde meno atte a risvegliare gli affetti occulti dell'anima nostra, che la voce raccontandole. …" ["… if he does this without speaking, only by means of his instrument, with its harshness and accents of musical pathos, as the inanimate strings are less suitable to awaken the hidden feelings of our soul than the voice that explains them."] Giving due consideration to the pictorial and literary tastes of our natural philosopher, we would understand better why he could never have abandoned, in the heavenly orbits, the abstract perfection of circles for those asymmetric ellipses. These were filling churches and palaces, while baroque shapes were being flaunted under his nose by his rival astronomer of Northern Europe. And yet the forms of a lute, like the one that he probably loved playing, were to remain distant from the circle, and closer, rather, to the ellipse.[49]

He even declared at times that he had not conducted the experiment because he knew what the result would be. What did he answer to those who asked if he had dropped a ball from the top of the mast of a ship in movement, to verify the law of inertia? He replied that he did not need to carry out this experiment, because he knew that it would drop exactly at the base of the ship's mast.[50] Galileo was capable of soaring up much higher than that mast, because he dwelt up there, in the heaven of the ideas. Here, he brought support to his imagination, using the instruments that made it possible for him to connect one idea to another. From one of these, he derived some sure consequences through processes of reasoning that followed a course he maintained clear and controlled, and not uncertain or unreliable like the context of life that surrounded him. Therefore he used Euclid's geometry. Up there, he had learnt to read a new book.

In his *Il Saggiatore* our scholar of nature affirmed: "La filosofia è scritta in questo grandissimo libro .... (io dico l'universo), ma non si può intendere se prima non si impara ad intendere la lingua .... Egli è scritto in lingua matematica ed i caratteri sono triangoli, cerchi, ed altre figure geometriche … senza questi è un aggirarsi vanamente per un oscuro laberinto". ["Philosophy is written in this great book … (I mean the universe), but it cannot be understood unless one first learns the language … It is written in the language of mathematics, and its characters

[49]Galileo Galilei 1964, pp. 5, 10, 17ff. At most, he could construct water-clocks. Cf. Settle 1996. Galileo Galilei 1968, XI, p. 342. Holton 1993, pp. 154–155 and 167–174. Lundberg 1992, p. 219.

[50]Galileo Galilei 1996, II, pp. 185–186. Cf. Feyerabend 1973, pp. 63–73; Feyerabend 1979, pp. 58–77.

are triangles, circles and other geometrical figures … without these, it is a vain wandering through a dark labyrinth"].[51] Our renowned scholar from Pisa wanted to distinguish the sciences from the *Iliad* and the *Orlando furioso*, and thus geometry, from the language of the cosmos, became the criterion of demarcation from the rest of culture. Wasn't this a continuation of the influence of Plato? Galileo Galilei often repeated his profession of faith in mathematics, defending it from all the possible objections brought against it.

In his *Dialogo sopra i due massimi sistemi del mondo [Dialogue on the two general systems of the world]*, he made Simplicio say "… con Aristotele che nelle cose naturali non si deve sempre cercare una necessità di dimostrazione matematica. […] … queste sottigliezze matematiche … son vere in astratto, ma applicate alla materia sensibile e fisica non rispondono: perché dimostreranno ben i matematici con i loro principî… che *sphera tangit planum in puncto* ma come si viene alla materia le cose vanno per un altro verso: e così voglio dire di quest'angoli del contatto e di queste proporzioni, che tutte poi vanno a monte quando si viene alle cose materiali e sensibili." ["… with Aristotle that in natural things, it is not always necessary to seek a mathematical proof. […] … these mathematical subtleties … are true in the abstract but, when applied to sensible, physical matter, they do not correspond: because mathematicians will well demonstrate with their principles … that *sphera tangit planum in puncto* [a sphere touches a plane at a point], but when it comes to matter, things take a different course: and thus I affirm that these angles of contact and these ratios all come to nothing when we are dealing with material, sensible things."] In order to convince Simplicio to fly with him up among the angelic beings that populate heaven, Salviati–Galileo tempted him with a mercantile comparison. He answered him decidedly that "… quello che accade in concreto, accade nello stesso modo in astratto. [Come] i numeri astratti [corrispondevano] alle monete d'oro e d'argento ed alle mercanzie in concreto, [ma a patto che quando si andassero a calcolare] gli zuccheri, le sete e le lane …" ["… what happens in the concrete realm happens in the same way in the abstract. [As] abstract numbers [corresponded] to gold and silver coins and merchandise in the concrete realm [provided that when they went to calculate] the sugars, silks and wools…"] they made the necessary tare for the crates and the packing. Thus, when the philosopher-geometrician wanted "… riconoscere in concreto gli effetti dimostrati in astratto, [bisognerebbe che] difalchi gli impedimenti della materia." ["… to recognize in the concrete realm the effects shown in the abstract, [it would be necessary for him] to subtract the impediments of matter"].[52]

In his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze [Discourses and mathematical demonstrations regarding two new sciences]*, Galileo repeated, after including his mathematics even in the title, that "… tutte le ragioni della meccanica hanno i fondamenti loro nella geometria." ["… all the reasonings of mechanics have their bases in geometry"]. He even tried to go beyond his own

[51] Galileo Galilei 1996, I pp. 631–632.

[52] Galileo Galilei 1996, II, pp. 27–28 and 260.

geometrical knowledge, which he had largely derived from the ancients, Euclid and Archimedes. He recalled that Plato wanted his pupils to be "… ben fondati nelle matematiche …". ["… well grounded in mathematics …"]. But unfortunately, he too ended up by getting lost in the labyrinth created by the paradoxes of infinity, which awaited a less Pythagorean, more Aristotelian concept of the *continuum*. Before them, our philosopher was blocked, he felt an aversion, and his literary style furred itself. The atomistic explanation of a problem of cohesion between bodies had led him into the treacherous marshland of an equally atomistic idea for 'real' numbers.[53] From the Greek and Latin tradition, he had inherited at the same time the best results and the prohibitions regarding irrational numbers and infinity. And yet, in spite of these aspects which became for us the defects of his language, our Pisan scientist remained the inventor of mathematical models when he said that it was necessary to "subtract" the impediments of matter. In opening the door to the modern scientific scenario, he also revealed its main limit.

How was it possible to understand what could be safely ignored, without compromising the phenomenon studied? Because it subsequently happened (to him, as well) that some material impediments were deducted, which should have been included, on the contrary, among those things that determined the event to be analysed. In this connection nowadays, we would speak about "stability". In other words, this means that if you intend to ignore something, you must also demonstrate that the effect of the approximation remains, in proportion, minimal: 'minimal' modifications in the causes must correspond to 'minimal' modifications in the results. Thus, passing to the limit, in this case it would be possible to arrive at reliable explanations. Furthermore, stability (in one of its numerous variants) remains a fundamental condition to reproduce the observation. As the parameters of the environment, where the experiment is conducted, are always subject to small variations (the temperature varies, a comet passes, the earth's crust moves, a pleasant lady doctor coughs, …), when stability is not present, the experiment cannot, strictly speaking, be repeated, and in theory, the comparison between events becomes impossible. Without stability, mathematicians and physicists would speak of different, incommensurable events, without any possibility of reaching any agreement. After the studies of Henry Poincaré (1854–1912) or Aleksandr Ljapunov (1857–1918) up to René Thom (1923–2002), the problem of stability (in particular also of the solar system) was taken very seriously (as a rule, with negative results).[54]

We find ourselves faced with two different points of view, which deserve to be studied in their relative worths and weaknesses, that is to say, in the pertinence to their different aims. Now, the objection of Simplicio (Aristotle) is as intelligent and profound as the answer of Salviati (Galileo). The latter could only count on his own intuition and his own fortune to understand what was the wool to be maintained and what the packing to be eliminated in the (mathematical) comparison of money and gain. Sometimes his attempts were successful, other times they were not. One

[53] Galileo Galilei 1996, II, pp. 571, 707, 598–623. Tonietti 2004a, pp. 94–100.

[54] Tonietti 2002a.

brilliant result was achieved by ignoring the friction of air for the fall of solids, and thus he succeeded in arguing by means of geometry that the spaces traversed were proportional to the square of the times of their falls. But the phenomenon of meteorites that burst into flames due to friction with the air escaped his notice, unlike that of Aristotle. Galileo reduced the problem of floating to the specific weight of the body, and to theArchimedes' upthrust.[55] He believed that he could disprove Aristotle's argumentations based on the form of the vessel, with results that boat constructors would do well not to follow. Our great compatriot believed that the tides were a proof of the movement of the earth, which shook the sea like water in a basin, and he made fun of the obscure, hidden influences of the moon, which we shall soon find, on the contrary, in the following sections.[56]

While the dynamics of Aristotle or Leonardo da Vinci had been one of fluids, in mediums with friction like air or water, movement in Galileo Galilei regarded, on the contrary, solid bodies in a void. This new physics should, then, have projected the Earth into the Heaven, as if fired from cannons. However, as their effect did not seem to be sufficient, what was needed was a mathematical abstraction of a different power, so as to make matter levitate among eternal laws. Our man from Tuscany often expressed himself in a fine, clear Italian which is comprehensible still today (except when he attacked the ineffable infinite), but Latin proved to be necessary, above all in the more technical mathematical passages and argumentations: symbolism still had to wait.

In their textbooks, all the students in the world learn that in the following centuries, the point of view proposed by Galileo Galilei was to prevail. Subtracting the impediments of matter was to become the orthodox, dominant scientific approach in Europe. From then on, scholars will believe that scientific truths should be born from procedures that separate. Otherwise, bodies would not move as they are thought to move, but as they are seen. Instead, by suitably eliminating air and friction, feathers would fall like cannon-balls, and would no longer float down capriciously through space. Thus, through this distinction, knowledge led to the separation from the senses and from common sense. By subtracting here and separating there, we end up by gradually cutting the links with the earthly world where we live. In the end, we acquire a more perfect knowledge, constructing another world apart which is in harmony with our ideas: this only happens in laboratories where the experiments are conducted from now on.

Galileo Galilei's first trial was concluded in Rome in 1616. The theory that the Sun was immobile at the centre, and the Earth revolved around it, was condemned as heretical by the Holy Office, and in particular by the Jesuit cardinal, Roberto Bellarmino (1542–1621). Variously known as heliocentricism or Copernicanism, this theory overturned the Ptolemaic system, which taught the opposite and had been taken as 'true' by the Church because it was considered to accord with certain passages of the *Bible*.

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[55]Galileo Galilei 1996, I, p. 451 and II, p. 642.

[56]Galileo Galilei, *Dialogo sui massimi sistemi*, "Ultima giornata"; in Galileo Galilei 1996, II.

Consequently, Galileo was not allowed to teach, defend, or in certain documents, even to discuss this doctrine. It is a particularly famous event, which has been the subject of numerous studies, although these hardly ever arrive at the same conclusions. And yet the story is often vulgarised as the clash between the obscurantism of the Roman Catholic Church and the light of scientific reason, or the attempt of political and religious power to prevent the freedom of research between the truth and error, and so on. Such an important historic episode, however, does not deserve a similar caricature, and in any case, respecting its complexity, my interpretation in part is different.

We are helped to observe some of its original aspects if we take into consideration, on the one hand, the behaviour followed by the Jesuits in China, during those very years of the trial, and on the other, if we consequently observe in greater detail the importance of the religious arguments presented by the parties involved, including those of Galileo Galilei.

Thanks above all to Matteo Ricci (1552–1610), the Society of Jesus had succeeded for the first time in penetrating into that huge country, making itself accepted, and establishing its bases. The empire of the Ming dynasty (1368–1644), which we have already seen with prince Zhu Zaiyu, was about to come to an end in China, and the missionaries would have to face up to the relative court astronomers. But here, the political power was not in the hands of the Jesuits, as it was in the Papal States, with the consequences that we shall see.

High imperial officials were converted to Christianity, and at the same time were instructed in the European sciences. With the help of one of them, Ricci had even succeeded in translating Euclid's *Elements* (the first six books) into Chinese, under the title *Jihe yuanben [The original roots of how much it is]*. According to this Jesuit's strategy, Euclidean geometry and Ptolemaic astronomy were to offer a convincing tactical prelude to the higher, celestial truths of the Catholic faith. When translating and publishing in 1574 his first Latin edition of the *Elements*, his master, Christophorus Clavius (1537–1612), following Plato, had sustained that "The mathematical disciplines elevate the soul and stimulate the strength of the mind, to contemplate divine things".[57]

Starting from Macao and the area around Canton, the route followed was initially fairly successful, allowing them to arrive as far as the imperial court in Beijing.[58] Thus Western sciences arrived in the Country-at-the-Centre, taken there by the Jesuits, and there they found other, different sciences. Between our missionaries who offered their Ptolemaic astronomy and the imperial experts at the astronomic office of the calendar, there were meetings and direct clashes. The eclipse of 1610 had been calculated by the Western men of the Church with greater precision than it had been by the imperial astronomers.

Matteo Ricci wrote in his diary of the little appreciation the Chinese, which he had met, felt for the mathematical sciences. He told them that he was bringing not

[57]Clavius S.I. 1589, p. 15.

[58]Tonietti 2006a, Chap. 4.

only the One True God, but also the only true theorems regarding geometry and the true movements of the stars. "… sì alla matematica come alla medicina non si applicano se non persone che non possono studiar bene le loro lettere per puoco ingegno e habilità; e così stanno queste scientie in bassa stima e fioriscono assai poco." ["… both to mathematics and to medicine, the people who apply themselves are only those who cannot study their literature, due to their limited intelligence and ability; and thus these sciences enjoy little esteem and are limited in their flourishing."] The only Chinese art that was recognized was that of succeeding in casting magnificent astronomic instruments in bronze: "… grandi e belli …" ["… large and fine …"]. To clarify, for the sake of contrast, the reader can make his/her own (anachronistic) choices between the heavens of the Jesuits, made up of solid spheres, and the empty sky of the Chinese, the round Earth of the former and the square one of the latter, the four fixed Greek and Western elements and the five Chinese phases in continuous transformation into one another, between the Euclidean theorem of Pythagoras and the Figure of the string.[59]

Even if they had gained a knowledge of the Chinese cultural differences, these Jesuits of the seventeenth century looked down on them from the heights (literally) of their religious and scientific certainties, and transformed them into inferiority, and errors to be corrected. Criticism was even expressed in the following terms by an imperial official whose name was Xu Guangqi (1562–1633). He had been converted to Christianity and became a pupil of the Jesuits, collaborating in the translation of Euclid's *Elements*. "The *gougu* leaves out explanations, and it can only be seen".[60] That expression of Xu's, "it can only be seen", referred to the *Xiantu* [the Figure of the string], which we know, on the contrary,[61] really succeeds in demonstrating the famous connection between the three sides of a right-angled triangle. Now, for him, too, the glorious Chinese mathematical tradition would have become flawed.

For the Jesuits, the only true theorem of Pythagoras was the one proved inEuclid's *Elements* at the end of Book 1.The sentence quoted above is taken from a book entitled *Gouguyi*, which means *The right gougu*, where right-angled triangles are dealt with in the Western, Euclidean manner. In other words, Euclid was presented to Chinese scholars as the way to make their geometry "correct". It could now be placed on sure bases, provided by the European procedure of proof: that is to say, as a linear discourse which starts from the axioms, postulates and theorems already previously demonstrated, with the aim of arriving at the affirmation of certainties with precision. As Matteo Ricci wrote, this should take all doubt away.

Thus the procedure should be step by step, as if climbing a ladder towards the Heaven of the eternal truths. The image of the ladder was explicitly included byRicci in his "Introduction" to the translation of Euclid's *Elements*. Here he wrote that, however much intelligence they had been endowed with by Heaven, scholars were

[59] See above, Part I, Chap. 3. Tonietti 2006a.

[60] Tonietti 2006a, Sect. 5.3.

[61] Part I, Chap. 3.

to use "this book [the *Elements*] like a ladder". In the *Bible*, the ladder was the means seen by Jakob in a dream to arrive in Heaven.[62]

It should be only thanks to this ladder that the scholar should succeed in reaching the truth he seeks. But anyone who does not follow this course will not find it, and will remain in error. Thus the theorem of Pythagoras is true, while the right-angled triangle *gougu* and the "Figure of the string" are open to doubt. These did not appear to the Jesuits of yesterday (and to many colleagues of today) to be so precise, well-founded or general. The *gougu* should become *yi*, that is to say, "right and correct," and should acquire significance only if it were adapted to the ladder constructed by Euclid.

After Ricci's death, another of his converted pupils, Li Zhizao (1565–1630), collected the 20 books printed by the missionaries under the title *Tianxue chuhan [The first collection of heavenly knowledge]*, where he included the translation of the *Elements*, together with articles on religious subjects. Thus the "Heaven" of the title, for him, meant the one inhabited by the God of the Christians.[63] The general problem of Christianity in China has been studied by Jacques Gernet.[64] Also Ad Dudink has shown the close relationship that the Jesuits created between the evangelisation of China and their relative scientific novelties. Manuel Dias wrote in 1615, "Not only did the physical eye see only a fraction of the phenomena, but it was also not able to see the Lord of Heaven, who can be seen only through the phenomena he created. Therefore astronomy is an introduction that paves the way to paradise or Heaven".[65]

Chinese scholars remained down here, because their trust in the Earth was greater. European scholars sought perfection in Heaven. By them, the latter was kept separated from the former for centuries. And even when, following the scientific revolution of the seventeenth century, the Europeans tried to unite the two, they studied the Earth by projecting it into the higher celestial world, as Galileo did.

We have already seen[66] that it was quite possible that the *veritas* [truth], both for the Jesuits and for Galileo, would not fit in easily with the Chinese culture, where there was no tendency at all to divide and to discriminate by means of rigid, absolute laws. In Europe, at the time, books were full of truth and falsehood, good and evil, soul and body, friendship and enmity. The Western philosophies of sciences theorised the distinction between primary qualities and secondary qualities, like misleading sensations. In the seventeenth century, European scholars believed that scientific truths derived from procedures that made a separation: "subtracting" the impediments of matter was really what Galileo did.

The Jesuit missionaries took to China precisely that Classic of the West, in which all these differences between the two distant cultures had found their deepest

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[62] *Genesi*, **28**, 12.

[63] Tonietti 2006a, Sects. 4.1 and 4.3.

[64] Gernet 1982 and Gernet 1989.

[65] Dudink 2001, p. 213. Gernet 1982, passim.

[66] Part I, Sect. 3.6.2.

roots: the *Bible*. This was the setting of the famous story about the tree whose fruit Adam and Eve had been told not to eat. Otherwise, "your eyes would be opened and you would become like God, knowing Good and Evil"[67]. However, this page presents something different from the current interpretation, which sustains a generic prohibition of knowledge. The rabbis who can read Hebrew, on the contrary, explain that the original word does not mean "to know", but rather "to mix". Thus the Biblical prohibition regards "mixing Good and Evil". Now the Western Classic has also taught us not to mix the True and the False, and even, as we know, to beware mixing female with male. Over time, this kind of carnal 'knowledge' become proverbial. If they had not been convinced to be converted to Christianity, those imperial officials would not have been happy to abandon the Confucian (or Taoist) art of mixing, amalgamating and connecting things together in order to understand them.

In the following years, however, the initial relative success of the Jesuits was to turn into a heavy defeat. The Chinese officials converted to Christianity, among whom the most influential was Xu Guangqi, together with the Jesuits, tried to carry out the reform of the Chinese calendar. But this should have ended up by involving the organisation of the Rites, which had represented, ever since their 1,000-year-old institution, one of the most important pivots for the orderly functioning of the empire. Confucius had said in the *Dialogues* (Book II): "He who governs virtuously is like the pole star, which remains stable in its place, while all the others revolve around it". Astronomic calculations served for the calendar, which regulated dynastic successions. Imperial power was considered to be a "heavenly mandate". Thus the Jesuits had been too presumptuous and dared too much: they ended up by being arrested in 1616, and were expelled in 1617, with the confiscation of their possessions. Some Chinese scholars openly declared that those mathematical sciences served to introduce Christianity into China.

On the side of the political and religious power, the Jesuits in Rome, with Gregory XIII, had recently reformed the calendar, and as judges were imposing the absolute Ptolemaic truths about the cosmos on Galileo Galilei. At Beijing, on the contrary, they were under accusation in the dock; there they were orthodox, here they had become heretical, and now here suffered what they had practised in Rome. In China, the Jesuits tried to defend themselves from the accusations by sustaining that their astronomic theories were only an instrument to calculate *tuibu* [literally, "a stage in deducing", i.e. mathematics] the different trajectories and movements of the planets, which did not require the real existence of the nine orbits.

Our Europeans believed that they could use the same tactics in China as had proved to be so effective at home. The *De rivolutionibus orbium celestium [On the revolutions of the heavenly orbits]* of Copernicus had been tolerated, with the *imprimatur*, and even the dedication to Paul III, because it was presented as a hypothesis; it was addressed to mathematicians, and the procedure was admitted, seeing that (perhaps?) it was better for calculations. Professional astronomers could

[67] *Genesis*, **3**, 5.

thus (freely?) discuss it. But woe to those, like Galileo Galilei, who claimed to transform the mathematical hypothesis of Copernicus into real *veritas*. The absolute *veritas* must remain an exclusive prerogative of the Holy Scriptures, as interpreted by the Holy Office, and not by heretics, whether they were Lutherans or otherwise. In *Psalm XIX*, David extolled the firmament. God, and therefore the Church, had established what place the Sun occupied in it. Also Galileo Galilei believed in God and in the absolute truths that derived from Him. But our famous natural philosopher claimed to interpret this same passage in a heliocentric, Copernican sense. "*Deus in Sole posuit tabernaculum suum* come in sede nobilissima di tutto il mondo sensibile." ["God placed his tabernacle in the Sun, as in the most noble place of the whole sensible world"].[68] Galileo incurred the wrath of the Holy Office because he claimed to oppose his absolute truth to that of the Jesuits, and furthermore he dared to back it up, not with arguments of pure natural philosophy, but taking his stance on ground that was forbidden to him, because it was the dogmatic field of the Holy Scriptures.

We shall leave aside here the faulty, insufficient experimental arguments like that of the tides, proposed by Galileo to try to demonstrate the movement of the Earth, and also the better ones regarding the choice of the reference system from which the phenomenon was to be observed. Our mathematician would like to condemn appearances. The Earth appears to be still because we are set on it, but if we were on the Sun … And he made use of the example, in Copernicus, of the boat that moves with respect to the river bank. "… che serve non a dimostrare la verità della posizione, ma la non repugnanza tra 'l poterci parere, quanto ad una semplice apparenza del senso, la Terra stabile e mobile il Sole, ben che realmente fusse il contrario." ["… which serves not to demonstrate the truth of the position, but the non-rejection that, to the simple appearance of the senses, the Earth may seem to be still and the Sun to move, whereas in reality the opposite is true"]. For Galileo, this argument would be insufficient to demonstrate the movement of the Earth.[69] Sure enough, after making the movement relative to the observer, it would be necessary to justify why the viewpoint from the Sun is to be preferred. For our renowned natural philosopher, the movement observed from our star should, on the contrary, lead us to an absolute truth. The religious question remained, therefore, of who had placed his tabernacle in the Sun.

Let us return to China, though. Cunningly, having changed their position in the process, the Jesuits now made their Ptolemaic system play the role of a mathematical hypothesis only, losing the prerogatives of the absolute truth. Therefore, they hoped the Chinese imperial officials would treat them in the same way in which Copernicus had been tolerated in Europe. Then the Chinese would thus allow them to continue their work of religious, cultural and commercial penetration. But they were expelled

[68]Tonietti 2006a, pp. 175–176. Galileo's letter to Mons. Pietro Dini of March 23rd, 1615. Galileo Galilei 1968, V, p. 303. Cf. D'Elia 1947.

[69]Galileo Galilei 1968, V, p. 370.

all the same, because they were felt to be a threat for the empire, just as Galileo was at that time for the Church, which was facing up to heretics and schisms.

The Ptolemaic system was true and real in Rome against Galileo Galilei, but would it become a convenient mathematical hypothesis at Beijing? The *Societas Jesus* presented a double truth. Should we include among its merits also the invention of relativism? In time, the adjective "Jesuitical" came to assume also a derogatory sense of "astute, false, hypocritical", as we may read in dictionaries. We shall see below what another Christian natural philosopher like Blaise Pascal (1623–1662) thought of the matter.

Hauled before the tribunal again in 1633, and faced with the threat of torture, Galileo Galilei is said to have repudiated his mathematical and naturalistic truths, and submitted to the authority of the Roman Catholic Church. However, he muttered under his breath the legendary "And yet it moves!" and remained convinced of the movement of the Earth, thus suggesting to Bertold Brecht (1898–1956) the excellent aphorism: "Blessed are those peoples who do not need any heroes". Protected by the Grand Duke of Tuscany, he went on with his research very quietly, though the results were prudently published in Holland.

I have no intention to go over Galileo's trials again, if for no other reason, to avoid offering some amiable colleague new pretexts to accuse me of wanting to repeat them.[70] But I do not wish to put on trial our poor Jesuits, either, who, in spite of themselves, became those who were persecuted in China. In any case, they suffered a defeat in the field where they felt they were best prepared. The military terminology will be forgiven for a religious order whose project was to declare war on errors and heretics, even using the rank of "general" in their hierarchy. In their excessive preoccupation with their absolutes, which however became relative when it suited them, they did not understand a crucial point: the difference between Chinese culture and their own was much greater than they could imagine.

The European environment was dominated by typical clear-cut dualisms, such as soul and body, good and evil, friends and enemies, man and nature, transcendence and immanence, and so on, arriving at truth and error. Here, downgrading a truth to a hypothesis avoided a head-on confrontation (and the relative serious consequences) with another truth that was incompatible with it. In the Chinese culture, on the contrary, they generally preferred to combine things together, until an overall organism was created, and human beings were not thought to transcend the social and natural context where they live. The dominant Confucian tradition, for better or worse, adapted the rules to the people, and not vice versa, the people to laws. The circumstances thus toned down the contrasts between the truth and error. Thus the Jesuitical stratagem of downgrading the Ptolemaic truths, that were absolute in Rome, to mathematical hypotheses in China proved to be ineffective and useless. They were contrasted, on the opposite side, not by the equivalent truths of Chinese cosmology, but rather by the practices of a 1,000-year-old system of the calendar, sustained by the officials in power.

[70] Feyerabend 1985. Stengers 1989. Hammond 1992. Redondi 2004.

In spite of this episode, it should be maintained that, in the end, the Western sciences imposed themselves in China, too. Of course. Therefore, we now face the question of understanding its reasons and the historical circumstances in which the truths and the errors were decided by arguments as convincing as those heard in the ecclesiastical tribunals.

Starting from 1618 in the north-east, the Manchu were invading China, and winning battles, conquering some important Chinese cities. As a result, the Chinese now needed that military competence which the prevailing Confucian tradition had neglected, or relegated to the lower ranks of the imperial hierarchy. Matteo Ricci had realised that warlike activities. considered so noble in the Christian West, did not enjoy an equal consideration there in China. And the Jesuit had included this among the differences to be taken into account. When introducing the translation into Chinese of the *Elements*, however, he had explicitly written that it was essential for generals of the army to be precise, and thus it was important for them to study mathematics. Anyone who knew history was bound to know that those who disposed of the newest, most ingenious weapons possessed the means to win battles, or better defend themselves.

Ricci's expressions might seem to us pure advertising blurb, and for him they must, in any case, have remained theoretical, albeit useful with interlocutors who appreciated above all the practical side. But this was not the case with the other translator of the *Elements*, the convert Xu Guangqi, who followed them literally. Starting from 1619, this imperial official was responsible for the army, because he was considered to be an expert on arms, with the task of defending the capital. He insisted that the armament should be renewed by importing arms from the West, and that the military experience of faraway lands should be used in the war.

It was above all Sun Yuanhua (1581–1638), a pupil of Xu, who was to prove to be the most active, productive element in the noble Christian art of killing enemies. He pressed for the use of telescopes, built fortifications, and even went so far as to cast cannons, and master the art of shooting with them. Here, Euclid's theory of ratios, and the art of performing calculations with a pen on paper, instead of sticks, were exploited to obtain the quantity of gunpowder needed. Sun wrote books on military subjects, such as *Jingwu quanbian [Complete collection of military matters]* and *Xifa shenji [Divine machines with the Western rules]*. Sun was successful in his enterprises: his cannons were used victoriously in battle against the Manchu. More cannons were ordered from him, and he became responsible for their casting. Naturally, Xu and the others spoke in favour of the purchase of the cannons, and of the recruitment of Western experts from Macao. Sun ended up by commanding an army of 8,000 soldiers.

In this way, the Jesuits sneaked back into China, at first only unofficially, and a host of famous characters arrived. In 1621, Johannes Schreck (alias Terrentius, 1576–1630), a member of the Academy of the Lincei and a colleague of Galileo, arrived in China. Then in 1622, Adam Schall von Bell (1592–1666) Arrived. These two obtained permission to stay in the Country-at-the-Centre as military experts. With the help of a Chinese collaborator, in 1643 Schall produced a compendium on firearms, and how to use them in warfare. Others, like Guido Aleni (1582–1649),

again started to convert to Christianity those Chinese who showed an interest in geometry. One of these converts, Han Lin (1600–1644), began to study military sciences. and was instructed in the use of cannons. There was a growth in literature about military topics, written by the missionaries who were Ricci's successors and by those who they directly converted to Christianity.

The needs of warfare had forced the Ming empire to open up to the West in an unprecedented manner. Thanks to Schreck, this new opening now led to the entry of trigonometry. He translated Galileo's *Compasso geometrico militare* [Military geometrical compass], and the *Misura del cerchio* [Measure of the circle] by Archimedes. A committee was set up to translate Western scientific books: this was composed of three or four Westerners and some Chinese who were almost all converted. Xu and Sun assumed increasingly important imperial responsibilities. The former again put his hand to the reform of the calendar. As its director, he excluded an elderly Chinese scholar who would have liked to take part without conforming to Western methods. Ricci's pupil set out the rules for the applications of mathematical sciences. (1) forecasts of floods and droughts, (2) dykes and canals, irrigation, (3) improvement of music, (4) military camps, arms, fortifications, (5) financial administration and taxes, (6) building of houses and bridges, (7) mechanical instruments, (8) geographical maps, (9) medical astrology, (10) the measurement of time. But in spite of the help of the Jesuits and the European scientific experts, in 1644 the Ming lost control of the empire to the Manchu, subsequently called Qing in Chinese (1644–1911). Apart from Xu Guangqi, who died of old age in 1633, the other Chinese converts met with a particularly violent end, as a result of the war.

However, the presence of the Jesuits continued also under the Qing, the last empire, destined to endure until the formation of the Republic in 1912.[71] The pictorial style followed in the *Zhoubi* for proofs was taken up by Mei Wending (1633–1721). Yet he now gave this a linear written representation, even copying some characters from the *Jihe yuanben* by Ricci and Xu. He could no longer ignore the Western mathematical sciences, but he was to find another way of bringing them back to China, and positioning his Country-at-the-Centre at the centre of the Earth. He believed that the Chinese must, in all cases, have arrived first; then their inventions would have spread everywhere, arriving in Greece. Also Euclid, therefore, might have re-elaborated in his own way ideas which must have come from the Country-at-the-Centre. Lastly, the Jesuits would have brought that Chinese geometry, clothed in Greek apparel, back to its real country of origin. In the meantime, unfortunately – Mei sustained – the Chinese scholars would have forgot their ancient books, and were therefore unable to recognize them as those brought from the West.

We may easily harbour the suspicion that this giddy historical hypothesis had simply been invented for reasons of national prestige. Not only are documents lacking, as usual, to prove the single origin of all universal geometry in China.

[71] Tonietti 2006a, Chap. 5.

On the contrary, we have extant printed texts, where any interested literate could have found again, more or less easily, those ancient mathematical treatises of the Han period, as besides the interlocutors of Matteo Ricci did.[72] Anyway, Mei Wending would have tried to reconcile Chinese tradition with Western mathematical novelties. He brought again Euclid's geometry toward the tradition of the *gougu* in order to make it comprehensible: "while they [the Westerners] make a secret out of it teaching about God, and we [Chinese] reject it as a *yixue* [foreign doctrine]".[73]

The translation of Euclid's *Elements* was finished half-way through the nineteenth century. In fact the ultimate and final exportation of Western mathematical sciences to the Country-at-the-Centre, which alas! in that period had discovered itself off-centre, took place as a result of the stimulus of, and following, the infamous opium wars: the most shameful and darkest page of European economic globalisation, the one that in other periods, with less hypocrisy, were called colonial imperialism. The preface to the complete translation of Euclid was written by no less than the famous warlord Zeng Guofan (1811–1872). Zeng and the movement *Yangwu* [foreign affairs] sustained that, in order to restore the power of the Qing, it was necessary to make use of Western sciences and techniques.

Unfortunately, the events that took place in China in the seventeenth century have generally been ignored or underestimated by science historians (except Sinologists), above all by scholars of Galileo and the so-called scientific revolution. But on the contrary, they make it possible for us to reformulate, with greater understanding, the crucial questions that lie at the basis of the relative historiographic discussion.

What circumstances influenced the evolution of the sciences in Europe? What role did a transcendent religion like Christianity play? Why does war break out so often? What is hidden behind the continual claim to possess the truth, whether scientific or religious, when those who believe in it do not hesitate to contradict it or adapt it to their own advantage, depending on the circumstances? Can we avoid answering these questions, if we wish to understand how, from time to time, groups of orthodoxy are established, and heretics are put aside, when attempts are not made to eliminate them completely? If we could know, and take into due consideration the historical circumstances, why can we not arrive at the point of thinking that the results of scientific research are not so objective and absolute as they are commonly believed to be? Even in the truths of science, do we not end up by believing above all by faith?

The hard nucleus of the dispute between Galileo Galilei and the Church was: who possessed the monopoly of absolute truth, controlling the means to arrive at it, and discriminating between truth and error? That was clearly written in our mathematician's correspondence with Benedetto Castelli, Piero Dini and Christine of Lorraine. The defenders of the Ptolemaic system wielded blows with the Holy Scriptures, like the expression "Sun, stand thou still" used by Joshua in the *Bible*. They "… si tengon sicuri d'avere in mano l'assoluta verità della quistione che

[72]Tonietti 2006a, pp. 170–171.

[73]Martzloff 1981a, p. 34. Tonietti 2006a, pp. 182–197.

intendono disputare, …" ["… are sure about possessing the absolute truth of the question they intend to dispute, …"]. Let them say "… se loro stimano gran vantaggio aver colui che in una disputa naturale s'incontra a sostenere il vero, …, sopra l'altro a chi tocca sostenere il falso? So che mi risponderanno di sì, e che quello che sostiene la parte vera potrà aver mille esperienze e mille dimostrazioni necessarie per la parte sua, e che l'altro non può aver se non sofismi, paralogismi e fallacie." ["… whether they consider it a great advantage to have the one who, in a dispute about natural matters, argues sustaining the truth, rather than the other one, who is left to sustain what is false. I know that they will answer affirmatively, and that he who sustains the true part may have thousands of experiments and thousands of necessary proofs on his side, and that the other cannot have anything better than sophisms, paralogisms, and fallacies"]. Then, Galileo Galilei asked Castelli, "… perché, nel venir poi al congresso, por subito mano a un'arme inevitabile e tremenda, che con la sola vista atterrisce ogni più destro ed esperto campione?" ["… why, in coming to the discussion, do they immediately take out an inevitable, dreadful weapon, which, just at its sight, appals all the most accomplished and expert champions?"] In all these general considerations, the religious and naturalistic questions were inextricably mixed, becoming intolerable to suffer for the political and religious power of the time.[74]

For us, it is also interesting that Galileo Galilei's way towards the "absolute truth" always brought him close to music. "Chi è quello che non sappia, concordantissima essere l'armonia di tutti i veri in natura ed asprissimamente dissonare le false posizioni da gli effetti veri? Concorderà, dunque, in ogni spezie di consonanza la mobilità della Terra e stabilità del Sole con la disposizione di tutti gli altri corpi mondani e con tutte le apparenze, che sono mille, che noi ed i nostri antecessori hanno minutissimamente osservate, e sarà tal posizione falsa e la stabilità della Terra e la mobilità del Sole, stimata vera, in modo alcuno non potrà con le altre verità concordarsi?" ["Is there anybody who does not know that the harmony of all true things in nature is perfectly concordant, and that false positions are harshly dissonant from the true effects? Will the mobility of the Earth and the stability of the Sun will be in agreement, therefore, in every kind of consonance, with the disposition of all the other worldly bodies and with all the appearances, which are innumerable, that we and our predecessors have observed in great detail? And will this false position, of the stability of the Earth and the mobility of the Sun, if considered to be true, in any way agree with the other truths?"][75] How this sentence could not bound to bring to our heads again the music of the spheres, so dear to Plato and the Pythagorean sects?

We are venturing along untrodden paths that lead us to listen to music, and also to observe other, faraway, exotic scientific cultures, also in order to help us answer

[74]Galileo Galilei 1968, V, pp. 285 and 281–370. Feyerabend 1985, pp. 250–256. Cf. Redondi 2004.
[75]Galileo Galilei 1968, V, p. 356.

these problems and succeed in breaking away from the well-known commonplaces found in so many books.

> ... you will know the truth and the truth will set you free.
>
> John **8**,32

8.3 Johannes Kepler: The Importance of Harmony

In 1619, Johannes Kepler (1571–1630) published his *Harmonices Mundi Libri Quinque [Five books on the harmony of the world]*. With the surprising (for us of the twenty-first century) idea of continuing the *APMONIKA* of Claudius Ptolemy, which had remained incomplete, he wrote a complex treatise of geometry, music, astrology and astronomy. His theory for the division of the octave, and the justification of consonances was highly original, because it was based on the "demonstratio" ['constructibility'] of regular polygons with a certain number of sides. In "Book V", lastly, he included his third law of planetary movement. "Sed res est certissima exactissimaque, quod *proportio qua est inter binorum quorumcunque Planetarum tempora periodica, sit praecise sesquialtera proportionis* mediarum distantiarum, id est *Orbium* ipsorum; ..." ["But it is absolutely certain and exact that the *proportion between the periodic times of any two planets is precisely the sesquialterate proportion of their mean distances, that is, of the actual spheres*, ..."].[76] In post Cartesian symbols, we might write today that for the periods of revolution T and the mean distances from the Sun R:

$$T_1 : T_2 = R_1^{3/2} : R_2^{3/2}$$

Thus this famous result was to be found in a book that was also about the theory of music.

Western modern sciences were now transformed into the art of perceiving intuitively how much, and what, could be omitted in a phenomenon, in order to make it absolute, and transport it into the realm of pure ideas. Attempts were made to achieve results that were certain and universal. And yet, depending on the individual, the tendency was to "subtract" different impediments, and consequently not to construct the same model for the same phenomena. The history, culture and interests of various subjects made the music that spread from the heavenly spheres sound different. Kepler did not live in the same manner, did not reason in the same way, and did not share the same values, as Galileo Galilei. Apart from Copernicanism that they had in common, they often arrived at contrasting conclusions. The German believed, unlike the Tuscan, that it was the mysterious influence of the Moon that moved the tides, not the movements of the Earth, just as the 'chaste goddess' regulated the female cycle and the growth of the plants. For him, all the stars, and

[76] Kepler 1619 [1969], V cap. III, 8, pp. 189–190. 1997, p. 411. The Italics are in the text.

the planets in general, influenced the Earth, and even our minds, which they affected by means of their harmonious movements. Could a better proof have been offered than the one provided by music?

In his *Mysterium cosmographicum [Mystery of the cosmos]* (1596) Kepler wrote: "In this booklet, my reader, I propose to demonstrate that the most excellent and great Creator, in creating these movements of the world, and setting out the heavens, had reflected on those five regular bodies, famous from Pythagoras and Plato to our days, and had proportioned to their nature the number of the heavens, their ratios and the reason of their movements." [...] "... I admire more than anything else analogies, my most faithful masters. They know all the secrets of nature, and must not be at all neglected in geometry". "The main aim of all investigations into the external world will have to be to discover the rational order and the harmony that God has set on it, which He reveals to us in the language of mathematics."[77]

Also for him, the harmony of the cosmos was expressed in mathematics, and again, it was above all Euclid's geometry. However, Kepler read it in his own way, arriving at surprising results: he succeeded in driving the five Platonic solids into the orbits of the six then known planets, as follows: by analogy, he placed the circular orbit of Mercury around the Sun on the sphere inscribed within the regular octahedron, and the orbit of Venus, again circular, on the sphere circumscribed around the octahedron. This last sphere was also inscribed within the icosahedron, and so on, placing the circular orbit of the Earth between the icosahedron and the dodecahedron, that of Mars between the dodecahedron and the tetrahedron, that of Jupiter between the tetrahedron and the cube, and lastly that of Saturn on the last sphere possible, circumscribed around the cube (Fig. 8.3).

To Kepler, this mathematical model of the solar system must have seemed particularly harmonious, but only in a qualitative way. That is to say, in trying to make it quantitative, in relationship to the astronomic measurements, he must have realised that those geometrical forms were not the only ones possible, and therefore our original German astronomer proposed others. In his *Astronomia nova [New astronomy]* (1609), he distributed a good number of planetary observations along elliptical orbits, and no longer along the traditional circles, which had remained such also for Copernicus and his followers. He had come into possession of those numbers and angles at Prague, by fraud,[78] as he himself related, while he was an assistant to Tycho Brahe (1546–1601), subsequently taking his place as director of the observatory. The orbit of Mars, above all, must have seemed to him too eccentric to be placed on a circle. Ellipses did away with need for the epicycles, eccentrics and equants to which Copernicus and Galileo still had recourse. Thus he enunciated his first two famous laws: the orbits covered by planets are ellipses around the Sun, which takes one of the foci; the radius vector between the Sun and the planet sweeps equal areas in equal times. The planet thus moves faster at the

[77] Kepler 1938, v. I, pp. 9, ... and passim.

[78] With the permission of the emperor, but without that of the heirs.

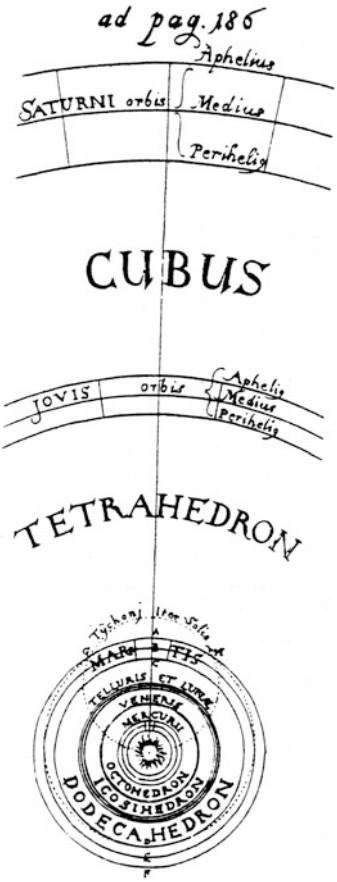

Fig. 8.3 How Kepler distributed the regular polyhedra among the orbits of the planets (Kepler 1619, V p. 186)

perihelion, when it is close to the Sun, and more slowly at the aphelion, when it is furthest away.

But in so doing, hadn't he destroyed that perfect Greek beauty of Platonic solids, to use figures that were not equally harmonious? Undoubtedly, it cannot have been easy for him to give up the sphere and the circle, seeing that these geometrical figures maintained the divine quality of Heaven. Actually, Kepler immediately dealt with the problem of how to reconcile the elliptical orbits again with the harmony of the cosmos. And this became the even more surprising story of the third law. Totally unsatisfied with the solution found, and not at all convinced that the adventure could finish there, with a better agreement of astronomic data, Kepler then wrote the most interesting and most important book for our history: the *Harmonices Mundi Libri Quinque*.

The not enough historians who have drawn close to our German scientist would quickly underline the great attention that this work should deserve, and what a serious defect it is to ignore it. But then few have studied the text with the necessary

care. There are histories of scientific thought that speak about everything (including Vladimir Ulianov, known as Lenin, 1870–1924) for thousands of pages, but then dedicate exactly nine pages to Kepler![79] The *Harmonices* ... is treated even worse, because almost all confess candidly that they have not read it. It was more often to be found in the libraries of musicians like Alban Berg (1885–1935).[80] In Italy, the anastatic reprint was brought out in a series of books about music. Until quite recently, its translations existed only in German, and (a mediocre one) in French; the English only published the fifth chapter. In Italian a paltry compendium came out, which was too partial, and questionably edited. We follow the original Latin, and the complete English translation which appeared a few years ago.[81]

The book deserves to receive more attention, just because in it Kepler expressed the idea of a strong, crucial link between geometry, astronomy and music. In the first two books, he expounded a theory about the 'constructibility' of polygons, and regular solids, also demonstrating their properties of filling the plane and space. Having shown that the heptagon does not possess the necessary quality of 'constructibility', Kepler presented, in Book III, his geometrical division of the monochord, and therefore he proved himself capable of excluding the dissonances generated by the division into seven parts. In Book IV, he explained how the planets and the stars influenced the souls of men living on the Earth, by means of the harmony generated by their movements, similarly to what music is actually able to accomplish. In Book V, every planet was assigned its own particular melody: fewer notes for Venus, because its orbit is almost circular, a longer sequence for Mars, because its orbit is fairly elliptical. The Earth sang (and continues to sing, unfortunately) one strident soft semitone like *mi* − *fa*, *mi* − *fa* "MIsery and FAmine, MIsery and FAmine" (Fig. 8.4). The third law, seen above, was almost hidden without any emphasis towards the end.

Kepler had thus succeeded in establishing his famous law, while again trying to model the heavenly spheres according to the proportions of music. He had found the sesquialtera 3:2 of the fifth again, in the ratios between times and distances.

While he reflected on the question, he also read Galilei, however not his colleague Galileo Galilei, rather his musician father, Vincenzio Galilei, who, as we have seen, had assumed a stance in the musical controversies of the period, with his work *Dialogo della musica antica et moderna [Dialogue between ancient and modern music]*. In the context of the Camerata dei Bardi, our Florentine composer and theoretician sustained the emerging accompanied monody of "recitar cantando" [reciting in song], which preluded lyric opera: in other words, the music practised by the ancients. However, Kepler undoubtedly preferred the modern music of the period instead: that is to say, polyphony. And, as he had transformed the orbits from circular into elliptical, he had to give up the unison of the classical conception, according to which movement would produce a single note. Now, he had finally demonstrated

[79] Geymonat 1970, II, pp. 105–107 and 508–514.

[80] Tonietti 2004, pp. 665–669.

[81] Kepler 1619 [1969]; 1939; 1952; 1979; 1994; 1997. Holton 1993, pp. 3–23.
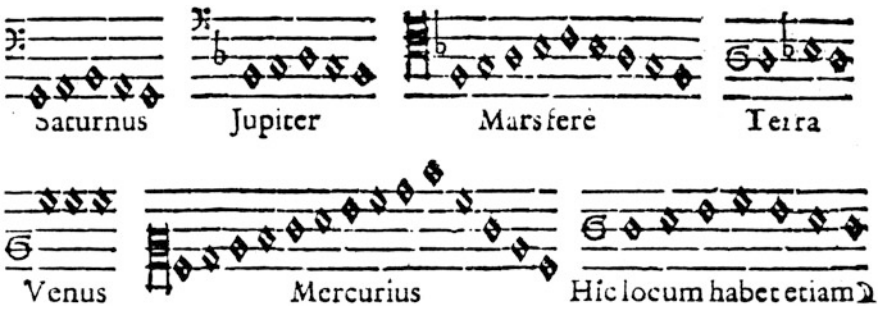
Fig. 8.4 How Kepler imagined the melodies of the solar planets (V, p. 207; 1997, p. 440; Kepler, facsimile 1969, lib. V, pp. 186/187)

that the different melodies, sung by the various planets, harmonised in a general composite polyphony in following their orbits. Was it not true that the heavenly ellipses behaved up there in an analogous manner to the music written down here by the most renowned composers to elevate souls? Macrocosm and microcosm were thus in perfect harmony in singing God's praises.

The harmony of the cosmos had now finally been regained and maintained. Above all, it appeared to be very interesting that Kepler divided the monochord in an original manner by means of geometry, moving away from the usual well-known considerations about the numbers 1, 2, 3, 4 of the classical Pythagorean tradition. He still used the general Euclidean approach of ratios, like Galileo, but he obtained a music that resounded to be in part changed. He had few doubts that the Third law indicated the conductor of the orchestra had to be the same for all the planets, and should thus be situated in the Sun. Without any fear of betraying a trust in some occult agent, our German scientist believed that it was a kind of influence analogous to magnetism, or light. Thanks to his faith in harmony, an important step forward had been taken towards the modern construction of the cosmos. The next step would be taken by a scholar using other mathematical instruments, in order to make it quantitative. But for this aim, it was necessary to learn to use other symbols in geometry, other ideas that came in part from far away, from environments which would not necessarily all continue to express themselves in Latin, which remained Kepler's main written language.

For our German astronomer, the harmony of the cosmos was to include all kinds of phenomena, nothing excluded. "First, then, you will remember that the hard third arose from the pentagon, and the pentagon uses the division into extreme and mean ratios, which form the divine proportion.[82] However, the splendid idea of generation is in this proportion. For just as a father begets a son, and his son another, each like himself, so also in that division, when the larger part is added to the whole,

[82]The golden section $1{:}x = x{:}(1-x)$, indicating the whole as 1 and the larger part as x.

the proportion is continued: the combined sum takes the place of the whole, and what was previously the whole takes the place of the larger part.[83] Although this ratio cannot be expressed in numbers,[84] yet some series of numbers may be found which continually approaches nearer to the truth [the real value]; and in that series, the differences of the numbers from the genuine terms (which are not countable, but inexpressible) by a wonderful alternation, breeds males and females, distinguishable by the members which indicate sex. Thus if in the first place the larger part is 2, and the smaller 1, the whole is 3.[85] Here, plainly, 1 does not stand to 2 as 2 stands to 3[86]; for the difference is unity, with the result that the rectangle of the extremes 1 and 3 is less than the square of the mean, 2.[87] Then, by adding 2 to 3, the new total becomes 5; and by adding 3 to 5, the total becomes 8, etc. The rectangle of 1 and 3 creates a female, for it falls short of the square of 2 by unity (Fig. 8.5);

The rectangle of 2 and 5 creates a male, for it exceeds the square of 3 by unity; the rectangle of 3 and 8 a female, for it falls short of the square of 5 by unity. Again from 5 and 13 arises a male, in respect of the square of 8; from 8 and 21 a female, in respect of the square of 13; and so on infinitely.[88]

Such is the nature of this division, which relates to the construction of the pentagon; and God the Creator has shaped the laws of generation in accordance with it; in fact, in accordance with the ratio of inexpressible terms, which is genuine and perfect in itself [divine proportion], he has shaped the ratios of the seeding of plants which have been commanded individually to have their own seed within themselves. In the accordance with the combined ratio of pairs of numbers (of which the falling short of one by unity is compensated by the excess of the other), he has shaped the coming together of male and female: what is surprising, then, if the progeny of the pentagon, the *dura* [hard] third 4:5 and the *mollis* [soft] 5:6 moves minds, which are the images of God, to emotions which are suitable for the business of generation? Here it must be repeated, from Chap. III, that, although 1:6 comes from the hexagon,

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[83] $(1 + x):1 = 1:x$; that is to say, $x = \frac{1}{1+x}$.

[84] In other words, it cannot be represented in a 'rational' manner, because $x^2 = 1-x; x^2+x-1 = 0; x = (-1 + \sqrt{5})/2; x = 0,6180339\ldots$.

[85] $1 + 2 = 3$.

[86] $1:2 \neq 2:3$

[87] $3 \times 1 = 2 \times 2 - 1$.

[88]

$$5 : 3 \neq 3 : 2; 2 \times 5 = 3 \times 3 + 1$$

$$8 : 5 \neq 5 : 3; 3 \times 8 = 5 \times 5 - 1$$

$$13 : 8 \neq 8 : 5; 13 \times 5 = 8 \times 8 + 1$$

$$21 : 13 \neq 13 : 8; 21 \times 8 = 13 \times 13 - 1$$

$$\ldots$$

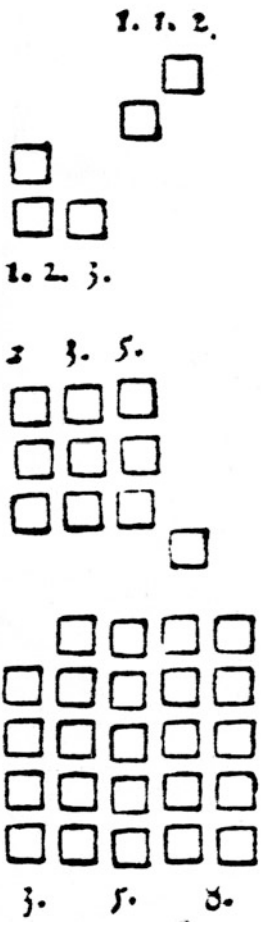

Fig. 8.5 How, in his arrangement of the *squares*, Kepler represented the numbers of sexual generation by means of Fibonacci's series (Kepler, ibid. lib. III p. 77)

yet its remainder 5:6 is consonant, not on account of the hexagon, but on account of its derivation from three tenths of the circle, by the doubling and halving of terms. Thus, this remainder also, and its progeny, the *minor* third, comes from the pentagonal class of figures. On establishing, then, that the association of two thirds represents the association of male and female, it is now no trouble to assign to each sex its own third. For the *major* third will prove manly, the *minor* feminine, since the proportion of their actual bodies is also the same as that of the material and spiritual powers. And as the *major* comes from a figure with an uneven number of sides, that is to say, the pentagon, but the *minor* originates from the decagon which has an even number of sides, this is also in agreement with the views of Pythagoras, who said

that uneven numbers were male, and even ones female[89] (which is confirmed by this study of excesses and shortfalls, since the odd is also in excess) with the result that the former is considered to be of the masculine sex, the latter of the feminine."[90]

In other words, using post-Cartesian and post-Arabic algebraic modern notations, Kepler 'showed' that the fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \ldots$ became increasingly closer to the number $x = \frac{1}{1+x}$, that is to say, to the continuous fraction capable of defining an irrational:

$$\frac{1}{2} = \frac{1}{1+1}; \frac{2}{3} = \frac{1}{1+\frac{1}{1+1}}; \frac{3}{5} = \frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}; \ldots$$

Going on to infinity, we would obtain

$$x = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}}}$$

thus $x = \frac{1}{1+x}$; that is to say, $x^2 = 1 - x$ from which $\frac{x}{1-x} = \frac{1}{x}$, and in other words, $1 : x = x : 1 - x$. Therefore, x represented the value of the divine proportion. Not only so, the fractions that approximated to its value as a continuous fraction were composed of ratio between two adjacent terms in Fibonacci's succession, just the one inspired by the love-making rabbits. Furthermore, the divine proportion entered into the construction of the pentagon. Here, Kepler seems to want to render heavenly also those activities which others in his period would have considered all too animal and earthly.

Establishing unexpected relationships between the pentagon, the divine proportion and Fibonacci's succession, Kepler seized "the law of generation", valid for the seeds of plants, and for the union between male and female. This law did not appear to be the prisoner of whole, rational numbers, but rather tended to the ineffable, the divine, the infinite. The pentagon also achieved that division of the monochord which justified the new consonant harmonic ratios of the third, borrowed from Ptolemy, and made current in that period by Zarlino. Thus, even his

[89] Five was the Pythagorean number for marriage.

[90] Kepler 1619 [1969], III cap. XV par. VIII, pp. 76–77; 1997, pp. 241–242; translation slightly changed.

mathematical model for generation derived from the search for a harmony in every aspect of the cosmos, even biology, that was inspired by music. And with this model, he explained why the activity tending towards generation was stimulated by the intervals of the major third *-do/mi-* and the minor third *-mi/sol*. Kepler interpreted the world through mathematical relationships, which illustrated its harmony. It regarded all phenomena, and not just astronomic and physical ones. Even between human beings, animals and plants, our German astronomer assumed geometrical relationships, which were to be in agreement with those of the musical notes and the planets, in a global conception.

For music, Kepler made reference to the Pythagoreans, to Ptolemy, to Boethius, and so on, all the way down to Zarlino and Vincenzio Galilei. He explained how and why he could not wholly accept their proposals about the origin of harmonious proportions and consonances. In contrast with the Pythagoreans, who reduced harmony to whole numbers, he invoked the ear. They did not justify why certain numbers were acceptable and others were not. On the contrary, he offered the causes. "Cum enim intervallorum Consonorum termini sint quantitates continuae, causas quoque, quae illa segregant a Dissonis, oportet ex familia peti continuarum quantitatum, non ex Numeris abstractis, ut quantitate discreta; …" ["For since the terms of the consonant intervals are continuous quantities, the causes which set them apart from the discords must also be sought among the family of continuous quantities, not among abstract numbers, that is in discrete quantity; …"]. He thus finally broke out of narrow pattern of the *quadrivium*. He could then explain, by means of continuous geometry, that consonant intervals were "scibiles" ["knowable"] and dissonant ones "aut improprie scibiles, aut inscibiles" ["either improperly knowable, or unknowable"].[91]

Having curved and closed in a circle the vibrating string that generated sounds, our German astronomer divided it in accordance with the ratios previously fixed by regular polygons. For him, the "constructible" polygons became the cause of "knowable" intervals, and thus of consonances. He knew that the musicians of his period had by now gone beyond the limits of the Pythagoreans, making a large use also of the intervals of the third and the sixth, only to remain silent at the moment of the new heavenly harmonies. The garments worn by Kepler were still those of Euclid, woven with definitions, axioms and propositions, but he used these to obtain different results, seeing that he explicitly wrote that he repudiated the numbers of the Pythagoreans. The regular polygons with 5 or 6 sides thus justified the new consonant relationships of the third and the sixth, whereas those with 7, 9, 11, … sides gave dissonances.

But it was not only as a Platonic that he presented his "sublime" contemplation of the new axioms of musical theory; for him, it was also "… Fideique Christianae analoga, …" ["… analogous to the Christian faith …"]. This geometry, therefore, eternal like the God of Kepler, provided the models from which the laws could be derived. Spirits, souls and minds, as images of God the Creator, rejoiced in the

[91] Kepler 1619 [1969], III, pp. 7–9; 1997, pp. 137–139.

same proportions that could be found in the senses, bodies and movements. "… per materiae quantitatem, inter infinitas proportiones non harmonicas, occurrerint etiam harmonicae istae suis temporibus, & sic non in ESSE sed in FIERI consistant." ["… due to the quantity of material, among an infinity of proportions which are not harmonic, those harmonic proportions also occur at their own time, and thus subsist not in BEING but in BECOMING."] For these proportions guaranteed by God, Kepler provided models, both with the heavenly movements [in Book V], and with the "Radiationibus Astrorum" ["radiations of the stars", in Book IV] and also with the human soul, which takes pleasure from musical notes.

Considering the resonance between two vibrating strings, our natural philosopher from Northern Europe explained that the vibrations were propagated from the first one, when it was plucked, also to the second one, left untouched, but in unison; its cause was: "… species immateriata corporis chordae, …" ["… an immaterial image of the body of the string …"]. He wrote of "species", and not "ictus" [strokes] as Maurolico, Benedetti or Galileo Galilei did, as we have seen above. Kepler believed that the relative theory was insufficient to make the strokes coincide in order to explain the consonances. Otherwise, the best consonance would have been playing in unison. For him, instead, it was not necessary to count the (discrete) number of strokes that arrived at the ears with the sound, but rather to follow that "species" of the vibrating string as far as the mind, where it was judged by the tribunal of the spirit.[92]

Following his own ears, Kepler had discovered the errors of the ancients, and insisted above all that he had discovered the causes of harmonious relationships. "… ut certus sit, ea, de quorum causis laboramus, sensuum experientia certissima niti, nec sponte mea (cujus criminis Pythagorei in parte rei sunt) esse conficta, proquam veris obtrusa." ["… so that he can be sure that those things, whose causes we are struggling over, rest on the surest test of the senses, and are not improvised inventions of my own (a crime of which the Pythagoreans are partly guilty) which have intruded into the place of truth."][93] He had undoubtedly found the same musical intervals as Ptolemy and Zarlino, but now he felt sure about them, as he had justified them in a balanced way, between the senses and geometry.

Thus our natural philosopher cannot have concerned himself only with an abstract, inaudible music of the spheres, because very often he indicated on Guido D'Arezzo's stave the notes corresponding to the numbers of the intervals. He even arrived at the triads of the major and minor chords, refusing, however, any Pythagorean interpretation of them based on the number three.[94]

Kepler remembered that incommensurable magnitudes existed in geometry, and therefore it was a vain hope to expect to find a "… minimum Elementum …" ["… smallest element …"] with which the other consonant or melodious intervals

[92]Kepler 1619 [1969], III, pp. 13–15; 1997, pp. 147–150. Here *species* is misunderstood as "emanation".

[93]Kepler 1619 [1969], III, p. 28; 1997, p. 165. Translation slightly changed.

[94]Kepler 1619 [1969], III, pp. 30–31; 1997, pp. 168–170. Dickreiter 1973, pp. 153–154.

could be formed. Therefore it was necessary to accept that "… Intervalla consona (praeterquam quorum unum est alteri Multiplex) esse, ut proportiones illas ipsas, incommensurabilia; …" ["… Consonant intervals (except in cases where one is a multiple of the other) are incommensurable, like the actual proportions; …"]. In the example that 1:4 was not commensurable with 2:3, we can see the arrival of the notorious *comma*. For Kepler, music was made up of consonant intervals, which preceded any division into smaller intervals. Music was not destroyed by breaking it down into quantities; on the contrary, the relationships between notes possessed a qualitative nature. Breaking down a consonance would have meant ruining it. Thus tones, semitones and diesis arose from the consonances, and not vice versa, as the ancients believed.

With this (Aristotelian?) idea in his head, that the quality of the whole could not derive from the quantitative union of the single parts, our revolutionary astronomer of the seventeenth century was able to go back over the more traditional melodious intervals, including the various *comma*, *limma*, and diesis of Ptolemy's treatise. But he continued to repeat his accusations against the Pythagorean "… superstitione …" ["superstition"] of numbers, which were to be substituted with his geometry. Albeit with the due criticism, he followed Ptolemy, and ignored the Platonic-Pythagorean scale.[95]

Of the three Greek genres, Kepler cancelled the enharmonic one, and reinterpreted the diatonic and the chromatic genres as "durum" [hard] and "molle" [soft] through the decomposition of the fifth into a "hard" and "soft" third. Consequently, in a similar way to Zarlino and other musicians of the period, also our astronomer contributed to the transformation of the Greek-ecclesiastical modes into the subsequent 'major' and 'minor' keys.

In Chap. VIII of "Book III", Kepler states his position regarding the controversy that we have seen running through European culture for centuries, which had recently been inflamed again by Vincenzio Galilei. Here, then, he wrote that in practice, musicians were to temper the notes of the scale on their instruments, and to make all the tones equal, dividing them into two semitones that were likewise equal. And now the astronomer took as his explicit target Vincenzio Galilei, who he knew to have been inspired by Aristoxenus, and tried to demonstrate that the hearing should reject this solution which was most convenient for players of music. He calculated the numbers of the lengths for the strings of the lute according to Vincenzio, who had taken the constant ratio 17:18 as the semitone. This was clearly an approximation to the correct number for the tempered semitone. For a string 100,000 long, decreasing by the geometrical progression of $\frac{17}{18}$, he obtained the length of 50,363 for the higher octave, and not 50,000, in accordance with the due ratio of 1:2, as in the classical Greek scales taken by Kepler.

[95]Kepler 1619 [1969], III, pp. 33–37; 1997, pp. 173–179. Here *superstitione* is translated as "fascination".

He also noticed the imperfection of the tempered fifth 67,025, instead of his Greek-Pythagorean 66,667.[96] "… & haec differentia sonorum facile deprehenditur ab auribus, …" ["… and this difference between the notes is easily discerned by the ears, …"]. "… sciunt boni Arithmetici falsum esse, cum 1:2 & 17:18 sint incommensurabiles." ["… good arithmeticians know this to be false, since 1:2 and 17:18 are incommensurable"], that by repeating 17:18 12 times, we obtain 1:2. "… auditus, si ratione acuatur … probans partes, quas mei numeri signant, repudians Mechanicas primi ordinis … Ego quidem usum ejus Mechanicum agnosco, ut organis eadem pene libertate in tensionis, quae est in humana voce, possimus uti; ad speculationem vero, imo ad Naturam cantus dignoscendam perniciosam exsistimo: quoque hoc efficiat, ut Organum ingenuitatem humani cantus vere nunquam assequatur." ["… the hearing …, if it is sharpened by reason, …"] approving in the table ["… those parts which my figures represent, and repudiating the mechanical figures [of Vincenzio Galilei] in the first column. … I indeed recognize its mechanical function, so that in instruments we can enjoy almost the same freedom of tuning as can the human voice. However, for theorizing, and even more for investigating the nature of melody, I consider it ruinous; and the effect of it is that the instrument never truly attains the candour of the human voice."] At the beginning of Chap. IX, however, our 'good arithmetician' admitted: "Non sine lite res est; sed sequor ego rationes necessarias." ["The matter is not uncontroversial; but I am following necessary calculations."][97]

After all the observations written in favour of geometry and continuous magnitudes, like those suitable also for the treatment of music (and not only astronomy), nobody should imagine that Kepler was not capable of extracting the roots of numbers, and did not know how to divide the octave into 12 exactly equal parts, either. After all, he also had a direct knowledge of Aristoxenus. He simply did not want to do so, therefore, and preferred to remain close to Zarlino in his variant of the classical Greek *diapason* [octave], although he had found and justified it otherwise. Why?

From time to time, he repeated, even in this musical "Book III" as well, that harmony had been fixed by God, also for the movement of the planets. "Rursum autem hoc discrimen utriusque generis harmoniae, Deus ipse in motibus planetarum expressit, ut lib. V audiemus." ["Now again, the distinction between the two kinds of harmony [hard and soft] has been expressed by God himself in the motions of the planets, as we shall hear in Book V".] Thus the basic reason that guided our versatile scholar of Northern Europe was to become clear only in astronomy. All that he thought and did in the theory of music was to prove to be consistent with the novelties imagined for the movement of the planets and vice versa. "… Proportionibus, totoque apparatu Harmonico ex Musica, opus habeo ad explicandas causas proportionis Orbium coelestium, Eccentricitatumque & motuum

[96]For Vincenzio's tempered fourth, the table shows a printing error: 75,242 instead of 75,142, compared with the number 75,000 of the ratio 3:4.

[97]Kepler 1619 [1969], III, pp. 46–50; 1997, pp. 195–200 . Kepler 2009, pp. 20–31.

in Apsidibus." ["... I also labour over the proportions and the whole harmonic panoply in music, in order to explain the reasons for the proportions of the celestial circles, and their eccentricities and their motions at the apsides."][98]

Historians of music may find it interesting to study the variants introduced by Kepler with respect to the Greek modes, the ecclesiastic modes, Ptolemy and the other theoreticians of his time.[99] We may limit ourselves here to the observation that our mathematician continued to maintain the tortuous theoretical complications of the orthodox Greek tradition, refusing the route chosen by contemporary musicians and composers. If he, too, like them, had accepted the equable temperament re-proposed by Vincenzio Galilei, as if by magic, the system would have been simplified. What Kepler succeeded in achieving with the orbits of the planets contained in only three laws, he did not succeed in doing with earthly musical harmonies.

It was a curious paradox that, thanks partly to music, our astronomer had made the image of the solar system more simple. And yet, at the same time, as a result of the link imagined with astronomy, he had to maintain much of the ballast of the Pythagorean-Ptolemaic theories of music, in a period when musicians instead, freeing from it, would have given rise to a brilliant season of creativeness. They sought variety in the modulations between one tonality and another, which were considerably facilitated, above all by the possibility of transposing intervals on the different notes of the scale. But our natural philosopher from Northern Europe would have preferred to stop all those transpositions which were incompatible with his system. "Quaeras ultimo, quid impediat illas transpositiones, toties jam a me rejectas? Videntur enim aures illas non impedire ..." ["Finally, you may ask what prevents these transpositions which I have already rejected so often? For the ears seem not to prevent them, ..."]. And in the end, Kepler had to invoke "naturae leges" ["laws of nature"] as a support for his system. But which laws? Those of astronomy, of sounds or of music?[100]

In the wake of a tradition that went back to Plato's *Republic*, our astronomer of the seventeenth century was not averse to expressing his opinion about the effects provoked by the various tones, intervals or modes on people. Having reduced the genres to "hard" and "soft", he identified the former with the 'male' and the latter with the 'female'. In support of this, it was clearly not sufficient to quote the classic episode in which the singer Timotheus was driven out of Sparta because his chromatic tempered melodies made the young men effeminate and unsuited for warfare. "Nam ut foemina ad patiendum potissimum facta est, mas ad agendum, praesertim in generationis negocio, sic Molle genus passionibus animi foemineis, Durum actionibus virilibus accommodatur; ...". ["For as woman is made chiefly to be passive, and man to be active, especially in the act of generation, so the

[98]Kepler 1619 [1969], III, pp. 44, 58, 60, 85, V, pp. 233–234; 1997, pp. 189, 212, 215, 254, 476–478; passim.

[99]Kepler 1619 [1969], III, Chap. IX–XIV; 1997, pp. 200–237. Dickreiter 1973.

[100]Kepler 1619 [1969], III, p. 74; 1997, p. 236.

'soft' kind is fitted for the feminine emotions of the mind, and the 'hard' kind for masculine activities; . . . "] To prove this, then, he elaborated the explanation quoted at the beginning, which culminated in the expressive Fig. 8.5, where the 'soft' minor third stood for womanliness, and the 'hard' major third for virility.[101]

At the end of his *Les six livres de la république [The six books of the republic]* (1576), Jean Bodin (1530–1596) expounded a theory which used the three means, the arithmetic, geometric and harmonic mean, to explain and justify democracy, aristocracy and monarchy, respectively. In the "Political digression on the three means" at the end of "Book III", Kepler resumed, and criticised the Frenchman for various reasons, partly mathematical, but often ended up by finding that the harmonic means, like those found for the monarchies, gave the best results, also in political questions. Some historians or political scientists may still find some of its details interesting. Among these, it is worth underlining that in an authentic democracy, the fundamental criterion for choosing should be by drawing lots: a simple procedure, which would immediately improve the political class of the planet, and would solve countless problems. Beyond this, in our particular history of sciences, the digression would throw further light to clarify certain characteristics of our astronomer. Plato's *Republic* continued to make its presence felt, with all its weight, as did also, partly, Aristotle's *Politics*. Ratios still remained the fundamental criterion to understand the phenomena, in this case those of politics.

Music facilitated the ability to extend ratios and analogies to peace and war. "Hic cum Ptolemaeo (. . .) ex generibus harmoniarum & cantus, Molle quidem Paci daretur, Durum bello; dissidiaque generum eadem quae in civitate utrorumque temporum." ["Here in agreement with Ptolemy (. . .), of the kinds of harmonies and melody, the soft would in fact be given to peace, and the hard to war; and the dissensions between the genres are the same as those in the state of both times."] Thus, the dissonances played an important function, not only to give a colour to the melodies, but also in relation to the events of the world, in accordance with the plan of divine providence.[102]

And it was even more significant that, thanks to the ratios between numbers, the laws of the State and of jurisprudence merged in the pages with those of the whole cosmos. These were all guaranteed by God, and this thought gave rise to the habit of indicating as laws (of Kepler, of Newton, of inertia, of movement, of gravitation, . . .) those rules, models, or relatively constant forms of behaviour in the movements of the planets, in vibrating strings and in other cases, too. We shall soon find the two last aspects again, in the other German natural philosopher and jurist of our history: Leibniz.

In "Book IV", our North European theoretician of music broke away from earthly sounds, to consider the stars, which influenced earthly phenomena by emitting rays. The harmony always remained the same, immutable and valid for everything, but now it changed its name: for sounds, the word 'octave' was used, for the rays

[101] Kepler 1619 [1969], III, p. 77; 1997, p. 240.

[102] Kepler 1619 [1969], III, p. 101; 1997, p. 276.

emanated by heavenly bodies in the same ratio, the term used was the 'aspect in opposition'. Before giving us, with relative exactness, particulars about the 'aspects' of the planets among themselves, in other words the angles formed between them, Kepler was anxious to point out that the relationships between sounds needed the mind and the soul in order to be achieved, otherwise they would remain separate. "… in genere relatio omnis, sine Mente nihil est, …" ["… in general, every relationship is nothing without the mind …"].[103]

Thus our natural philosopher appears to recognize the true harmonies, which the sense of hearing brought him, because their ratios, if correct, corresponded to the eternal, universal archetypes deposited by God in the soul and in the mind. These archetypes, or paradigms, revealed the pure, secret harmonies of things, and in order to attain them, of course, it was necessary to make use of the mathematics expressed in proportions. Kepler's archetypes did not exist outside the soul. One way or another, harmonies could be adequately represented only by regular figures inscribed within the circle, and present in the mind. For this reason, among Plato, Aristotle and Ptolemy, our mathematician preferred to cite the neo-Platonic Proclus at length. In him, he even found Christian cues, and he interpreted Plato's *Timaeus* as a comment on Moses' Biblical *Genesis*, transcribed, however, in accordance with Pythagorean philosophy. He even succeeded in remembering the *Acts of the apostles*, where Paul spoke to the Athenians about his God. The archetypes of all the numbers (1, 2, 3, 4, 8, 9, 27) and of the circle, as a model of all movements, would lie, according to him, in the soul.[104]

Of Aristotle, he rejected the image of the soul as a white sheet of paper waiting to be written on, but he shared his criticisms of the Pythagoreans. As regards the continuous magnitudes of geometry, for Kepler numbers remained secondary, and he refused their symbolic interpretations (present also in Plato and Bodin), like that of the climacteric number 7. The "Archetypalis Harmonia" he included in his figures, and these were to be "Neque tantum scibiles sed etiam scitas …" ["Not only knowable, but also known, …"] by the mind, because they dwelt in it. The senses were not sufficient, then, to establish their criteria. "Geometria ante rerum ortum Menti divinae coaeterna, Deus ipse (quid enim in Deo, quod non sit Ipse Deus) exempla Deo creandi mundi suppeditavit, & cum imagine Dei transivit in hominem: non demum per oculos introrsum est recepta." ["Geometry, which before the origin of things was co-eternal with the divine mind and is God himself (for what could there be in God which would not be God himself?), supplied God with patterns for the creation of the world, and passed over to Man, along with the image of God; and was not, in fact, taken in through the eyes."][105] From his treatise on optics, *Ad Vitellionem Paralipomena [Integrations to Witelo]* (1604), our Christian natural philosopher again assumed the sphere as the image of the Holy Trinity, Father, Son and Holy Spirit.

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[103] Kepler 1619 [1969], IV, p. 109; 1997, p. 291.

[104] Kepler 1619 [1969], IV, pp. 114–117; 1997, pp. 298–302.

[105] Kepler 1619 [1969], IV, pp. 118–119; 1997, pp. 303–304. Tangherlini 1974.

By horoscopes, we mean the ascendants, or rather the aspects, that is to say the angles at which the planets and the constellations of the zodiac can be observed from the Earth at the moment of birth. About 800 of these, cast by our renowned astronomer during his whole lifetime, are extant. He proved to be particularly good at this, and met with success; consequently, he was sought, both for the compilation of almanacs with weather forecasts, and to foretell political and personal events. For 1618, he had foretold turbulences in May, and on May 23rd, the famous Defenestration of Prague marked the beginning of the Thirty Years War. In 1608, he was working for the famous general Albrecht von Wallenstein (1583–1634), horoscope included. Interpreting again the same horoscope at his request in 1625, Kepler predicted that the terrible confusion in the country of March 1634 (between Catholics and Protestants, with various armed forces, Swedish, Spanish, French, Italian, imperial, and others, running amok all over Central Europe) would have serious consequences for our imperial general: Wallenstein was assassinated on February 25th, 1634.

The event that this astronomer was not only capable of foreseeing the observable position of the planets with great precision, but even claimed to derive predictions about the cold weather in winter, and the destiny of men and empires, has always created embarrassment for historians of scientific progress, who are used to distinguishing astronomy from astrology. With Kepler, these experts are forced to leap acrobatically from one document to another, and from one written work to another, usually ending up by concluding that he was forced to do so, to earn a living at the court of the powerful. And yet, nevertheless, emperors and nobles often did not pay him the sums that they owed. But when he was about to choose a second wife for himself, he had no problem in admitting the influence of the stars on him.[106] Thus it is anachronistic to distinguish between Kepler, the astronomer, and Kepler, the astrologer.

Actually, he did not declare himself either in favour or contrary to astrology: the title *Tertius interveniens [A third possibility exists]* is already fairly indicative. He refused the determinism of judicial astrology, for which one's whole destiny is decided by the horoscope, but he admitted that the stars influenced earthly phenomena. On the other hand, Isaac Newton, if not Galileo Galilei, was to follow him in explaining the tides as due to the influence of the Moon on the sea. It was possible to believe in the influence of stars without falling into superstition; it was sufficient to understand the reasons of it.

His philosophy consisted of assigning the form of the circle also to the soul, in order to find in it those musical ratios justified by geometry, which would be the same also for the heavenly movements. "Valde enim confirmant hanc Philosophiam Geneses, cum videamus characterem confluentiae radiorum coelestium ad idem punctum, velut ex circulo communi imprimi in nascentis animum; …". ["For this philosophy is strongly confirmed by horoscopes, as we see that the character of the concurrence of celestial rays at the same point, as if from a common circle,

[106] Kepler 1992. Kepler 1997, pp. 308–309.

is imprinted on the mind of a newborn; …"].[107] The proof of this was that the harmonies touched the minds of everyone, including those who were ignorant of philosophy and geometry. Amorous attractions were thought to be of the same nature.

Depending on the angles at which the stars were observed from the Earth, the archetypes of the human soul would come into resonance with the heavenly ones, and were influenced by them. Kepler attributed a soul also to the Earth; consequently, as a result of the same reasons and the same mechanism, also meteorological phenomena on the Earth, such as clouds, wind, rain, heat and cold, were believed to be influenced by the astrological aspects. "In Terrae enim corpore ponenda est haec anima: quia nec anguli harmonici radiorum in ulla alia parte mundi quam in Terra existunt & opera Naturae, quae ad configurationes radiorum sequuntur, ex Terrae visceribus, montiumque cavernis ortum trahunt." ["For this soul must be placed in the body of the Earth, as the harmonic angles of the rays exist in no other part of the world but on the Earth, and the works of Nature which follow upon the configurations of the rays take their origin from the bowels of the Earth and the caverns of the mountains."][108]

However, our astronomer made a distinction between music, as played by musicians, and the alternation of the various angles in the movement of the planets. In the *continuum* of sounds, there were jumps from one note to another, and the infinite possible intervals between the two were passed over in silence. In the world of the planets, on the other hand, the discordant moments mixed with the harmonious ones, which happened only at particular instants, although they generated more lasting effects. "Breviter, configurationes praecinunt, Natura sublunaris saltat ad leges hujus cantilenae." ["In brief, the configurations [of the planets] sing the leading part, sublunary Nature dances to the laws of this song."][109]

Even if in a slightly different way than the one thought for music, our German-speaking astronomer-astrologer again used his geometrical ratios of the first two books, to select the aspects, that is to say, those configurations which influenced the earth: 180° (opposition), 90° (quadrature, from the square), 120° (trine, from the triangle), 60° (sextile, from the hexagon) and so on. He even took pains to remind us that the term "aspectus" derived from an Arabic translation of the Greek $\sigma\chi\eta\mu\alpha\tau\iota\sigma\mu\acute{o}\varsigma$ [appearance], used by Ptolemy both "… in *quadripartito [Tetrabiblos, Four books]* …" and "… in *Opere magno [Almagest, Great work]* …" and "… inque *Armonicis [Armonikon, Harmony]* …". The abstraction of the Greek Schema became, for the Arabs, a far more concrete "face": in German *das Angesicht*, which Italians would call – Kepler wrote – *Mascaras* [masks].[110]

Kepler specified here that no perfect correspondence existed between the musical consonances and the astrological aspects. They would have different origins, seeing

[107] Kepler 1619 [1969], IV, pp. 121–122; 1997, p. 308.

[108] Kepler 1619 [1969], IV, p. 131; 1997, p. 323.

[109] Kepler 1619 [1969], IV, p. 132; 1997, p. 325.

[110] Kepler 1619 [1969], IV, p. 133; 1997, p. 326.

that the circle that generated the former eventually had to be opened to become a vibrating string. Instead, for the latter, a more important role was played by the congruences of the second book between the figures, and in any case, an aspect was an angle. Thus, the only consonance that corresponded perfectly to an aspect remained the octave, generated by the division of the circle into two parts, which would also produce the opposition of 180°. On the other hand, the trine would not correspond to the fifth, but only to the interval of the octave plus a fifth; the quadrature would not correspond to the fourth, but to the double octave, and so on. Although the same numbers were not counted for the aspects as for the musical harmonious divisions, all the same a great affinity remained between the two, because they both were born from the "noble" figures of the circle.

For music, marriage would have been admitted – Kapler continued – between a noble patrician, like the fourth part of a circle cut away from the square, and a plebeian woman like the remaining three quarters. But with the aspects, there would be no female remainders, and thus no marriages could have been combined. "Contra in Meteorologia mos est alius. Nam ut quisque ipse nobilis seu ortu seu meritis (Scibilitate scilicet aut Congruentia) ita plurimum valet authoritate, reliqui volitant velut umbrae: foeminarum ratio habetur nulla." ["On the other hand, the fashion in the study of the Meteorology is different. For as each nobleman is worth himself by his origin or by his merits (that is, by the knowability or congruence), so [the aspect] has great power by its authority, and the rest flit like shadows. No account is taken of females."][111]

He previously wrote, for the *Ephemerides [novae motum coelestium] [New almanac of heavenly movements]*, that "... Creatorem Deum, aut ex Harmoniis Cantus infra Octavam, libro III descriptis, desumpsisse leges ordinandorum Aspectuum; aut ad coelestes Aspectus attemperasse aures hominis, Concordantiarum illarum judices." ["...: God the Creator either took the laws for ordaining the aspects from the harmonies of the music within an octave, which are described in Book III, or attuned the ears of man, which are the judges of those consonances, to the heavenly aspects."][112] But the author of these almanacs, however successful they were, must have had doubts about too close a correspondence between two partly different phenomena. Thus he searched more carefully for the causes. One reason was that he wanted to make a better defence of his third position, in other words intermediate between the out and out denigrators of astrology and the followers of judicial astrology. Among the former, he quoted also Giovanni Pico della Mirandola (1463–1494). Thus he had partly modified what he had written, starting from 1606 in *De stella nova [On a new star]*.

To combat the adversaries of both sides, Kepler endeavoured to demonstrate that the Earth possessed a soul, like living creatures. because certain characteristics of this soul could be observed externally. In this. he was similar to Leonardo da Vinci, who, however, he could not gain a good knowledge of, because his codices were not

[111]Kepler 1619 [1969], IV, p. 155; 1997, p. 355; translation different from ours.

[112]Kepler 1619 [1969], IV, p. 151; 1997, p. 350.

yet in circulation in his times.[113] Among the arguments proposed by Kepler, which are most convincing for us today, because they have been taken up by the subsequent orthodox evolution of sciences, we may read: "… ille mirabilis fluxus & refluxus Oceani semidiurnus, qui etsi ad Lunae motus se accomodat, sic ut probabile mihi sit visum, in Praefatione commentariorum Martis, undas a Luna trahi, ut ferrum a Magnete, virtute corporea unitionis corporum; …" ["… the wonderful twice daily ebb and flow of the ocean; yet it fits in with the movements of the Moon in such a way that it seemed probable to me, in the preface to my *Commentaries on Mars*, that the waves are drawn, as iron is by a magnet, by a corporeal virtue of the combination of bodies; …"].[114] Among the most vertiginous and bizarre analogies, here we find rocks and minerals that spout from the bowels of the earth, as if from a pregnant woman, with figures and images impressed on them like those present on newborns.

Of course, also the soul of the Earth has a circular form, like the zodiac, containing its regular archetypes. He preferred to call the essence of the soul *'ενϵ′ργϵιαν* [energy], which he compared to the flame in movement, and to God. "Deus quippe est substantialis Energia, …" ["God, of course, is substantial Energy, …"].[115] This is how the circle of the zodiac could succeed in influencing not only the circular soul of man, but also the meteorological phenomena of the earth.

As an example, our astrologer-astronomer offered his readers his own horoscope, comparing it with his capacities. Briefly, he was a Capricorn with a strong presence of Jupiter, so much so that "… magis delector Geometria in Physicis rebus expressa, quam abstracta illa, & Saturni siccitate prae se ferente, magis inquam Physica quam geometria; et quod Luna gibba in clara frontis Taurinae constellatione impleat Animae Phantasticam facultatem imaginibus, quarum tamen multas Naturae rerum consentaneas re ipsa expertus sum, velut ex Procli Paradigmatibus delapsas." ["… I delight more in geometry expressed in physical things than in the abstract, and showing in its appearance the dryness of Saturn, more, I say, in Physics [natural philosophy] than in geometry; and because the gibbous Moon in the famous constellation of the brow of Taurus would fill the fantasizing faculty of the soul with images, though I have experienced that many of them agree with the nature of things, as they have been taken from the patterns of Proclus."]

But he also added immediately that astral influences by themselves would not be sufficient to explain many other important events that had happened to him, which depended, rather, on circumstances like the studies carried out, the places where he had lived, the emperors he had met, and other things, too. The only effect of the horoscope "… instigavit animum ad laborem indefessum, auxitque desiderium sciendi; breviter, non inspiravit animum non ullam dictarum hic facultatum, sed excivit." ["… urged my mind on to untiring toil, as well as increasing my desire for knowledge. In short, it did not inspire my mind, or any of the faculties stated

[113] In out great epoch, the Earth, considered as a single, complex living organism, has been resumed, and dubbed the 'Gaia hypothesis'.

[114] Kepler 1619 [1969], IV, p. 162; 1997, pp. 365–366.

[115] Kepler 1619 [1969], IV, p. 163; 1997, p. 367; here *Energeia* has been translated as "activity".

here, but roused it."] As a direct attack on deterministic astrology, Kepler described a person, born under his same constellation, who first of all was a woman "... inquietissimo sane ingenio, sed quo non tantum nihil proficit in literis (non mirum hoc est in foemina), sed etiam totum turbat municipium suum, sibique author est miseriae deplorandae." ["... a temperament which was certainly very restless, but from which she not only draws no advantage in book learning (that is not surprising in a woman), but she also disturbs the whole of her town, and is the author of her own lamentable misfortune."][116]

This is enough as regards astrology, but Kepler's character will continue to accompany us, because he took pleasure in revealing it also through more scientific arguments. We can easily imagine that all this brought into discussion the complex religious problems of the age, including predestination, free will, original sin, grace, etc.: all elements disputed by Catholics, Lutherans and Calvinists. Which God did Kepler believe in? He quoted Luther's horoscope, in the version of Girolamo Cardano, deriving from the conjunction of many planets (Venus, Mars and Jupiter) his authority over the people and his favour with princes. He ended "Book IV" with a prayer to the Triune God for the salvation of souls. At the beginning of "Book V", he confessed "... me vasa aurea Aegyptiorum furari, ut Deo meo Tabernaculum ex ijs construam, longissime ab Aegypti finibus." ["... I am stealing the golden vessels of the Egyptians to build a tabernacle to my God from them, far, far away from the territory of Egypt."] The sentence was a clear reference to the *Bible*. But how to restrain ourselves from interpreting it as indicating that he, like a new Moses, had stolen the astronomical observations of Tycho Brahe, to construct a different planetary system?

Now, more than 20 years later, Kepler thought back over all his astronomical studies. He made modifications where necessary: "... pluribus sit opus principijs, praeter quinque regularia corpora." ["... more basic principles are needed in addition to the five regular solids."] Then he again took up his initial work, *Mysterium cosmographicum*, to correct the proportions between the distances of planets, and to justify them in another way. Among other things, he seized the opportunity to create marriages between the Platonic solids. By transforming (in duality) the faces of one into the vertices of the other, and attributing a sex to them, the vertices of the male solids, like the cube and the dodecahedron, corresponded to, and could enter into, the faces of the female octahedron and icosahedron. Instead the tetrahedron was transformed into itself, and was thus a hermaphrodite (Fig. 8.6).[117]

As in the Greek and Latin tradition still present in Maurolico, our Central European astronomer combined the numbers of the vertices, the edges and the faces in the ratios of music. But as he had been forced, by the precise observations of Tycho Brahe, to abandon circular orbits in favour of the eccentric ones of the ellipse, he was in search of other elements related to the movements of the

[116]Kepler 1619 [1969], IV, pp. 170–171; 1997, pp. 376–378 and pp. xxi–xxiii. Cf. Kepler 1984, pp. 87–90. Cf. Caspar 1993, pp. 76, 181–185 and passim.

[117]Kepler 1619 [1969], V, pp. 179, 181, 187; 1997, pp. 391, 396–397, 407.

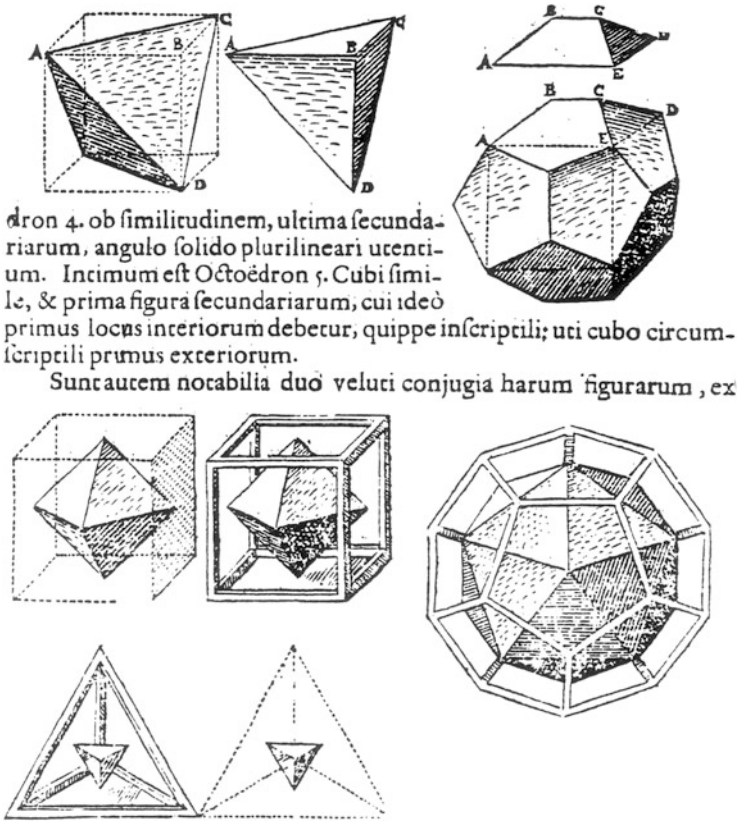
dron 4. ob ſimilitudinem, ultima ſecunda-
riarum, angulo ſolido plurilineari utenti-
um. Intimum eſt Octoëdron 5. Cubi ſimi-
le, & prima figura ſecundariarum, cui ideò
primus locus interiorum debetur, quippe inſcriptili; uti cubo circum-
ſcriptili primus exteriorum.
Sunt autem notabilia duo veluti conjugia harum figurarum, ex

Fig. 8.6 How Kepler coupled the regular polyhedra, attributing male, female and hermaphrodite properties to them (Kepler, ibid. lib. V p. 181)

planets, that could reconstruct the desired harmony. In solving this problem, he also nonchalantly corrected the ratio that he had written in *Astronomia nova*, where the times taken by planets to complete one full orbit had been made to increase with the square of the mean distance from the sun. They would have increased too much as they moved away from our star, and as a result, here in the *Harmonices* ... the proportion became the sesquialtera (intermediate between the linear proportion and the quadratic one) of the third law, as we have already seen at the beginning of this section.[118]

Anyway, faith in the music of the spheres continued to burn brightly in the heart of our German mathematician. He was now only deciding what notes the planets should emit if they moved in accordance with the new laws invented by him. The heavenly harmonies would not be lost, because they would be found here

[118]Kepler 1619 [1969], V, p. 189; 1997, p. 411.

among the ratios of angular velocity. He excluded the periodic times, the mass and the planetary distances, because among these physical elements he could not find any harmonious ratios, though he continued tenaciously to search for them. "Cum autem Deus nihil sine Geometrica pulchritudine constituerit, quod non ab alio priore quadam necessitatis lege sit nexum; … […] … oportet igitur vel in his Moris, vel si quid est ijs prius in Mente Opificis, inveniri concinnitates Geometricas." ["Since, however, God has established nothing without geometrical beauty unless it is bound up with some other, prior law of necessity, … The geometrical harmonization must therefore be found either in these times, or in something prior to them in the mind of the Maker."][119]

Between the maximum distance of the Earth from the Sun (aphelion) and the minimum distance (perihelion) of Venus, there would exist even the ratio 1,018:719, which, equalling 1.4158, would be a good approximation of $\sqrt{2}$:1, that is to say the *semidiapason*. However, Kepler ignored it, like other approximations to the octave and to the fifth, because he was searching, rather, for harmony between the movements, as music is born from these. In the end, being he guided by instinct and by reason, he had the fate to find the notes among the angles followed every day by the planets, as seen from the Sun: the source of all movement. "… ut ita haec apparentia, beneficio luminis ad corpus Solis perlata, cum ipso lumine recta in creaturas, instinctus hujus participes, sic influere possit: sicut libro quarto, schema coeli in foetum, beneficio radiorum diximus influere." ["… so that this appearance, brought by the agency of light to the body of the Sun, can along with light itself influence directly the living creatures, who share in this instinct, just as in the fourth book we have stated that the pattern of the heavens influences a foetus by the agency of the rays."][120]

Effectively, among the seconds of the arc covered every day by the planets, our obstinate and skilful mathematician succeeded in identifying major and minor thirds, fifths, octaves and other intervals. For example, between Saturn at its perihelion and Jupiter at its aphelion, the ratio 2′15″:4′30″ of the octave; between the aphelion and the perihelion of Mercury 38′1″:25′21″ (2,281″:1,521″) the fifth. The fourth can be found in the movement of the Moon. Not all the intervals prove to be perfect, but – Kepler wrote – it would have been difficult to perceive the margin, although in the case of Mars and Jupiter, it arrived at a diesis. Jupiter and Mars also would produce the dissonant interval 1:7. Between aphelion and perihelion, each single planet was said to have produced one note after another in monody, like the ancient Greeks. Modern (in that period) polyphony, instead, comes from combining the various planets together.

Our Central European astronomer translated the ratios between angular velocities of the planets into the notes of the scale, which he then transferred to the stave, distinguishing the hard genre from the soft one. There would no longer be any reason to marvel at the beautiful melodies composed by men, because they would

[119]Kepler 1619 [1969], V, p. 194; 1997, p. 419.

[120]Kepler 1619 [1969], V, p. 198; 1997, p. 424; translation a little different.

do "… nihil aliud quam Dei Creatoris simias agere & ludere veluti drama quoddam ordinationis motuum coelestium." ["… nothing but aping God the Creator, and, as it were, acting out a particular scenario for the ordering of the heavenly motions."] In this way, Kepler assigned to the planets the melodies of Fig. 8.4 and to the Earth, a strident 'soft' semitone, better interpreted as *mi fa mi*.[121]

Having discovered the "… genuinus Archetypus fabricae Mundanae …" ["… true archetype of the construction of the world …"], our German-speaking mathematician then addressed the "modern" musicians of his time, to propose to them "… prima universitatis exempla genuina …" ["… the first true offprints of the universal whole …"] offered by nature. As he declared, having read Vincenzio Galilei's *Dialogo della musica antica et moderna* [Dialogue between ancient and modern music], modern composers were, for him, the musicians of polyphony with several voices, those composers that the Tuscan was pushing to one side in that period, to make room for the monodies inspired by the ancient Greeks. Kepler, in fact, had referred several times to the Flemish polyphonist, Roland de Lassus, whose compositions, "In me transierunt", "Ubi est Abel" and "Tristis est anima mea", he quoted in his text, together with notes and titles".[122]

Roland de Lassus (1530/32–1594) had flooded Europe, from Italy to the Flanders, with his 2,000 sacred and profane compositions, madrigals, motets and masses. Having eventually settled down at Munich, he became the precentor of the Catholic Counter-Reformation. He concentrated, in time, on writing spiritual madrigals, and became so famous in his epoch that he was called 'the prince of music'. In the years of his education, and often subsequently, he visited Italian musical environments, which were already enriched by the presence of other famous Flemish composers. He had met not only Giovanni Pierluigi da Palestrina, but also Adrien Willaert, Cyprien de Rore or Jacques Archadelt. Even the Protestant Churches were glad to have contact with him. If necessary, they changed the texts to be set to music. "La lettre, accommodée à la Musique d'Orlande … estoit sotte, lascive et profane, … j'ai rendu ces chansons honnestes, et chrestiennes pour la pluspart …" ["The text set to music by Roland … was frivolous, lascivious and profane, … I have made these songs honest and Christian for the most part …"]. For this characteristic of his, he would undoubtedly have been appreciated also by Kepler.

And yet that fine tenor voice from Mons represented the peak of polyphonic culture of the sixteenth century; at the same moment, the radical innovation of the accompanied monody was appearing on the European scene, as elaborated by Vincenzio Galilei, by the Camerata of the Bardi, and lastly by Claudio Monteverdi (1567–1643). Two aspects present in the work of Lassus had interested Kepler. The first was the profound Christian faith, transcending a fleshly reality, though there was no desire to renounce its joyful presence. Together with about 50 Masses, a 100 or so Magnificats and innumerable penitential motets, our musician had added profane motets, Italian madrigals, popular Villanelle and French songs about

[121] Kepler 1619 [1969], V, pp. 204, 207; 1997, pp. 435–436.

[122] Kepler 1619 [1969], V, p. 208, III pp. 64, 72, 75 and 84; 1997, pp. 441, 221, 234, 239 and 253.

licentious topics. Preferring to set verses of Petrarca to music, we may identify the second aspect as a common classical-style platonism.[123]

Right from the moment of creation, God was thought to have made the world sing in polyphony with several voices, the same that whispered to the human mind through the ears. Thus, astronomic tables in hand, our religious philosopher prepared to demonstrate what music the Maker had composed for the different planets. He even invited composers to make use of it. "Anne flagitium fecero, si a singulis hujus aetatis Componistis artificiosam aliquam Motetam pro hoc Elogio exigam; Textum huc aptum Psaltes Regius, caeterique Sacri libri suppeditare poterunt. Verum heus vos in coelo plures quam sex non concordant. … […] … Qui propius Musicam coelestem exprimet hoc opere descriptam, huic Clio sertum, Uranie Venerem sponsam spondent." ["Shall I perhaps be committing an abuse if I demand some ingenious motet from individual composers of this age for this declaration? The royal psalter and the other sacred books will be able to supply a suitable text for it. Yet take note that no more than six parts are in harmony in the heavens. … If anyone expresses more closely the heavenly music described in this work, to him Clio pledges a wreath, Urania pledges Venus as his bride."][124]

Between Mars, the Earth and Mercury, consonant harmonies occurred quite often, but to obtain them between four or five planets, it would be necessary to wait for centuries, or tens of thousands of years. As a result, the consonant harmony between all six would occur only once, at the beginning of time, and would characterise creation. Among the various angles covered every day at the various points of their orbit, Kepler found those that generated the chords *mi* − *sol* − *si* and *do* − *mi* − *sol* in the hard genre, and in the soft genre, *mi*♭ − *sol* − *si*♭ and *do* − *mi*♭ − *sol*. The harmonies were classified as hard and soft, because they referred to the male Earth or to the female Venus. If it were possible to silence this talkative goddess who is an eternal source of disagreement, with only five planets, and thus only five voices, the chords obtained would be *sol* − *si* − *re* and *sol* − *si*♭ − *re*.[125] Our very human imperial mathematician must have really fallen in love with his sexual reading of the two genres, already seen above and re-proposed here. Between Earth and Venus, there would be discord, when the husband accomplished his virile duties, while Venus remained distant in her gynaeceum. Yet sometimes the planet Earth descended in its perihelion towards Venus "… voluptatis causa, positis clypeo paulisper & armis operibusque viro convenientibus: tunc enim Harmonia mollis est."

[123] Borren 1944, p. 65 and passim.

[124] Kepler 1619 [1969], V, p. 208; 1997, p. 441. As I have done so far, at times I will modify the 1997 translation to make it closer to the Latin text, without continuing to point this fact out.

[125] The chords lend themselves to a modern harmonic interpretation as 'hard' major tonalities or 'soft' minor ones. However, this would be anachronistic, because Kepler referred to the Franco-Flemish polyphony that preceded the harmonic revolution of Bach and Rameau: from the Greek-ecclesiastical modes to major and minor tonalities.

[“… so as to make love, laying aside for a little while his shield and arms, and those tasks which are proper for man: for then the harmony is soft.”][126]

In our mediocre, wretched times, however, dissonances could not have been avoided. “Nihil igitur aliud sunt motus coelorum quam perennis quidam concentus (rationalis non vocalis) per dissonantes tensiones, veluti quasdam Syncopationes vel Cadentias (quibus homines imitantur istas dissonantias naturales) tendens in certas & praescriptas clausulas, singulas sex terminorum (veluti Vocum) ijsque Notis immensitatem Temporis insigniens & distinguens; …”. [“Therefore, the motions of the heavens are nothing but a kind of perennial harmony (in thought, not in sound) through dissonant tunings, like certain syncopations or cadences (by which men imitate those natural dissonances), and tending towards definite and prescribed resolutions, and towards each of those six terms (as with vocal parts) marking and distinguishing by those notes the immensity of time; …”]. This is the reason why man, “… Creatoris sui Simia …” [“… aping his Creator …”], invented in the end that polyphony which was unknown to the ancients. “… ut scilicet totius Temporis mundani perpetuitatem in brevi aliqua Horae parte, per artificiosam pluriumque vocum symphoniam luderet, Deique Opificis complacentiam in operibus suis, suavissimo sensu voluptatis, ex hac Dei imitatrice Musica perceptae, quadam tenus degustaret.” [“… so that he might play, that is to say, the perpetuity of the whole of cosmic time in some brief fraction of an hour, by the artificial concert of several voices, and taste, up to a point, the satisfaction of God, his Maker, in His work, by a most delightful sense of pleasure felt in this music, imitator of God.”][127]

For Kepler, Saturn and Jupiter would sing bass parts, Mars tenor, Earth and Venus contralto, and Mercury soprano. Having excluded from his theory both the distance from the sun and the mass of the planets, he tried to justify it through the number of the notes attributed to them. As everything was established by God by fixing the archetypes, in the end our German philosopher abandoned astronomical measurements to show that the planets must follow those exact ratios, and not other possible ones. The style remained that of Euclid, with axioms and propositions, but often the arguments recalled the Pythagorean sects, Platonic apriorisms and Aristotelian logic. *Breviter*, Earth and Venus would show small-scale eccentricities in their orbits, for the precise purpose of being able to sing major and minor sixths, of the hard and soft genres. Vice versa, Mars and Mercury should have to possess greater eccentricities, because of their intervals. Saturn and Earth would espouse the hard genre, Jupiter and Venus the soft one. In short, our Lutheran astronomer’s idea was that the eccentricities observed in the orbits of the planets derived from the musical harmony fixed by God at creation.

Music gave the harmonic ratios, and from these, Kepler now derived also the distances of the planets from the Sun, which he compared with the measurements, “… qui omnes valde prope accedunt ad illa intervalla, quae inveni ex observationibus Braheanis.” [“… all of them approach very closely the distances which I found

[126]Kepler 1619 [1969], V, p. 211; 1997, pp. 446.

[127]Kepler 1619 [1969], V, p. 212; 1997, pp. 446–448.

from the observations of Brahe."] There was a "… parvula differentia …" ["… tiny difference …"] only with Mercury, which the imperial mathematician justified by the scarcity of the relative astronomical observations.[128]

He explained here his preference in the end for the musical ratios of harmony to geometrical proportions, not on the basis of astronomical data, but by means of general philosophical arguments. Geometrical proportions are infinite, whereas harmonies are few. The former include also the incommensurable ratios. "At Harmonicae proportiones omnes sunt effabiles, omnium termini commensurabiles; ijque desumpti ex figurarum planarum certa & finita specie. Infinitas vero sectionis materiam, commensuratio vero seu Effabilitas terminorum formam repraesentat. Ut igitur materia Formam, ut rude saxum justae quidem quantitatis Ideam humani corporis, sic Geometricae Figurales proportiones Harmonias appetunt; …". ["But the harmonic proportions are all expressible [constructible], and the terms of all of them are commensurable. Also, they have been taken from a definite and limited class of plane figures. Now infinite divisibility signifies matter, but commensurability or expressibility [constructibility] of terms surely signifies form. Therefore, as matter strives for form, as a rough stone of the correct size indeed strives for the idea of the human form, so the geometrical proportions in the figures strive for harmonies."][129]

Consequently, Kepler did not take into consideration the rival musical theory of Aristoxenus, but rejecting it as we have seen above, because it was based on the incommensurable division of the tone and the octave. He had direct knowledge of this ancient Greek, because his manuscript had recently been published, together with those of Ptolemy, in the edition of the *APMONIKA* that he used.[130] Here in *Harmonices mundi* …, Aristoxenus was mentioned only twice: as a source used by Vincenzio Galilei to tune the lute in the equable manner, or so that Kepler would not be confused with the Greek musician when he, too, considered 12 intervals, which actually he did not make all the same.[131]

The whole argument, about how heavenly musical harmonies guided the movements of the planets, in the end led to the Sun. "Nam uti Sol in seipsum revolutus, per emissam ex se speciem, movet Planetas omnes, sic etiam mens […] omnes facit intelligi." ["For as the Sun, in its revolution about its own axis, moves all the planets by emitting a glance from itself, so also the mind … causes all things to be understood."] "… quietem Solis in centro mundi, pulchro responsu consequitur intellectionis simplicitas, …" ["… from the immobility of the Sun at the centre of the world follows the simplicity of the understanding by a beautiful relation …"]. "… in Sole vero Intellectum simplicem $\pi\tilde{\upsilon}\rho$ $\nu o\epsilon\rho\acute{o}\nu$ seu $No\tilde{\upsilon}\nu$ habitare, omnis Harmoniae fontem, quicunque ille sit." ["… surely on the Sun dwells simple

[128] Kepler 1619 [1969], V, p. 240; 1997, p. 486.

[129] Kepler 1619 [1969], V, p. 242; 1997, p. 489.

[130] Kepler 1997, p. 131.

[131] Kepler 1619 [1969], III, pp. 48 and 67; 1997, pp. 197 and 226.

Understanding, the 'intellectual fire' or 'mind', the source of all harmonies, of whatever kind they may be."][132]

"The epilogue in the Sun" was called "conjectural" by Kepler. He was trying to explain his philosophy of considering the Sun as immobile. And yet to us this would seem to be, rather, a chapter about God and the Christian religion. "IN SOLE POSUIT TABERNACULUM" ["In the Sun He has placed His tabernacle"] was the same Psalm of the *Bible* used by Galileo Galilei to defend heliocentricism.[133] Would the Central European Lutheran fare better or worse than the Tuscan Catholic?

We could easily expect an astronomer of the seventeenth century, who was a follower of Copernicus, to move away from the planetary model of Ptolemy. But the event that he continued to take his inspiration from him to continue his programme of research on the music of the spheres should come as a surprise to us. "… hic Ptolemaeus ante 1,500 annos materiam mei lib. V propriam tractare instituerit, si per suam astronomiam potuisset." ["… here [in chapter XV of *APMONIKA*] Ptolemy, 1,500 years ago, would have set about handling the subject matter belonging to my Book V, if he had been able to, through his own astronomy."][134]

That is to say, Ptolemy would already have started to explain the movements of the planets through music, although he would not have succeeded in this. Kepler, on the contrary, could now do so, because he had discovered the cause of movements in the Sun, and the reason for consonances between notes, no longer in the numbers of Pythagoreans, but in regular polygons.

The falsity of Ptolemaic astronomy was revealed in the "… imperfectio Symbolismorum illorum …" ["… imperfection of his symbolisms …"]. "… contra Ptolemaeum disputo, symbolismos scilicet illos potissima parte non esse necessarios, nec causales, nec naturales, sed poëticos potius & oratorios." ["… I disagree with Ptolemy, arguing of course that those symbolic associations are for the most part not necessary, because neither causal, nor natural, but rather poetic and rhetorical …"]. "… ostendi Ptolemaeum luxuriare exercitio comparandi poëtico vel oratorio." ["… I have shown Ptolemy exaggerates in using poetic or rhetorical comparisons."][135]

Kepler sustained that he had demonstrated the "poetic" conjectures about the music of the spheres handed down by the Pythagoreans and Plato, and taken up in the end by Ptolemy. How seriously our imperial mathematician took the astronomer-astrologer-musicologist from Alexandria can be even more clearly understood from his project of translating his works, commenting on them, and completing them

[132] Kepler 1619 [1969], V, p. 247; 1997, p. 496.

[133] Kepler 1619 [1969], V, p. 245; emphasis in the text. Kepler 1955, v. XVII, p. 326. Tangherlini 1974, p. 166. See above 8.2.

[134] Kepler 1619 [1969], V, p. 251; 1997, p. 503.

[135] Kepler 1619 [1969], V, pp. 249, 250, 251; 1997, pp. 500–502.

analytically, although this was only started, and not concluded. Unsatisfied with the current translation, he borrowed the Greek original.[136]

Among the natural philosophers of the seventeenth century, Kepler was not the only one who showed faith and interest in the music of the heavenly spheres; for example, the Jesuit Athanasius Kircher (1602–1680) has left us his *Musurgia Universalis*.[137] But he, like Ptolemy, did not meet with the unconditional approval of Kepler. His explicit criticism was reserved for the doctor from Oxford, Robert Fludd (1574–1637). This English scholar dealt also with practical music, whereas the Central European scientist studied above all the theoretical aspects. The former dressed himself in a certain obscure symbolism, typical of chemists and alchemists, and of the hermetic tradition of Paracelsus, whereas on the contrary, the latter – Kepler repeated here – sought the causes. Our mathematician, in the pay (so to speak) of the Holy Roman Empire (the Habsburgs), investigated, through the soul, the "noble theme" that the whole cosmos could be found in man, and that the Earth was a living organism. The English doctor dressed up the correspondences between Microcosm and Macrocosm with a formal rhetoric. Like the ancients, he continued to construct his cosmic music on the abstract numbers of the Pythagorean sects, and subsequent variants, down to the Rosicrucian sects. Their music of the spheres sounded different, both in style and in their results, and for a few years even a written dispute developed between the two.[138]

Kepler also mentioned Cicero's *Somnium Scipionis* with reference to the music of the heavenly spheres.[139] But now, he had substituted his dreams on this subject with his own mathematical proofs. He dreamt, instead, about people living on the Moon, as he wrote in his *Somnium seu opus posthumum de astronomia lunari [Dream, or posthumous work about lunar astronomy]*. He imagined that he was observing from there the movements of the Earth and the Sun, in support of Copernicanism. "This is the gist of all the *Somnium*: proving the movement of the Earth, or confuting the contrary arguments, based on subjective perception."[140] Experience by itself, and crude observation by itself were, for Kepler, "ignorant experience". Science would be born when this ignorance is allowed to die, and is, rather, the fruit of theoretical imagination.[141] His *Somnium* is thus to be compared with the *Discours de la méthode* by Descartes, and with the numerous subsequent scientific methods emphatically flaunted by a host of philosophers and science historians. All of his precious astronomical observations were not sufficient for Tycho Brahe, who,

[136]Kepler 1619 [1969], V, pp. 249–251; 1997, pp. 130–131, 190, 390–391 and 499–503. Stephenson 1994, pp. 37–38 and 98–117.

[137]Kircher 1650.

[138]Kepler 1619 [1969], V, pp. 251–254; 1997, pp. 503–508. Baumgardt 1951, pp. 181–182. Caspar 1993, pp. 290–293. Pauli 1955. Ammann 1967.

[139]Kepler 1619 [1969], IV, p. 106; 1997, p. 284.

[140]Kepler 1984, p. 117; cf. 137.

[141]Kepler 1984, pp. 67 and 75.

according to the imperial mathematician, did not possess sufficient imagination to detach himself from the ancient planetary systems.

The *Somnium* was, at the same time, Kepler's first and his last work. His first, because he penned the most ancient version of it in 1593, when he enrolled in the University of Tübingen, his last, because he continued to comment on it for the rest of his life, in 1609, 1611, and above all after 1620, 1621, 1622–1624, 1627, until his death in 1630, while he was still trying to have it printed. But it was only published posthumously by his brother-in-law and his son.[142] Yesterday (like today), when it was a risk to express one's opinions freely, it was better to take refuge in satire.[143] This does not make this booklet insignificant or misleading, but on the contrary, it makes it a priceless mine of information to arrive at the deepest convictions of the imperial mathematician. And actually, the clear defence of the Copernican system was immediately censured by all the Protestant professors of Tübingen (except one), who were as liberal and tolerant as their Catholic colleagues (and as those of today).

Thus, the Moon greatly attracted Kepler, who felt that its influence was exerted over the sea, over every body and every other thing, just as he believed that the Sun did, as well. "I define gravity as a capacity for mutual attraction, like the attraction of magnets. But the force of this attraction is greater between bodies that are close together than between those that are far away".[144] He imagined, by analogy with magnetic attraction, a universal gravitation, to which Isaac Newton was to give a quantitative formulation. The greatest force was exerted when the heavenly bodies were lined up in conjunction, 'with syzygies' as astronomers say. Our German natural philosopher, who wrote in Latin, used the term "copulis" ["with copulations"] for this phenomenon.[145] As he had elsewhere assigned sexual attributes to the planets, we may suspect that he was alluding here to more common relationships and unions of a completely different kind, those that the word has maintained in English.

Anyway, he dreamt. He dreamt that the inhabitants of the Moon called their country Levania, as in the Hebrew language. Considering the Earth from there, a planet four times bigger, which revolved rapidly on itself, he gave it the name Volva, from the Latin *volvo*, [to rotate]. But once again, our imperial mathematician chose, for his satire, a word that presented a salacious 'double entendre', also in Latin, which it is not necessary for me to spell out. We all know that in our dreams, we can take the liberty of behaving in ways that would be repressed if we were awake, or permitted only in analysis.

Our Lutheran natural philosopher considered the "… vow of virginity to be contrary to natural inclinations …" and if one would be forced to observe it, he should be cured with a "… well-tried medicament …: continuous, intense, ardent

[142] Kepler 1984, pp. 15–22. Caspar 1993, pp. 351–353.

[143] Kepler 1984, pp. 22, 70, 97, 99, 107, 109.

[144] Kepler 1984, pp. 57, 102–105, 161.

[145] Kepler 1984, p. 161.

speculation."[146] In his allegory, what was Kepler alluding to? To his own impossible dreams? To the rules imposed by the Catholic Church on its religious orders? To that joyful coupling of planets in their dance around the Sun, sustained by the Copernican system, but included among the mortal sins by the geocentric system?

In general by him, the Muses and the sciences were considered "… gentle and innocuous …", above all astronomy. But then Kepler specified, in order to avoid any accusation of lying, their negative aspects, too. "… medicine also teaches poisons, …, metaphysics, …, disturbs the dogmas of Catholicism with too many inappropriate subtleties, …, astrology sponsors superstition; optics deceives; music serves love; … arithmetic serves avarice."[147] He did not approve, therefore of all music; rules governing it should exclude the kind capable of influencing illicit passions. And here we are bound to hear echoes of Plato, Guido D'Arezzo, or Cardano, criticising music that was capable of corrupting young people.[148]

With the assistance of his witch-mother and the Muse Urania anyway, our astronomer-astrologer travelled to the Moon, and was able to observe his beautiful Volva (*alias* the Earth) which moved joyfully in various ways, while Levania (*alias* the Moon) would appear to be immobile. And then all immobilities would be apparent and deceptive. Backed up by an endless classical culture, the ability shown by Kepler in speculating on the phenomena of the Moon, and how they might be seen from there, can but leave us wondering in admiration. However, it is impossible not to notice that his argument cut both ways, and could be overturned to argue against heliocentricity. Of course, it would have been possible to dream of travelling to the Sun, and observing from there that the Earth was moving. However, why should the inhabitants of the Sun enjoy a special status called reality and truth, (the Sun stands still while, instead, the Earth moves), while the inhabitants of the Earth should suffer from the deception of appearances (the Sun moves and the Earth remains motionless)? Kepler did not go so far in his hot speculations as to distribute a certain value equally to the various points of view. For him, the viewpoint from the Sun could not be appearance, like the others, whether they were based on the Earth or on the Moon, because there, God had placed His tabernacle, as we have seen above. It was undoubtedly a question of flattery, in order to obtain help in collecting the eternal credits from the Court of Prague, that in 1618, Kepler even placed the councillor Matthias Wackher on the Sun. "If I were a pagan, ignorant of Christian teachings, I would say that on the Sun – the excellent globe of the universe – there would be a refuge open to all Wackher [valorous] souls".[149]

Albert Einstein (1879–1955), born three centuries after our imperial mathematician in the same region of southern Germany, he too dreaming of moving amid improbable reference systems, was to speculate about similar problems, but he was to invent the absolutes of Relativity.

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[146] Kepler 1984, p. 109.

[147] Kepler 1984, p. 83.

[148] See above, Volume I, Sect. 2.3, 6.2, 6.5.

[149] Baumgardt 1951, pp. 130–131.

Our Lutheran natural philosopher was not successful with his Levanias and his Volvas, which continued to mean something else, at least in neo-Latin countries. On the contrary, the term "satellite", introduced by him, was adopted with its current meaning, like the word "focus" for the point of the ellipse occupied by the Sun.[150]

Strena seu de Nive Sexangula [Gift, or on hexagonal snowflakes] was a booklet in which Kepler explained why snowflakes exhibited forms with a hexagonal symmetry. Here, he was looking for geometrical archetypes in the hexagonal cells of bees, in pentagonal flowers, and in the rhombohedral berries of the pomegranate. "Geometria est archetypus pulchritudinis mundi" ["Geometry is the archetype of beauty for the world"].[151] Our imperial mathematician distinguished the numerical symbols of the kabbala from the geometrical archetypes, in a letter of 1608 written to Joachim Tanckius. With the former he might play, in the latter he firmly believed, searching for their effects on phenomena. "But the geometry of [astrological] aspects is an objective cause, which impresses a certain impetus on the underground archaeum, from which all the phenomena mentioned in general derive; at times one, and at times another, depending on the circumstances."[152]

"... ne in Nive quidem hanc ordinatam figuram temere existere credo." ["... I do not believe that this orderly figure may exist by chance, in snow too"]. He connected the order impressed, even on the snow, with the soul, the image of the Creator. But in living creatures, order would derive from a purpose, which eliminates chance, remaining the reason. What could the purpose of snow be? "Respondeo, Rationem formatricem non tantum agere propter finem sed etiam propter ornatum, nec solum tendere ad corpora naturalia efficienda, sed etiam solere ludere in fluxis, quod multis fossilium exemplis patet. Quorum ego universorum rationem à ludicro (dum dicimus Naturam ludere) ad hanc seriam intentionem transfero: quod puto, Calorem qui hactenus tutabatur Materiam; ubi à circumstanti frigore vincitur: ..." ["I answer that the creative reason does not act only for a purpose, but also to embellish; it does not tend only to produce natural bodies, but it also enjoys itself, flickering lightly, as we can see in many examples of minerals. The reason for which I transfer in general from a joke (we say then, that nature plays its jokes) to this serious intention: as I believe that the heat which so far protected matter is overcome by the surrounding ice: ..."].

For snowflakes, our German natural philosopher ended up by discarding the Platonic octahedron, since it was a solid in space, and fell back, in the end,

[150] Kepler 1984, pp. 160 and 201.

[151] A scattered sentence between the *Mysterium Cosmographicum* and *De Stella Nova*, quoted by Robert Halleaux in Kepler 1975, p. 27.

[152] Kepler 1954, v. XVI, n. 493, pp. 154–165. Here he already wrote down some of those elements referring to music and sexual generation, in accordance with Fibonacci's numbers, seen above in the *Harmonices mundi* .... As regards the geometrical archetypes in God, in creation and in nature, see also Kepler's letter to Christoph Heydon of 1605 . Kepler 1951, v. XV, n. 357, p. 235. Quotations also from Halleaux in the introduction to Kepler 1975, pp. 26–29.

on hexagonal plane figures. Thanks to the "… Anima formatrix …" ["forming soul"], "… pugna calidi vaporis et frigidi aeris in planitie existit …" ["… a struggle exists between hot steam and icy air in the plane …"]. But why would forms with a hexagonal symmetry derive from such conflicts, which were more Heraclitean than platonic? After gradually eliminating other hypotheses, seeing that the struggle between opposites was to take place on a plane, Kepler reflected on the geometrical plane figures, concluding in favour of the hexagon. "… formatrix facultas sexangulum eligat, nulla materiae spaciorumque necessitate coacta, sed solum decentia hac invitata, quod alias sexangulum struat planitiem excluso vacuo, sitque (ex iis figuris, quae idem possunt) circuli simillima." ["… let the forming faculty choose the hexagon, not constrained by any necessity of matter or of space, but only attracted by this convenience: that furthermore, the hexagon builds the plane, excluding empty space, and that (among those figures that can do this) it is the most similar to the circle".][153]

The booklet clearly confirms our imperial mathematician's faith in geometry, as the model of the soul created by God for all things, living and not living. About 10 years before the *Harmonices* …, he already presented here some of the themes that were to be developed there. Among these, we may recall the search for harmony, in this case spatial symmetry, the properties of figures in constructing the plane and space, the relationship between the divine proportion (the golden section), Fibonacci's succession, and the pentagon, as an expression of the biological generation among plants and animals.

This brief essay allows us to glimpse Kepler's inventive process, as marked by deeply rooted themes, which returned in continuous variations and confirmations, until they received complete, satisfactory expression. He thus appears to be a tenacious scholar, who did not lose sight of the purpose he had set himself until he had reached it. He also liked joking and enjoying himself. That Christmas present for a close, and important, friend of his in Prague, was presented as a "nihil" [nothing], not only as insignificant as a snowflake, but also as an allusion to another short poem, appreciated by the addressee, the title of which was, appropriately, *Nihil*. The jokes that recurred in the text, between one who did "nothing at all" and such an illustrious character of the imperial court, found learned variants, and puns: between the Latin *nix* [snow] and the German *nichts* [nothing]. Perhaps this also concealed a satire against the void of nothing and the atoms, seeing that he also quoted the famous saying "nature abhors a void" of Aristotle. Even this "nothing" thus appeared to be full of valuable things, all the same. And there was bound to be, on our Volva, the story about a woman who was made pregnant by a snowflake, sufficiently lively, obviously, to perform the task. And after the initial "… lascivi Passeris lusum argutissimum simul et venustissimum ..", ["… penetrating, and at the same time, delightful amusement of the lascivious cock …"], the reader may imagine by himself whatever he pleases.

[153] Kepler 1941, IV, pp. 275, 278, 279; Kepler 1975, pp. 73–74, 79–80.

While he was a fluent writer on many subjects – somebody counted a total of 86 works written by him – the style used by Kepler is characteristic, and different from that used by other natural philosophers, and above all by future modern scientists. In connection with his own inventions, he also told us about the historical circumstances considered pertinent to the subjects presented. Thus, in his books about science, he revealed here and there some episodes of his life, which had been a particularly dramatic, adventurous one, not at all simple or comfortable.

Kepler began his *Dream* with the conflict between the emperor Rudolph II of Habsburg (1552–1612) and his brother, Matthias. "In the theses [his first dissertation on the Moon, of 1593], I referred this to politics, seeing that public affairs are exposed to frequent large-scale changes, and private affairs often flourish." Through horoscopes, he tried to justify the reign of Rudolph II. But when the latter lost power, and had to abdicate in favour of Matthias, Kepler adapted his writings, practising self-censure.[154]

He informed us about the disputes sustained at Tübingen regarding Copernicanism and the *Bible*. Addressing the Jesuit Paul Guldin (1577–1643) in the geographical appendix to the *Dream*, Kepler wrote of a "… turbulent epoch, in which the whole Court is troubled by thoughts of war." With the death of Matthias in 1619, now Ferdinand II had become Emperor. In 1623, he lived in Vienna, where Guldin taught mathematics.[155] The war had started in 1618, with the repression of Bohemian Protestants, and the victories of the Catholic Counter-Reformation; after various phases, it was to come to an end only in 1648, thus deserving to be called the Thirty Years War.

In Prague, he had listened to the singing of the Muslim muezzin in the retinue of the Turkish ambassador, and compared this with the Easter hymns of the Christian liturgy. He was forced to exclude the possibility of finding harmony in similar intervals "… insolitis, concisis, abhorrentibus …" ["… unusual, truncated, abhorrent …"]. "Nihil dicemus de stridulo illo more canendi quo solent uti Turcae & Ungari pro classico suo: brutorum potius animantum voces inconditas, quam humanam Naturam imitari." ["We shall say nothing of that grating style of song which the Turks and Hungarians customarily use as their signal for battle: imitating the uncouth voices of brute beasts rather than human nature."][156] The stave of a warlike Turkish melody was contrasted with that of a devout "Victima Paschali laudes …". In that period, the vast Ottoman Turkish empire also included Budapest and all Hungary, and was drawing dangerously close to Vienna. Our German scholar undoubtedly hoped that, with goodwill, some consonant agreement might in the end be found between Catholics and Protestants, but he believed that this was not possible with Islam.

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[154]Kepler 1984, pp. 167–168. Caspar 1993, p. 204. As regards how Kepler wrote his books, see Koestler 1991, p. 309.

[155]Kepler 1984, pp. 180–181 and 187.

[156]Kepler 1619 [1969], III, p. 61; 1997, p. 217.

Also in his treatises packed with axioms, propositions, theorems and corollaries, like the *Harmonices mundi* . . . , this famous astronomer and mathematician relates to us the circumstances of his inventions. "22 years . . . " passed between the *Mysterium cosmographicum [Mystery of the cosmos]* of 1596 and the *Harmonices* . . . , which was finished in 1618 and published in 1619. He gave the name of the person who had passed him the relative book by Ptolemy, in which, however, he did not find ". . . the true causes of the harmonies."[157] The stages towards the final goal were described in detail. ". . . 16 years ago . . . "; ". . . my appetite was particularly intensified and my purpose stimulated by the reading of the *Harmonicorum* . . . [*Harmony*] of Ptolemy . . . ". "Now, eighteen months after the first light, 3 months after the true day, but very few days after the pure Sun of that most wonderful study began to shine, nothing restrains me." ". . . and if you want the exact moment in time, it was conceived mentally on the 8th March in this year, 1618, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the 15th of May and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labour of seventeen years on the observations of Brahe and the present study, which conspired together, that at first, I believed I was dreaming, . . . ".[158]

He had received Ptolemy's book in 1607, and thus Kepler was assigning the dates of late 1616 and March 8th, 1618 to his other attempts to compose the harmony of the planets. At last, on May 15th, he "dreamt up" his third planetary law. He related the story of his discoveries in order to explain better why in the end he had abandoned Platonic solids for the harmony of music. Starting from Copernicus in 1594, he had searched for the suitable proportions, to establish that the distances between the planets were not equal, nor were they ". . . unequally unequal . . . ". Subsequently, he had first thought of plane figures, and then Platonic solids.[159] His interest in Ptolemy's theory of music was by no means ephemeral, and he translated a part of it from Greek into Latin in 1608. However, ". . . my change of abode, combined with a great many troubles . . . " prevented him from finishing this work. Nevertheless, his intention was to include at least something about Ptolemy as an appendix to the *Harmonices mundi* . . . : to demonstrate ". . . the imperfection of his symbolism . . . [derived from the] . . . falsity of the basic principles of Ptolemaic astronomy . . . ". Again, however, other problems had cropped up, and ". . . meanwhile the neighbouring Bohemian war broke out, . . . ". Roads were closed, printers were recruited as soldiers, *und so weiter*. Anyway, even without Ptolemy, the book was terminated on May 17th/27th, 1618 and published on February 9th/19th, 1619. Why the double dates, 17th–27th and 9th–19th? Because in that period, Europe was divided by wars of religion, and now had two calendars. Catholics, who followed the new Gregorian calendar, counted 10 days more, while

[157] Kepler 1997, pp. 163–164 and xxiii.

[158] Kepler 1619 [1969], V, p. 189 . Kepler 1997, pp. 389–391, 411. Caspar 1940, pp. 462–484. Cf. Caspar 1993, pp. 91–96 and 264–290.

[159] Kepler 1997, pp. 489 and xvi–xxiv.

the others, who had remained faithful to the Julian calendar, 10 days less. In contrast with theologians and his colleagues at Tübingen, our Lutheran astronomer preferred the new papal calendar.[160]

Thus, here and there, wars broke out. Not only with the Turks, but also other melodies were defined as warlike.[161] Signs of war were visible, above all, in the dedication to "… Iacobum Magnae Britanniae" [James I Stuart (1566–1625)], king of Great Britain, who was the son of Mary Stuart, a Catholic, but had been brought up as a Calvinist Protestant. His daughter had married the Prince of the Palatinate (Germany), Frederick, the champion of the Protestants in the fight against the Catholic Habsburg emperors. But rather than imprudently taking sides with the defenders of his own Protestant faith, our imperial mathematician always "dreamt" of harmony, both in the Heavens and on Earth. In spite of the conflicts of which he had direct personal experience among the Habsburgs, Kepler foresaw that there would soon be peace. "… ut intelligeretur ex cursu non inperturbato Pacis ornamentorum per has provincias, famam intestini belli sinistram, cum ipsa re procul dubio brevi extinctum iri: Dissonantiamque hanc paulo duriorem, ut in Pathetica Melodia, jam jamque in suavem Clausulam desituram." ["… he should understand, from the non-imperturbed progress of the preparations of peace throughout these provinces, that the sinister rumour of civil war would undoubtedly soon be extinguished together with its reality, and that this slightly too harsh discord, as in an emotional melody, is on the very point of resolution into a pleasing cadence."][162]

While he had hoped, for 20 years, to discover harmony in the movement of the planets, until he finally succeeded, he was not allowed by God, or by a cruel destiny, or by history, to live serenely through a period of peace. And yet, as dissonances make consonances more pleasant in music, in the same way, divine providence was to make use of dissonances in human affairs. The Christian king, James I, was admired by Kepler, because he succeeded in reconciling two hostile kingdoms like England and Scotland. On the contrary, the life of our natural philosopher appeared to be increasingly dogged by dissonances. "Vellem hic, Dissonantiam publicam, Vocum trifariam obstrepentium, paulo mihi mitiorem esse, ut audiri publice ex animi mei sententia possem: …" ["I should wish, here, the three-part public dissonance of clamouring voices to show me a little more moderation, so that I could make the results of my own thinking publicly heard, …"]. If he did not succeed in surpassing that public screaming, then his voice, too, would simply increase "… molestias absurdi concentus …" ["… the annoyance of a false chorus …"] "… proh dolor,

[160]Kepler 1997, pp. 498–500. Caspar 1993, pp. 228–232. Baumgardt 1951, pp. 27–28 and 118. In a letter dated August 4th, 1619, however, Kepler wrote that the *Harmonices mundi* … had not been published yet. Baumgardt 1951, p. 141. As regards the extent of Kepler's studies on Ptolemy, see Stephenson 1994, pp. 98–117.

[161]Kepler 1997, pp. 219 and 239.

[162]Kepler 1619 [1969], "Dedicatio" [a–b]. Cf. Kepler 1997, p. 2. Here the translation is different: "non-inperturbato" was changed into "uninterrupted". Caspar 1993, p. 288.

tumere adhuc vulnus decussatum, an malumus sacratiori foeliciorique vocabulo, cruciforme tumere inquam, multiplici labro; & nullo illorum connivente, medicinam hactenus irritam, omnibusque partibus irrisam; …" ["… ah! what sorrow – that criss-cross wound is still swollen, or if you prefer a more sacred and more felicitous word, the cross-shaped wound, is swollen, I say, with its multiple lips; and though none of them winks at it, the medicine has so far been useless, and mocked on all sides; …"]. And yet Kepler still hoped that "… Dissonantia haec diuturna (…) in meram & durabilem Harmoniam terminetur." ["… this enduring dissonance (…) will end in pure and abiding harmony."][163]

Again in 1621, when introducing the second edition of the *Mysterium Cosmographicum*, in the words "… Now that the weapons have been laid down [sic!] …", he still hoped for peace. He believed in it so strongly, that he misunderstood Plato's *Republic*, as if the Greek philosopher had indicated geometry as the way of peace, instead of training young men for war. "(A benevolent fate has made me do this work during the sharp dissonances of the last 2 years) …".[164] If he was led to invoke the mediation of the British king, James I, to confirm "… studium in se Concordiae & Pacis Ecclesiasticae & Politicae …" ["… the zeal for concord and for peace in church and state …"],[165] what dissonances may our devout German imperial mathematician have experienced at his own expense?

At the root lay the religious contrasts that he himself mentions in the three clamouring, rather than singing, voices of the Protestants, the Catholics and the Calvinists. These were to lead to the Thirty Years War. In this context, not at all serene, during the last few years of work on the *Harmonices Mundi* …, as if all this were not enough, a trial also started against his mother, who was accused of witchcraft. Kepler felt Lutheran, and unlike other friends and acquaintances, he would never be converted to Catholicism, in spite of the obvious advantages that he would receive from this. In imperial Catholic Austria, at Graz as well as at Linz, he was persecuted, losing his possessions and his positions, and was forced to emigrate from one place to another.[166]

The Lutheran theologians of Tübingen did not treat him any better either. He had never accepted their Formula of agreement about Christ and the sacraments, and had therefore been excluded from them. "The Jesuits clamour and the Calvinists clamour they are discredited by the Formula of agreement, as regards the article of faith about the person of Christ." Here, our conciliatory Lutheran invoked his own conscience, going so far as to defend, in following the Holy Scriptures and the Church Fathers, those other Christians of a different Church, in opposition to his own pastors. Banished from imperial Catholic Graz, he was never to obtain the desired, and requested, place at the Lutheran University of Tübingen, but was forced, against his will, to become the assistant of Tycho Brahe at Prague. Subsequently, at

[163]Kepler 1997, pp. 3–5. The idea is also found in Leibniz; See Part II, Sect. 10.1.

[164]Baumgardt 1951, pp. 128–129. Caspar 1993, p. 306.

[165]Kepler 1619 [1969] "Dedicatio" [c–e]. Cf. Kepler 1997, pp. 3–5.

[166]Caspar 1993, passim. Baumgardt 1951, pp. 53–57 and 172–174.

the death of the aristocratic Dane in 1601, he became the Mathematician of His Holy, Christian, Imperial Majesty, thanks to the recommendation of his Catholic friend, Georg Herwart von Hohenburg (1553–1622).[167] But this was to prove to be a position that gave more prestige than a good, regular salary. Kepler was forced to attend to the credits that he had accumulated over the years, for the rest of his life. They were not even paid to his heirs after his death.[168]

In brief, Kepler was a heretic for everybody, and for this reason, was subject to continual persecution, and without many friends. Certain Lutherans went so far as to sustain that the main reasons for the mental problems from which his first wife suffered were the religious ideas of our astronomer.[169] However, he also accepted "… the doctrine of Catholicism, not only with my heart, but also with my head …". "… I was introduced to the Catholic Church by my parents with the baptismal water … I remain in the Catholic Church, but by rejecting all that I do not acknowledge as Apostolic … I cling to the Catholic Church, even if she rages and beats …".[170] In 1619, prudently, in order to avoid the probable Catholic censure against Copernicanism, he suggested that Italian booksellers should sell his *Harmonices Mundi* … only to high-ranking prelates, to mathematicians and to metaphysicians. At Graz, in 1623, copies of one of his almanacs had even been burnt in public. At Linz, in 1625, his library was sealed off.[171]

He still wrote his books even more frequently in Latin, though he felt that he was German. Every now and then, he let slip a few words in his native language: "… *das Klavier* [in Gothic], & instrumenti genus Clavichordium …" ["… *das Klavier*, and the clavichord kind of instrument …"]. Elsewhere, when using the three means, arithmetic, geometric and harmonic, to discuss the best form of government, and to give a correct retribution, he displayed German habits, writing of "… nos Germanos …" ["… we Germans …"].[172] In instructions specially for Italian booksellers, he said that he had written "… as a German, after the German fashion, and with German frankness." He preferred to live in German-speaking countries, and refused a teaching post in Italy at Bologna, and a probable position in England. To be more precise, he felt at home only in Southern Germany, and postponed for some time a position as mathematician-astrologer at the University of Rostock, situated on the Baltic.[173]

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[167]Letter to Mästlin dated December 12th/22nd, 1616; Kepler 1955, v. XVII, pp. 203–204. Caspar 1993, pp. 49–52, 77–118, 122, 188–189, 258–264, 359–360, … and passim. Baumgardt 1951, pp. 58–65, 101, 106–107.

[168]Baumgardt 1951, pp. 87, 90, 96, 129–131, 138, 163, 171, 174, 179, 189, 192–195. Caspar 1993, pp. 104, 109, 122–123, 141–142, 156–159, 188, 225, 356–357, 362–363, … and passim.

[169]Baumgardt 1951, pp. 112–113.

[170]Baumgardt 1951, pp. 133, 172–173. Cf. Caspar 1993, pp. 112–113, 148, 337, … *passim*.

[171]Baumgardt 1951, pp. 133–136 and 150. Caspar 1993, pp. 299, 307, 317, 335–337.

[172]Kepler 1619 [1969], III, pp. 51 and 98; 1997, 202, 272.

[173]Baumgardt 1951, pp. 133, 144–148, 189. Caspar 1993, pp. 220, 236 and 347.

From his "Provocatio" [challenge] launched to all rival scholars, we may imagine Kepler's character, too. He invited mathematicians and philosophers to come forward, and try to modify his harmonies, finding others that were better, and closer to astronomy, if they could. For his part, he put it in writing that he would change "... illa, quae prioribus diebus oscitanti cura, vel properanti ardore praepostere concepta deprehendere potui." ["... anything which I could discover which was incorrectly conceived in the preceding days, if my attention yawned or my enthusiasm was hasty."][174]

Having arrived at the last section of the chapter, $49 = 7 \times 7$, "Epiphonema" [Envoi], he took his leave as if it were the Sabbath, the seventh day when God had terminated his creation, quoting the relative passage of the *Bible*. We cannot underestimate the religious faith of the imperial mathematician and astronomer, which not only marked his life, but which he readily exhibited in his works of astronomy. Now he presented himself as an imitator of God, in search of perfection in his works, just as He had filled the world with harmony. He addressed all those who, in the spirit of Christ, rejected every kind of sectarianism, rivalry, envy, and anger, in order to restore harmony among Christians. He prayed that God would keep the dissonances of discord away.[175] That part of the infinite, which his limited mind had succeeded in grasping, he had conveyed to those who had read his demonstrations. But he himself had behaved like a "... vermiculo, in volutabro peccatorum nato & innutrito ..." ["... miserable worm, born and nourished in a slough of sins, ..."]. He only hoped that all his demonstrations would serve to extol His glory, and to save souls, instead of misleading them.[176]

So much did it mean for him to be considered a devout religious scholar, that in the end, he even strained the dating of Proclus, so often quoted by him (fifth century, backdated to the fourth century). He thus succeeded in interpreting him in a Christian sense, in spite of the event that this Greek neo-Platonic philosopher had written against the new form of worship of his period, which was by now consolidated in Europe. But the forced interpretations and the historical errors only underline Kepler's profound intention to make his Christian faith compatible with his mathematical and natural philosophy: the truth of both would be to be sought by transcending the sensible world. Thus his *Harmonices mundi libri quinque* terminated with a hymn to religious and scientific transcendence, which was a paraphrase of a psalm of David, suitably modified. "... in saecula saecolorum. AMEN." ["... for ever and ever. AMEN"].[177]

Kepler knew very well that if he tried to reconcile the *Bible* and Copernicanism, he would run serious risks, and he would be criticised by theologians, and by both Protestant and Catholic colleagues. Thus he had already written to Michael Mästlin in 1598: "... we shall imitate the Pythagoreans also in their customs. If someone

[174] Kepler 1619 [1969], V, p. 241; 1997, p. 488. Caspar 1993, pp. 180–181 and 323.

[175] Kepler 1997, p. 452.

[176] Kepler 1619 [1969], V, p. 243; 1997, p. 491.

[177] Kepler 1619 [1969], V, pp. 245–248; 1997, pp. 493–498.

asks us for our opinion in private, then we wish to analyse our theory clearly for him. In public, though, we wish to be silent. … Let us behave, therefore, in questions of astronomy, in such a way as to maintain support for it, and avoid dying of hunger."[178]

From a character like this, historians, philosophers, mathematicians, physicists, astronomers and popular writers of our great epoch have considered themselves authorised to take whatever they needed, and to ignore the rest. Among the very few who have tried to depict a complete Kepler in the round, considering for him as significant harmony, religion, astrology, wars and the other dramatic circumstances of his life, the palm must be awarded, above all, to Max Caspar.[179] We refer to him, for brevity, so that our *Musical history* … (already too extensive) will not become an interminable history which is the prelude to a volume about the imperial astronomer. To Caspar, we refer readers who are interested, not in an apology, but in understanding without excluding at all the possibility of laughing or weeping. Here, we underline only those interpretations which are too distant from, or too close to, my own. I shall behave in the same way with the secondary literature, which fluctuates between metaphysical rational reconstructions and the novel.[180] And there is even an opera dedicated to him.[181] But, surprisingly, we are unable to quote a single film of which he is the protagonist, in spite of all those events that might lend themselves to a successful script.

Galileo Galilei's trials are well-known, but also Johannes Kepler may be said to have suffered the consequences of a trial, which indirectly involved his own good name, to which he was forced to dedicate all his energies. The person on trial was his mother, Katharina Guldenmann (1547–1622), who was accused of witchcraft. "The prisoner appears, alas [sic!], with the support of her gentleman son, Johannes Kepler, the mathematician." Our mathematician had to defend her, never doubting that the accusation of witchcraft might not be legitimate. Besides, he had also made her a character of his *Somnium* … on the Moon. We can well imagine why, despite the acquittal finally obtained for her with great effort by her son, she died only a few months after the conclusion of the trial.[182]

Many letters written by Kepler, totalling about 400, are extant, as well as 700 received by him. Through these, the various events have been reconstructed.[183] They reinforce the impression already received from the reading of his most famous works. Curiously, there is a singular coincidence between the fortunes that befell our imperial mathematician, who was able to use Tycho's observations to calculate the orbit (the first elliptic one) of Mars, the god of war, and the dramatic events

[178]Caspar 1993, pp. 68–69. Kepler 1984, pp. 68–69.

[179]Caspar 1993. Cf. Gingerich 1970.

[180]For example: Banville 1993; Koestler 1991; Lombardi 2000.

[181]Hindemith 1957/1961.

[182]In that area, and in that period, 38 poor women were sentenced to death for similar accusations. Caspar 1993, pp. 240–258. Baumgardt 1951, pp. 156–162. Banville 1993, pp. 154–156. Lombardi 2000, pp. 59–66.

[183]Baumgardt 1951.

that afflicted him, as a result of the numerous wars in the Europe of that period. His father was a mercenary soldier, and for this reason was absent from home. In the end, our mathematician was given hospitality at Sagan by the Duke of Friedland [sic! Land of peace], the famous General Wallenstein (1583–1634), who was fighting the Thirty Years War, with success, on behalf of the Habsburgs. This did not make it any easier for him to receive the payment of his salary, or to carry out his studies in peace.[184] But perhaps it fortified his obstinate efforts to find a harmony that was impossible on Earth, which could only be imagined in the Heavens. Kepler passed away at Regensburg, during his umpteenth vain attempt to find some money and a safe haven. He was buried outside the city walls, because he was a Protestant, and the war, which merrily continued, soon destroyed his tomb. As an epitaph, he had it written that only his body was there in the dark earth, whereas "Mens coelestis erat" ["My soul was from heaven"]. The follies of men could no longer affect him. However, he was persecuted until the very end. Two Lutheran journalists succeeded, all the same, in taunting him, both in an article about his "vain" spirit of peace, and by expressing doubts about his religious faith.[185] Here, too, and not only with his new astronomy, Kepler was opening up the way to our great modern ages.

From certain indications here and there, we may begin to suspect a little-disguised inclination of the imperial mathematician towards that sex which is, in general, the more beautiful, the stronger, and the more important for historical evolution. His two wives both complicated and gladdened his existence. More than the first one, made happy his life the second one, much younger than him, who he had married on the day of an eclipse of the Moon. Numerous daughters and sons were the objects of his affectionate thoughts. The count arrived at 13, but at least half of these were destined for a short life. As a student, he had played female parts on the stage at University. These troubled him, without doubt, but for ever he was to love the more "… polished, noble, dear, timid and so easily frightened, tender sex." Through horoscopes, he gives us information about his first experiences of love, which we cannot define as outstanding.[186] In this connection, the natural philosophers of his period displayed different attitudes and customs. In general, they came from the monastery tradition, and had maintained these habits.[187] Also in this respect, Kepler appears to be a heretic.

[184]Caspar 1993, pp. 34–36 and 338–350. Baumgardt 1951, pp. 174 and 179.

[185]Caspar 1993, pp. 355–361. Baumgardt 1951, pp. 192–197.

[186]Baumgardt 1951, pp. 20, 24, 94, 112–117. Caspar 1993, p. 43. Walker 1967, pp. 243–245. Koestler 1991, pp. 231 and 236.

[187]Noble 1994. In this book, which deals with the absence of female figures in the development of Western modern sciences, Kepler was not even mentioned, perhaps because he appeared to be such an exception for the author's thesis.

It has been observed that none of the scholars who were his contemporaries gave much credit to Kepler's inventions.[188] As we shall see, neither did Newton, although he was a debtor to him for some essential elements of his gravitation.[189]

In the *Somnium*, our inventor of the three planetary laws referred to a booklet entitled *Ignatius His Conclave*, a satire against Ignatius of Loyola (1491–1556), the founder of the Jesuits.[190] By force of circumstances, it was published anonymously in 1611, though its author was, in fact, John Donne (1572–1631).[191] This famous English poet and mystic had passed from a family with Catholic traditions, actually related to Thomas More (1478–1535), the adversary that Henry VIII caused to be executed, to the Anglican Church. He became a *protégé* of King James I of England, to whom we know that Kepler dedicated the *Harmonices mundi* .... Donne was afterwards appointed chaplain in the entourage of the legation sent to Germany in 1619, with the – unfortunately vain – aim of making peace. It might be possible that on this occasion, this English poet and preacher may have met our German astronomer: the latter, in these years, was wandering from one town to another, worried about his mother's trial, in search of peace, of money, and of printing works where he could publish his books. But it appears to be unlikely, seeing that the only place where they could by chance have met was Augsburg. Here, Kepler might have arrived in the summer of 1619, to let his book about comets appear, whereas Donne was there between the end of June and July.[192] Or at least, perhaps Donne had his *pamphlet* with him, in which he spoke ironically about the Jesuits of the period. Remembering Giordano Bruno burnt at the stake, even if they were relatively well protected, it would be more prudent to put certain things in writing anonymously, in the form of satire. Thus the humorous, ironic side of Kepler in the *Somnium* may have encountered the same element in Donne. In the *Ignatius* ..., he joked about how the Jesuits were treating Copernicus. This must have naturally attracted the attention of the astronomer, who was always in search of support, against such powerful adversaries.

The episode should be seen, not only as a curious detail, which is only of interest for learned professors of this subject. Rather, it reveals to what extent Copernicanism had by now arrived beyond the bounds of mathematicians' university lecture-halls, and was discussed practically everywhere by the educated classes: we even find it among the writings of an English churchman. It is clear, then, that the Catholic Church, and others, as well, were worried by this. Again in 1611, the renowned English poet described its decadence, in "An Anatomie of the World".

[188] Baumgardt 1951, pp. 51 and 124. Caspar 1993, pp. 135–138.

[189] Barone 1989.

[190] Kepler 1984, p. 70.

[191] Donne 1969.

[192] Kepler 1963, v. VIII, pp. 135 and 461. Caspar 1993, p. 301. Bald 1970, pp. 338–365.

And new Philosophy cals all in doubt,
The Element of fire is quite put out;
The Sunne is lost, and th'earth, and no mans wit
Can well direct him where to looke for it.

Then, the beautiful spherical proportions of the world had been lost, "... to finde out so many Eccentrique parts [...] For his course is not round ... [...] All their proportion's lame, it sinckes, it swels."

The best parts of beauty, proportions, are dead, like the moral parts with recompenses and penalties. Even pains were out of proportion.

Shee by whose lines proportion should bee
Examin'd, measure of all Symmetree,
Whom had the Ancient seene, who thought soules made
Of Harmony, he would at next have said
That Harmony was shee, and thence infer,
That soules were but Resultances from her,
And did from her into our bodies goe,
As to our eyes, the formes from obiects flow:[193]

Written in the same period as *Ignatius* ..., this poem reveals the same themes dear to Kepler: harmony as the soul of the world, a harmony that was being lost, at least on the Earth. The poem "Loves Alchymie" made reference to the music of the spheres, and as a result, the Ancient mentioned would be Pythagoras. In Donne's "Hymne to God my God, in my sicknesse", the poet takes part directly in the music of spheres at his death, rising into Heaven.[194]

While different in their native tongues and in their professions, were these two characters linked together only by a vague neo-Platonism? Was it only a singular coincidence? Perhaps there we are facing something more to it. The indications that Kepler knew Donne's *Ignatius* ..., and that the latter travelled to Germany at the beginning of the famous War, would deserve greater attention than has been reserved to them in relative literature. The English Ambassador on the continent (above all for Venice), Sir Henry Wotton, who had tempted Kepler to leave his homeland for England, had been an intimate friend of Donne's for many years. As a result, he knew some of the people who were in contact with Kepler directly. Even if they probably never actually met, they were only one step away, to use a modern way off putting it.[195]

In his anti-Jesuit *pamphlet*, anyway, Copernicus was banished to hell by the English churchman, like Lucifer by John Milton, and made to dialogue with the chief of the devils. The Catholic Church had rewarded Ignatius in 1609, placing him among the blessed, that is to say, in paradise, but here Donne turned him into

[193]Donne 2002, pp. 44, 48, 52.

[194]Donne 2002, pp. 124 and 144.

[195]Donne 1969, pp. 101, 129, 139. Bald 1970, passim. Only Nicolson 1940 and Edward Rosen in Kepler 1984, p. 70 have dwelt on Donne, the former in view of literary interests. Cf. Koestler 1991, p. 361. Cf. Gingerich 1970, p. 306.

Satan's gatekeeper. In the style of Dante, famous characters would have wished to sit close to the leader of the devils: Paracelsus, Machiavelli, Pietro Aretino, Columbus, Filippo Neri. Of course, it was a pretext to magnify the diabolical virtues of Ignatius, the Jesuits and the Roman Pope. Donne was doing his job as an Anglican preacher at the time of the Counter-Reformation, writing satires in profusion about the copious partisan theological literature written in that period. He immediately translated the Latin original himself into English; he made fun of, and popularised, religious themes or commonplaces, which were the object of ferocious attacks, and controversies that were not at all peaceful. Every now and then, a king, a prince, a churchman, witches or natural philosophers were assassinated, or put to death, if they said the wrong thing, or were in the wrong place at the wrong time. Controversies of the kind thus appeared to be far from theoretical opinions, but assumed aspects that were decidedly political and social. Two characters who have already come into our story crossed swords over these questions: the Jesuit, Cardinal Roberto Bellarmino (1542–1621), the protagonist of the trials against Galileo Galilei, and the King of England and Scotland, James I, Kepler's dedicatee. Donne's *pamphlet* corresponded to their official written disputes.[196]

In the English poet's satire, Ignatius proved to be such an expert in selecting those that did (or did not) deserve to go to hell, that he was about to steal the position from Lucifer himself. The Jesuits, the worthy heirs of Machiavelli, would have surpassed their master in their ability and dialectic tricks. Consequently, to avoid contrasts, they were relegated to the Moon, where they would organise another Church of their own, and their personal Hell.

The first one who knocked at that fearsome door was Copernicus, but then, in the end, the Jesuit excluded him, on the grounds that he was not sufficiently meritorious. It was true that he had derided Ptolemy, and closed him in a corner, he had set the Earth in movement, thus bringing Lucifer from the bottom of the universe up to Heaven. Would the door remain closed for the one "… totam mundi machinam versanti, & pene novo Creatori, …?" "'… who had turned the whole frame of the world, and … thereby almost a new Creator?'" Had he not displayed enough pride, like Lucifer? But Ignatius of Loyola intervened, in spite of his ignorance on the subject, seeing that he had learnt something from his Jesuits who, like him, had ended up down there. Maybe, as the Earth moved, sins and Hell would disappear? "Omnia ut ante credunt, peragunt. … etiam vera esse possunt, quae asseris.." "Do not men believe? do they not live justly, as they did before? … opinions of yours may very well be true." As is known, neither the devil nor the Jesuits loved the truth too much … In his opposition to the truth and to Copernicus, their Clavius would have deserved the honour, rather, of sitting next to Satan. Furthermore, this Jesuit with the new Gregorian calendar, by which "… Ecclesiae pacem, & civilia negotia egregie perturbavit." "… both the peace of the Church, & Civill businesses have beene egregiously troubled: …" "*Horrende Imperator, Mathematiculus iste*". "… this little *Mathematician* (dread Emperor) …". He perhaps might be allowed in,

[196]Donne 1969, pp. xix–xxix.

only when the Jesuits would succeed in obtaining an appropriate anathema against Copernicanism from the Pope.[197]

Thanks to their good offices, Rome did actually include the famous treatise by Copernicus in the Expurgatory Index in 1616, but all the same, Donne did not modify this page in the subsequent editions. In a letter, he expressed another judgement on Copernicus. "Copernicus in the Mathematiques hath carried earth farther up from the stupid centre; and yet hath not honoured it, because for the necessity of appearances, it hath carried heaven so much higher from it."[198]

"[Ne] fiat iniuria Galilaeo" "In order not to do any wrong to Galileo", nothing was said at the beginning about the planets or the stars. Only at the end did the Tuscan natural philosopher return, so that Lucifer could suggest him to the Pope as a valid helper in transferring the Jesuits up there, as he had observed the Moon from close up. Kepler deserved something different. "… cui (ut ipse de se testatur) a *Tychone Brahe* mortuo, *cura incessit, ne quid novi in coelis se inscio existeret*." "… who (as himselfe testifies of himselfe) *ever since* Tycho Braches *death, hath received it into his care, that no new thing should be done in heaven without his knowledge*." Here Donne was quoting almost word for word what Kepler wrote in 1606, when he referred to a star in the constellation of the Cygnus. In astronomy, he appears to have accepted more readily the compromise that Tycho Brahe did between Ptolemy and Copernicus.[199]

About 10 years later,[200] Sir Henry Wotton was to invite the German mathematician to go to England. But would the ambiguous irony of the poet and militant Anglican preacher, as regards Copernicus, Galileo and Kepler, be enough to convince our Lutheran German that he would find a friendly environment in London? That city would give him a warmer welcome, different from those of Tübingen and Prague, Graz and Linz, or Rome and Bologna? So it was that Kepler stayed at home.[201]

In his *Astronomia Nova*, our imperial mathematician wrote, "After the divine goodness had given us in Tycho Brahe so careful an observer, that from his observations, an error of calculation amounting to eight minutes [of an arc] betrayed itself, it is seemly that we recognize and utilize in a thankful manner this good deed

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[197]Donne 1969, pp. 12–19. Emphasis in the text.

[198]Donne 1969, p. 109.

[199]Donne 1969, pp. 6–7, 80–81, 102, xxx. Emphasis in the text

[200]Caspar 1993, pp. 252 and 256.

[201]The hypothesis that Donne may have drawn his inspiration directly from Kepler's *Somnium* for his *Ignatius*, who likewise ended up on the Moon, seems unlikely to me. Note 8, added by Kepler to the *Somnium* … much later, says, "Fallor an author [sic!, auctor] Satyrae procacis, cui nomen Conclave Ignatianum, exemplar nactus erat [sic!, esset] huius opusculi; pungit enim me nominatim etiam in ipso principio." ["I am not sure whether the author of an impudent satire, the title of which was *Conclave Ignatianum*, may by chance have come across a copy of this leaflet; he teases me by name, right from the beginning of the work."] I prefer to maintain Kepler's doubts. The relative dates and the limited circulation of a manuscript, which was only printed posthumously, make this possibility too complicated and too remote. Cf. Nicolson 1940 and Donne 1969, pp. 159–160.

of God's, that is to say, we should take pains to search out at last the true form of the heavenly motions."[202] These were the values of the intellectual context in which Kepler put together the first laws of his astronomic system. He had always sought the truth, without being satisfied with appearances, to be calculated by means of mathematics, as the prudent Copernicus had done. For him, the truth could not be guaranteed by anyone less than God in person. If His divine providence, through the influence of the stars, had subjected him to such a hard test, giving him such a dramatic life, it was also true that His infinite goodness had led him, by tortuous pathways, to meet Tycho and his treasure of precious measurements.

As we are not at the same level as Kepler before the universe, and we are incapable of revealing the secrets of destiny and chance, we historians must, with due modesty, limit ourselves to relating the attendant circumstances. These were subject to continual unexpected *coups de théâtre*, almost always completely independent of the will of our natural philosopher, who could have equally well been a Protestant theologian at Tübingen, instead of finishing up as an astronomer-astrologer at the imperial Court in Prague. Between one persecution and another, between one city and another, between one journey and another, between one printer and another, between one war and another, between one request for payment and another, between one period of mourning and another, between one marriage and another, between one dwelling-place and another, the known events of his life were so unstable and adventurous, that they would have severely tested the most judicially-minded, that is to say, the most deterministic, astrologer.

In 1605, he confided to his friend and protector, Herwart. "My goal is to show that the heavenly machine is not a kind of divine living being, but similar to a clockwork, insofar as almost all the manifold motions are taken care of by one single absolutely simple magnetic bodily force, … I also show how this physical representation can be presented by calculation and geometrically."[203] In presenting the world as a clockwork machine, Kepler was starting out in a direction which, in time, even if not immediately, was to meet with success among other natural philosophers, perhaps beyond his own intentions. For would it not be easier, as he thought, to attribute a soul to an animated cosmos, like a living organism, than to an inert mechanical device? Among all the other aspects present in our famous German astronomer's work, this one was to be preferentially amplified by many of the subsequent scholars. In this way, European mathematical sciences were to move farther and farther away from that image of the world as a complex organic being, completely inter-connected, which we found predominant in the approaches of the Chinese.

In 1608, Kepler wrote to the astronomer-astrologer David Fabricius (1564–1617), "Natam ais tibi filiam ex Geometria matre? Vidi: pulchra est. At meretrix pessima futura est, abductura quam plurimis meis filiabus ex matre Physica susceptis, maritos suos. […] Me igitur nuda duxit Natura, nullis instructa

[202]Caspar 1993, p. 128. Gingerich 1970, pp. 294–297.

[203]Caspar 1993, p. 136.

vestibus hypothesium; [...] ... denique ut novo ornatu placeret delicatis, nuditatem fastidientibus: imo vero ut pro Natura, generosissima puella, substitueretur spuria tua, meretricio ornatu et moribus, ad voluptatem comparatis, non ad ingenuitatem: hoc est Fabricio interprete, ut arti consuleretur, non alienatis philosophis." ["You say that a daughter has been born to you by geometry. I have seen her; she is lovely, but she will become a very bad wench, who will carry off the men of the many daughters that the mother, physics, has given me. ... Therefore, bare nature has guided me, free of all the attire of a hypothesis. ... and then so that [your theory], thanks to the new embellishment, could appeal to lascivious men, who disdain simplicity: or rather, in the place of this Nature, a highly respectable girl, your whore with her wench's apparel and her conduct leading to lust, not to virtue, is supposed to appear: that is, in the speech of Fabricius, one should address science, not crazy philosophers."].[204]

He too, of course, was subject to the weaknesses of the flesh, and even our devout natural philosopher was to allow himself some little casual (paid?) *affaires* with young and beautiful, more frivolous daughters of Geometry. Geometrical theory, curiously, was contrasted here, in the debate with his colleague, with a presumedly virtuous (repressed?) Lady Nature. Between what he was later to write in the *Harmonices mundi* ..., above all, as we have seen above, in "Book IV", and the world as clockwork, we do not find much consistency. We are forced, then, to imagine an evolution in time. We find even less agreement when he defines as a "whore" that daughter of Geometry who was to introduce him to pleasures that were not only theoretical, but also sensible, because they are found in Music, and included in the Third Planetary Law.

"Do with me what you want. Even if you were to pull one vein after another out of my body, I would have nothing to admit."[205] These words pronounced by a poor 70-year-old woman, Kepler's mother, registered in the proceedings of a trial for witchcraft, should be compared with the famous, mythical "And yet it moves" muttered by Galileo Galilei after his well-known recantation. When they were led in front of the torture instruments, the Italian natural philosopher and the German sorceress reacted differently. Of course, there were differences of character, sex, language, faith, culture, values, contexts, different worlds. But in the tribunals of the period, the battle for religious, scientific, or judicial truth, or simply for the plain truth, continued with every means available, including extreme violence.

"I ask God to make my spirit strong, so that I direct my glance at the pure truth, from whichever side it should be presented, ...". "I wanted to become a theologian; for a long time, I was restless. But now, see how by my pains, God is being celebrated in astronomy also." Kepler thought of himself as a priest who was reading the book of nature, in order to sing God's praises. We need to take into consideration his neo-Platonic philosophical positions, together with his theological convictions about the divine presence of Christ in the world, which were more Lutheran than

[204] Kepler 1954, v. XVI, pp. 205–206. Cf. Caspar 1993, p. 171. Translation slightly different.
[205] Caspar 1993, p. 255.

Catholic, and more Calvinistic than Lutheran. How did the incarnation of God the Son take place? In the bread and the wine of the Eucharist, was Christ present as 'transubstantial', 'consubstantial', or only as a symbol? In any case, Kepler refused the worship of the images of saints, the worship of the Eucharist, and the idea of God's presence everywhere. At this point, there can be little doubt that the truth imagined for all his life byKepler was absolute and transcendent. For him, the world was a corporeal "imago" [image] of God, and the soul was the incorporeal "imago". In this term "imago", he included not only the image, the representation the design, but probably also the phantom, the dream and the appearance of the Latins.[206]

Plato's transcendence of geometry should afford the best "imago" of the Christian Trinity, which became a sphere with its centre in God the Father, the visible surface in the Son, Christ, and the Holy Spirit in its rays.[207] Having created us in His image and after his likeness, "… God wanted to have us recognize these laws …", he wrote to Herwart.[208]

In our imperial mathematician, the evident recurring dualism between truth and falsity in sciences was inextricably linked with the dualism between good and evil, or soul and body, of his religious faith. With him, it is impossible to throw out all theology, astrology, the weather forecasts of his almanacs and music, in order to keep only his studies on astronomy. We would be making the same mistake that he imputes to others in his longer title of the *Tertius Interveniens*: "Das Kind mit dem Badt ausschütten" ["To pour out the child with the bath"].[209] Consequently, as is more common among science historians, we should not distinguish, either, between the Kepler of progress, who anticipated Newton, and the Greek-Aristotelian one, between the tireless calculator who compared his own numbers with Tycho's observations, and the animistic metaphysician, convinced that the whole world was harmoniously connected together by the beautiful proportions of geometry and Greek music, between the mathematician of precision and the Lutheran theologian, between the tenacious scholar poring over his books and the ironic, polemical vagabond of the courts and the parishes, between the intelligent astronomer and the affectionate husband, …

Even without Kepler's permission, if we wanted (diabolically) to mix good with evil, we might tell how he took possession of the precious papers left by Tycho Brahe. "Ego sic censeo de Tychone, Divitiis abundare, quibus non recte utatur, more plerorumque divitum. Danda est igitur opera (quod et ego pro mea parte, sed cum debita modestia feci) ut divitias illas illi extorqueamus, ut emendicemus, scilicet, ut vulget suas observationes sincere, et omnes." ["My opinion about Tycho is this: he has abundant wealth. Only, like most rich men, he does not know how to make proper use of his riches. Therefore, one must take pains (what I did

[206]Caspar 1993, pp. 336, 373–378. Banville 1993, pp. 176–180 and 150.

[207]Kepler 1619 [1969], IV, pp. 119–120. Caspar 1993, pp. 272 and 379. Pauli 1955. Tangherlini 1974.

[208]Caspar 1993, p. 380.

[209]Caspar 1993, p. 182.

myself, with proper sagacity) to wring out his treasures from him, to get from him, by begging, the decision to publish all his observations without reservation."] "Non diffiteor, me Tychone mortuo, haeredibus vel absentibus, vel parum peritis, Observationum relictarum tutelam mihi confidenter, et forsam arroganter usurpasse; adversis haeredum voluntatibus; …". ["I do not deny that when Tycho died, shamelessly, and perhaps arrogantly, I took charge of directing the remaining observations, seeing that the heirs were either absent or too unskilled; against the will of the heirs; …"].[210]

Whether they served as ephemerides for navigation, or as almanacs for weather forecasts, or were used to establish the festivities in delicate, controversial religious calendars, or for astrological predictions, astronomical observations had a great strategic importance. On the basis of them, General Wallenstein decided his military campaigns. They were like the data used for constructing supercomputers or nuclear weapons today. Apart from the disputes between natural philosophers and theologians, they all guarded them jealously. Galileo Galilei hid his observations on Saturn and Venus behind anagrams, which Kepler in vain sought to decipher. "Haec immatura a me jam frustra leguntur o y." ["These immature things have now been explained by me not at all or y."] By this, he meant: "Cynthiae figuras aemulatur mater amorum." ["The mother of loves imitates the figures of Cynthia [Diana, the Moon]."] He had thus observed the phases of Venus, which, revolving around the Sun, was illuminated by it, like the Moon.[211] Followers of the Christian religions should be well acquainted with the long story of sins and violence related in the *Bible* and in the *Gospels*; this is no reason for their faith to be shaken. But how many followers of the religion of science would be ready to include this theft in their history of progress?

Though I am aware of all the limitations, the choice to leave Kepler in his historical context allows me to avoid the apologetic and promotional caricatures, which bring with them inevitable hints of nationalism: like the German Kepler versus the Italian Galileo, the Frenchman Descartes, or the Englishman Newton. However, it would be useless to ignore that the budding scientific community was to filter only that part that we all know from school textbooks. To understand better how this happened, and the differences with respect to the characters of the following chapter, we shall take up again the comparison with the cultural and scientific values of the Chinese.

In 1623, the Jesuit Father Johannes Schreck (alias Terrentius), whom we have already come across, wrote a letter from faraway China, a copy of which was destined to arrive also in Kepler's hands. ["Let us consider correcting the Chinese calendar, in which the main point is the forecasting of eclipses and the precession of the equinoxes. In this, your Reverences could be of no little help in this task, if you sent at least a short letter on the subject, if it is burdensome to write more,

[210] Kepler 1945, XIII, n. 113, p. 292. Kepler 1951, XV, n. 357, p. 232. Caspar 1993, p. 87. Koestler 1991, pp. 339–340. Banville 1993, pp. 127–129.

[211] Koestler 1991, pp. 369–370.

and if you had anything new besides what is found in the supplement to Magini [Giovanni Antonio, 1555–1617]: especially about the diameters of the Sun, when they are at the distances from the Earth in the apogee and the perigee, and about the other things that are necessary to calculate eclipses. Perhaps some new hypothesis about the Moon will be presented. I have no doubt that during our absence, others will have been published: for example, Kepler's *Hypparchus*, or other works by [Galileo] Galilei. It would take too long to wait for these books; they are held up, or destroyed, much more frequently than they are delivered, by the majority of means of consignment; it is easier to send a letter, and it arrives here more quickly, without doubt within 3 years. After 2 years, after completing my course of Chinese grammar, there will be the advantage, both of understanding and of writing many things. Now, however, I will indicate just a few.

The Chinese start their histories with Yao, the first famous king; they sustain that these are certain: although they count six kings from Fochi [Fu Xi] to Yao; their histories are full of fables, and no indication of dates is certain. Furthermore, from Yao to the present day, they count about 4,000 years, advancing by cycles of 60, as we do by 15 and 28. In that period, they observed the solstice close to the cusp of Sagittarius, I don't remember the degree: as a result, up to this point, it would have already advanced by 52°. This is in good agreement with the Tycho's results. Shortly after Yao, an extraordinary eclipse was recorded which, they say, touched the second degree of Scorpio. They indicate the year; I don't know whether the hour and the magnitude are known. This matter must be deferred until I talk to the royal mathematicians in Beijing. They have a similar arithmetic to ours, and in the quantities of the year they used fractions with precision up to the times of Yao. They also have certain geometrical problems; recently, I have seen 15 of them which are more than 3,000 years old. Among these, the first [problem] is the penultimate [proposition] of the First [Book of the *Elements*], that is to say, the theorem of Pythagoras.]"

The event that the problems seen in the Chinese book by Terrentius totalled "15" makes it practically certain that the Jesuit had in his hands the *Gouguyi [The right gougu]* by Xu Guangqi, which included exactly that number.[212] Therefore, he had not actually studied what the Chinese had really written, some time before, about the fundamental property of right-angled triangles: the *Zhoubi*. On the contrary, he knew the reduction of the *gougu* [right-angled triangle] to the Chinese translation of *Elements*, written by Euclid, but now reinterpreted here by Ricci and his collaborators converted to Christianity.[213]

> They divide the zodiac into 28 constellations. They call the heart of Scorpio the Dragon's heart, and the tail of Scorpio the Dragon's tail; they call the Canicula [the Dog Star, Sirius] the Wolf, and Capricorn the Ox. They call the last star of Ursa Minor [the Pole Star] the King, because it was once immobile, close to the Pole; all the others were worshipped. A

[212] See above, Sect. 10.2; Tonietti 2006a, Chap. IV, pp. 158–170.

[213] Kepler's editors failed to identify the books implicitly quoted in the passage. Kepler 1993, v. XI 2, p. 513.

> few days ago, I took a book containing all the method for the calculation of eclipses. It has not been copied yet. After I have studied it, and understood it, I will be glad to communicate it to Europe. In their calculation, they are not wrong by more than two or three quarters. What an amazing thing that it did not change more in 4,000 years. .... 1623.[214]

As it deals with all the same problems, the Jesuit's letter was almost a summary of another one written by Sabatino De Ursis (1575–1620) to a third brother, with the same aims: to correct the Chinese calendar, to calculate eclipses better, and to obtain pertinent books to be translated. This had been written in Portuguese more than 10 years earlier, in 1612, carefully explaining the differences between the Chinese calendar and the European one, which had recently become the Gregorian one of 1582. For Chinese culture, its importance lay in the marking of actions, forms of behaviour and ceremonies. If the days are not correct, "... todo o Reino fica emgannado ..." ["... all the kingdom is deceived ..."]. For this reason, the Office of the Calendar counted on the "... maiores da Corte ..." ["... leading members of the Court ..."] and was under the direct control of the Ministry of Rites and the Emperor himself.

For a long time, since before the foundation of the Empire, as we learn from the *Zhoubi [The gnomon of the Zhou]*, of which we have already spoken, Chinese functionaries were able to establish the solstices and the equinoxes by measuring the shadow of the sun projected by the graduated *bi* [the pole, the Greek gnomon]. Thus, they had also discovered that it varied from 1 year to another: the Western precession of the equinoxes, called *suicha* [difference of the year] in Chinese. Observing the phenomena of the earth, united to the sky, through the gnomon, and accepting their continuous transformations, with their *lifa* [rules of the calendar] the Chinese imperial functionaries re-established the calendar every year, measuring again whether the winter solstice arrived at the same moment, or not. They had already corrected it several times.

For the reasons that we already know well, some Jesuits had learnt the astronomy and mathematics that were current at that time in Europe, in order to take part in the corrections, and propose their new calendar. Since the time of the Yuan [the Mongols of the thirteenth and fourteenth centuries], the Chinese emperors had maintained a second Office of Astronomy, which carried out its calculations following Arabic traditions. Why should it not be possible, reflected De Ursis, with the help of the converts to Christianity like Xu Guangqi, to replace the Chinese tables and calculations with their own? That is to say, in other words "... como fizerão com as dos mouros ..." ["... to do with us as they did with the Mahomettans ..."]? The eternal theories and the "absolute" truths to which these Jesuits made reference were, at that time, those of Christophorus Clavius and Antonio Magini, and thus took their inspiration from Ptolemy, or perhaps partly from Tycho Brahe, but definitely not from Copernicus.

Like Matteo Ricci, who had recently died, also Sabatino De Ursis relied on his greater precision in calculating the exact moment of the eclipse. Consequently, he

[214] Kepler 1993, v. XI 2, pp. 300–301.

had started to find some support in the imperial Office, for the new corrections to be introduced following the theories of the Extreme West. But we can understand from the letter that obstacles were already blocking the success of the operation. While a first petition had received approval, the emperor's authorisation was slow to arrive. "A este 2° memorial nã̃o deu despacho nigum até agora el Rei. Dicem, porque no mesmo memorial se pedia desse maior dignidade aos dous mandarins que se havia̋o de applicar a isto;" ["This second memorial has not been sent yet by the King. They say that the reason for this is to be sought in the event that in the same memorial, a promotion in position was requested for the two mandarins who were to take charge of this;"]. We do not know whether the diplomatic excuse stemmed only from the Chinese, or also from the Jesuits. However, we have already written that everything ended up in the general expulsion of the Jesuits from China. They must have forgotten that the astronomy of the Arabs had penetrated into the Country-at-the-Centre, because Kubilai Kan had become emperor. Thus they should first have converted the Ming emperor to Christianity, in order to change the basic calendar of Rites, and not vice versa.

De Ursis thus had to leave Beijing, and took refuge at Macao, where he died a few years later. We also know the reasons why the Jesuits were re-admitted to the Empire-at-the-Centre. It is clear that they continued to search for the necessary instruments to succeed in their conquest. The novelties from Europe were now called Galileo Galilei and Johannes Kepler. Everybody desired to use the former's "cannon", or "eye-glass", or "telescope" to use the new name coined by the Accademia dei Lincei. These *wangyuanjing* ["lenses to see far away"] were presented to the Emperor in 1634. "With an instrument of the kind, it is not only possible to observe the heavens, but also to see objects at a distance several *li*, as if they were in front of the observer's eyes. It is extremely useful to observe the enemy within the range of cannon-shots." And thus we again encounter the war between the Ming and the Qing invaders, to which western sciences owed so much of their penetration into China.[215]

Schreck's letter was to take more than 4 years to arrive in Europe, and more time was necessary for it to arrive in Kepler's hands. He was able to reply only in 1627, commenting on it, sentence by sentence, with brief considerations. Among these, we find the following:

["... if the Chinese use astronomic tables in the form of a calendar, I ask, what kind of tables have they used so far, and who was the author of these? 14 years ago, more or less, I read the itinerary of a man of the church, through the land of the Tartars and the kingdom of the Mongols [...] to Cathai or, as I believe, China itself. This author says that for the Chinese, there are two tribunals [the Offices] that are representative of astronomers, who have the power, like priests, of fixing the solemnities: the greater authority is that of the tribunal from which the most reliable forecasts of eclipses come. The author judges that the others use the doctrine of

[215]De Ursis 1612; entirely quoted and translated in D'Elia 1947, pp. 74–117, on pp. 105, 109, 107, 86, 62.

Hipparchus. But there are two other works containing tables, which it would seem to be useful to mention to Father Terrentius, so that he can investigate whether there is any indication of these among the Chinese. The first is the calculation of the Turks, based on the years from the Hijra, for which see Scaliger [Julius, 1484–1558] and others. The Saracens will fill up the whole of the Orient with their inventions and teachings. [...]

The other work is the Persian tables, which use the Jesdagirdis years [which began in 632 A.D. with the Persian king, Jesdegerd III], for which see Scaliger and Christmannus [Jacob, 1554–1613]. These appear to have been only tables for the movements of the Sun and the Moon, and for eclipses. If I am not mistaken, these were copied by Arzachel [al-Zarqali, 1030–1100], and consequently, in the end, knowledge of them was incorporated into the Alfonsine tables, since these also recalled the Persian order of the months. [...] If the kingdom of the Persians thus arrives as far as the Indus, and the Indies border on China, it must be determined whether this knowledge of the tables ever penetrated into the tribunals of the Chinese. .... [....] .....

Perhaps something similar to these approaches may have been appreciated in that period by the Chinese, and accepted, in order to prepare some image of their year. [...] If the shortest cycle of the eclipses is then null, also the longest one will be useless for the periods of the State; it remains [to be understood] how the Chinese use any table for the movements of the Sun and the Moon, and how they try to correct them. [...] Irritated by the many epicycles of Tycho, ..., he will be able to teach the Chinese these eccentrics [of the Moon]; [...] however, if we observe the nature of things, they are not perfect, but degenerate into an elliptic trajectory .... if the Chinese also desired to know for what reasons the Moon abandons its perfect eccentric [circle] in the remaining intermediate positions, Father Terrentius could teach them from my *Commentaries on Mars* and from the *Epitome Astronomiae Copernicanae [Summary of Copernican Astronomy].* There is no circle in the intention of the heavenly mover, but [the ellipse] arises from a material, that is to say, geometrical, necessity; consequently, the trajectory becomes elliptic as a result of the convergence of two causes of the movements, one of which attracts the Moon away in a circle from its setting to its rising, and the other, maintains it in equilibrium in straight lines from the Earth and towards the Earth. The first of these causes has its origin in the non-material species of the Earth. [...] The Moon, in truth, is not fixed to the world with nails, but swims in it, almost freely. This convergence, I say, of two movements gives rise to the elliptic trajectory, which can be demonstrated by a suitable geometrical method. This is the reason why, in the title in bronze of the *Tabulae Rudolphinae [Rudolphine Tables]*, the word "Magnetica" is added on the right-hand side of "Geometria". If the Chinese were instructed in a more complete manner ... then they could be directed towards the other figure on the right, that is to say, the Stathmica [steelyard, from the Greek for "to weigh" "] (Fig. 8.7).[216]

[216]Kepler 1969, v. X, pp. 7 and 21–22. The allegory of Magnetism is at the top on the right, holding in its hand also the needle of the magnetic compass. The one next to it shows the steelyard, which

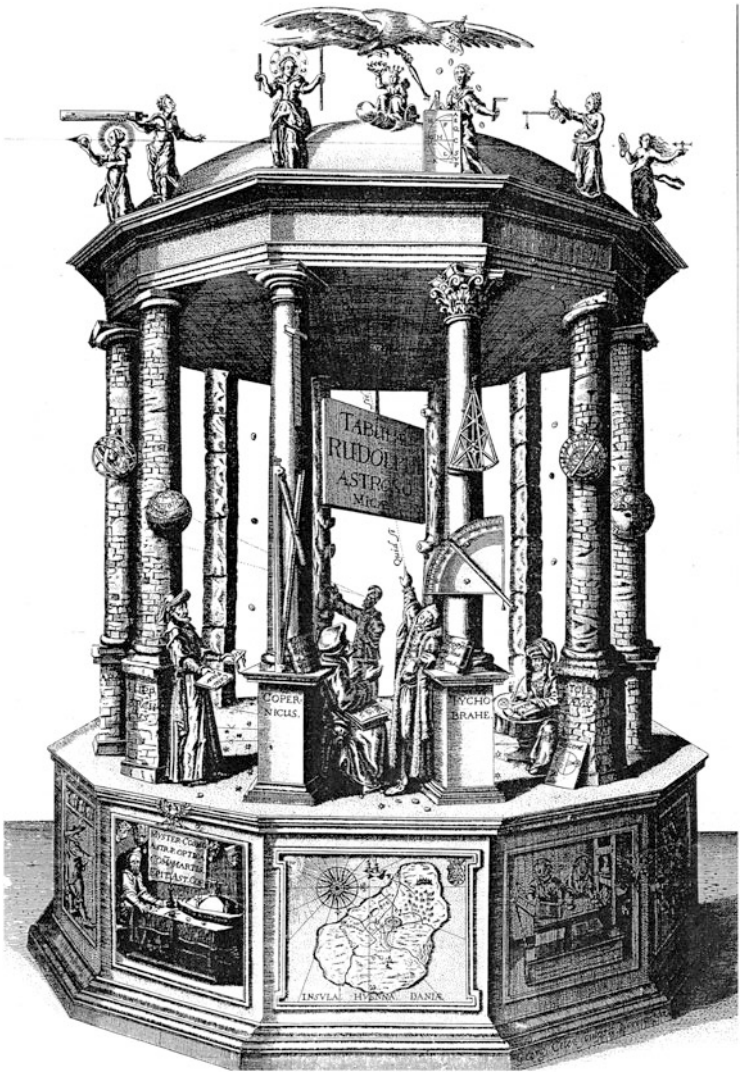

Fig. 8.7 Title page of *Tabulae Rudolphinae* with the allegories imagined by Kepler (Kepler, Opera Omnia vol. X 1969 title page)

Of the *Hypparchus*, Kepler could only quote the effects contained in the *Tabulae Rudolphinae*. Actually, they had not yet been [and in reality, never were] published, "… if you desire the demonstrations. But there will be a forthcoming edition, if

symbolises the way in that the sun has a stronger influence on the near planets, but less on the farther ones. This is followed, in order, by the Geometry of triangles, the numbers of Logarithms, the Telescope and lastly, the Physics of light.

God grants me life. [...] The reason for the sexagesimal cycle appears to be simply arithmetic elegance, not heavenly movements. [...]" Our European astronomer doubted that the Chinese had kep eclipses for 4,000 years, or that they had used fractions for a long time. "Fractions for the quantities of a year cannot have been stably known ever since the times of observations began: these fractions need to be learnt many centuries earlier. In the meantime, if they have become accustomed to using fractions of years, good; then they will appreciate the fractions of the *Tabulae Rudolphinae*. [...] Already, 1,900 years ago,Euclid made a collection of problems or of theorems known to the ancients. Thus it is confirmed that the Chinese spring from the Ionians, doubtlessly transported there through the Red Sea or the Persian Sea to the islands of the Orient, and from there to the nearby continent."

Kepler commented on what Terrentius said about the theorem of Pythagoras in Chinese, in accordance with the current Western tradition. "... that Greek had received the grace to excogitate it from his star, therefore he offered a hecatomb of oxen." The imperial astronomer traced the 28 Chinese constellations back to the Arabs, who would take the number of days and houses from the Moon. ["It would be surprising if all the Chinese doctrine of the stars were not taken from the Arabs, who were the most recent".] He accused Terrentius of believing an idiocy, if he thought that the Pole Star was closer than it is now to the pole of the world. ["... but if this is the teaching of the Chinese, this proves it is false that this information would be very ancient among them. [...] ... is it not true perhaps that the Chinese boast, if Terrentius himself makes reference to their experience of certain eclipses? Rather, let them first demonstrate that their stories and their observations of a nature dating back to 4,000 years ago are true. In fact I expressed the suspicion above that those eclipses derived from a retroactive calculation. [...]

Let Father Terrentius communicate to us the height of the pole of that place, or the exact height of the Sun precisely at midday [the latitude] ... let him also communicate the customs of the people, and what have that is fixed and established in ordering the days of the year, and what they allow to be movable and variable; which beginning of the year they observe, whether the day of the year is certain, or whether the beginning changes, like the Saracens. And then we will help to accommodate it in a kind of year which both corresponds to the sky and is acceptable for the Chinese. Lastly, we will help to convince people, not so much of the laws of our calendar, but of all the gentle yoke of Christianity. ... Ratisbona [Regensburg], December 1627."]

On the occasion of its publication, on January 15th 1630, the imperial mathematician on loan to Wallenstein also added a short appendix, in which he predicted the eclipse of the sun which was to take place on June 10th [1630][217] of the Gregorian calendar. With the dedication to the great General, the little "commentatiuncula" were at last printed at Sagan. "... the farthest limit of Asia, the kingdom of the Chinese, presents a request to the European mathematicians; the mathematicians ask me ... for advice about how to regulate times in the Orient, and to reply to

[217] An eclipse of the sun actually took place on June 10th, 1630; Espenak 2008.

the letters from China …". The military victories of his valorous soldier and host had opened up routes towards the seas and oceans, also conquering some of the Baltic regions. "The wars have opened up the Ocean to you, peace will be suitable to maintain its use … For the former, you needed the labours of Hercules … or the rivals of Saturn's courses, you will be able to arrive at the latter by pouring over that dirt a pleasant thunderstorm, with the nearby size of Jupiter, with justice, I mean, with equity, and friendship, by means of alliances, with clemency, …".[218]

Thus, in spite of the event that they had generally condemned various Copernicans (except Jesuitically, perhaps, in the silence of their minds), and in particular Galileo Galilei, certain Jesuits did not refrain from exploiting their competence when it was convenient to do so. But the Tuscan astronomer, who was less Jesuitic, would not be pleased, and did not answer the letter.[219] If they wanted his precise calculations for eclipses, they would have to take, together with them, his absolute truth for them. Otherwise, nothing doing. Thus, through the German Jesuit Fathers, the request was passed to Kepler, who was less susceptible, and answered. Together with the *Tabulae*, however, he too offered his model of elliptic orbits, his books and his own theory.

His letters also allow us to understand better to what extent Kepler defended his absolute truth, not only against the rival European truths, but also against the Oriental ones. Thus, as a good astronomer and a militant Western mathematician, he imagined that the Chinese, too, must have derived their astronomical and mathematical doctrines from the Greek cradle, via the Arabs and the Indians. Also fractions, which were so much more convenient, not only for calculations in the field of astronomy, could not have been invented by the Chinese in ancient times. Sure enough, the European imperial mathematician believed that it was necessary, in any case, to go through the same inexorable march, which had taken centuries here in the West: from Greek geometrical proportions to Arabic numerical fractions, which finally passed into Europe. Even the Chinese could not possible have learnt them in any other way than from Pythagoras and Euclid.

In this case, the absolute of scientific truths had been transferred into history, as well, which thus became a compulsory course, without options. Clearly, how could the stars have influenced and led the Europeans and not the Chinese? Here, the idea of a single, universal progress mingled with a weak version of astrology. And, *natürlich*, Kepler modelled all these convictions on his Christian religious faith, equally suited for China, because it was equally universal.

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[218]Kepler 1993, v. XI 2, pp. 299–314. Kepler 1959, v. XVIII, n. 1055 pp. 309 and 534–535, n. 1122 pp. 416–417 and 562. Cf. Caspar 1993, pp. 331 and 350.

[219]D'Elia 1947, pp. 39–48. Pasquale D'Elia S.I., however, underestimated the persecution suffered by Galileo Galilei, and attempted to find in the Society of Jesus some manifestations of sympathy for the Copernicans (though hidden, and ambiguous expressions, such as "… Dominum Galileum mathematicum tradat." ["… Mr. Galileo is to be treated like a mathematician."]). He covered up the expulsion of the Jesuits from China in 1616 in an appendix, even calling it a "pause", pp. 71–72. Cf. Schuppener 1997. The *Rudolphine Tables*, which were a list of numbers, actually arrived in China. Szczesniak 1949. Hammer 1950.

From Terrentius, we learn now that, in spite of the disastrous expulsion of 1616, and undoubtedly thanks to the new merits acquired by means of cannons on the battlefield, the Jesuits continued to hope that they might impose their new calendar, also in China. To beat the Chinese astronomers, they now tried to use the most famous Copernicans' results for eclipses. But the Jesuit, who returned to China, should have written that the Chinese were formidable adversaries, who had been capable of calculating with relative precision for a long time. Both he and Kepler understood that the European times would have to adapt to some Chinese customs. Was it an astute way to attain that Higher purpose declared by both of them? In the face of Chinese scientific culture, the differences between the Ptolemaic and Copernican astronomies had now become secondary, with respect to the main task.

With Galileo Galilei and Johannes Kepler, European scientific values were consolidated around dualistic polarities, as we saw in Chap. II, which were foreign to that faraway Oriental country, predominantly complex and immanent. Even more than Galileo Galilei, thanks to his religious faith spelled out clearly everywhere, and practised with a profound conviction, Kepler justified his scientific laws with the transcendent values that he had found in the Greek neo-Platonic mathematical tradition. Although the imperial mathematician also knew the results obtained by the Arabs, he continually made reference to his favourite world of classical Greece. He expressed his laws in communion with God, by means of the "unique and eternal" geometry of Euclid, reinvigorating it in the comments of Proclus.[220]

He did not love algebra. "Objiciat hic mihi aliquis doctrinam Analyticam, ab Arabe Gebri denominatam Algebram, Italico vocabulo Cossam: videntur enim in ea determinari posse omnis generis Polygonorum latera." ["Here, somebody could oppose to me the Analytical art called Algebra after the Arab Geber, its Italian name being Cossa [thing, unknown]: for in this art, the sides of all kinds of Polygon seem to be determinable."] For example, this was ably done by his friend "Instrument maker [*mechanicus*] to the Emperor", Jost Bürgi (1552–1632), an expert on clocks, and a great calculator of logarithms. Of him, he reported the calculation for the regular polygon of seven sides, which wanted to be excluded, because it was not constructible, and not 'knowable' according to his criteria. He changed Bürgi's symbols, which derived from the Italian tradition, using others "commodius" ["more convenient"], and obtained, in the end, the expression equal to "nihil" [zero].

$$7 - 14ij[x^2] + 7iiij[x^4] - 1vj[x^6].^{221}$$

But the procedure that moved through equations prompted the criticism of our natural philosopher. He underlined the differences and the limitations compared with his beloved geometry. "… plane nihil commune habere cum Definitionibus

[220] Caspar 1993, p. 380.

[221] Kepler 1619 [1969], p. 36. Kepler 1997, pp. 66–69.

nostris … […] … Haec omnia erant factu & possibilia, & faciliora quam dictu, ut norunt qui circinos tractant. Quidne facilius quam rectum angulum … facere, & .... scribere circulum …? .... […] … & miser Calculator, destitutus omnibus Geometriae praesidijs, haerens inter spineta Numerorum, frustra cossam suam respectat. Hoc unum est discrimen inter Cossicas & inter Geometricas determinationes." Algebra [" has absolutely nothing in common with the Definitions we gave above, … All this is possible and easier to do than to explain in words, as anyone knows who is used to handling compasses. For what is easier than to construct a right angle … [or to] draw the circle … ? … the unhappy Calculator, robbed of all Geometrical defences, held fast in the thorny thicket of Numbers, looks in vain to his *cossa* [algebra]. This one is the distinction between Algebraic and Geometrical determination."]

Kepler listed other elements that discriminated between algebra in the Italian version and his Greek geometry, underlining, above all, the discrete, numerical aspects of the former, compared with the continuous quantities of the tradition of the latter. He did not omit to point out that with irrational numbers, the algebraic procedure could only be approximate and inexact. "Concludimus igitur, Analyses istas Cossicas alienas esse a praesenti contemplatione; nec ullum constituere gradum scientiae, cum ijs comparabilem, quos explicavimus in superioribus." ["So we conclude that these Algebraic Analyses make no contribution to our present concerns; nor do they set up any degree of knowledge that can be compared with what we discussed earlier."][222]

The other "Arabic art" opposed by Kepler was the deterministic, predictive conception of astrology, because it was practised, above all in accordance with the style of that other culture. "… this Arabic art amounts to nothing … one must separate the precious stones from the dung, one must glorify the honour of God, …". Among the reasons for his continuous wanderings as in exile, we must also include the Turks, who belligerent were pressing on the borders of the Holy Roman Empire of the Habsburgs. Whether it was out of fear of an invasion, or of the religious differences between Islam and Christianity, Kepler knew, and also undoubtedly used the results of the Arabs in the fields of optics and astronomy, but tended to underestimate them too much.[223] He probably only considered them to be a stage in the course of the absolute truth from Greece to himself.

Faithful to the harmony of music, Kepler also placed on the scale pan of the balance the explanations that connected phenomena together, like the influence of the moon on the sea, or of planetary conjunctions on the weather. Thanks to that, he had encountered some success with the public, and scraped together some money. Together with these, however, the procedures that separated them were increasingly gaining importance. "I am a Lutheran astrologer, throwing out the chaff and keeping the grain."[224] He wanted, where necessary, to distinguish the precious pearls from

[222]Kepler 1619 [1969], I pp. 34–40; Kepler 1997, pp. 66–79.

[223]Caspar 1993, pp. 59, 75, 144, 157, 161, 329, 339.

[224]Caspar 1993, pp. 45 and 183. Kepler 1997, p. xxi.

the acorns, even if in reading the works considered here, I have not come across the famous Latin saying, *Nolite proicere margaritas ante porcos* [Do not cast pearls before swine]. Now, however, he was the one who was doing to the Arabs what we have seen the Arabs had done to the Indians. With the other subsequent characters who were exiting the scientific revolution in Europe, the balance would soon shift towards the art of separating, distinguishing and subtracting, in order to be able to invent transcendent laws more easily. In this, the contribution of the Arabic scientific culture would not be ignored.

We have seen that the whole *Harmonices mundi libri quinque* were based on the distinctions between constructible and non-constructible, knowable and non-knowable polygons, between consonances and dissonances, between hard and soft, male or female intervals, between soul and body, *und so weiter*. Kepler had transformed possible mathematical procedures into transcendent, eternal, universal laws. Chance was excluded. Would the dinner prepared by his wife, Barbara, be so tasty by chance? Would the ingredients mingle together by themselves? "... if pewter dishes, leaves of lettuce, grains of salt, drops of water, vinegar, oil and slices of eggs had been flying about in the air for all eternity, it might at last happen by chance that there would come a salad."[225] If his beloved first wife had prepared that dinner, was it not probable that conjunctions between planets, new or old stars, and astral movements, with the relative propitious or inauspicious consequences, likewise had an author?

In the abundance of secondary literature about Kepler, the curious reader can discover by himself both other interpretations close to my own, and those that are more distant. More frequently, he will find contemptuous judgements of modern university professors, whose salary is guaranteed, inflicted from the heights of their chairs in physics, astronomy and the history of the sciences, on poor Kepler. Just a few examples. For Owen Gingerich, the correspondence between the notes of the scale and the velocities of planets is a "shambles", because these were approximate, and in any case, by adjusting the numbers, it would be possible to find some kind of correspondence with any note "if you hammer away hard enough". Consequently, the whole construction is "... the edifice of a madman...", "... part fantasy and chimera ...".[226] But everybody knows that the academic world of today carefully eliminates those who, for any reason, are considered madmen or heretics, preferring to offer work to the thousands of Gingeriches.

Bruce Stephenson moves along the same lines, albeit with tones that are at times apparently softer, given the popularising nature of his work. "... Kepler's harmonic theories were fanciful, even for their time." "... poetic metaphor, but not as scientific truth ...". He considers still valid the judgement passed on Kepler in that period by the Jesuit, Giovan Battista Riccioli (1598–1671): "... the evidence ... to establish those harmonies seems poetical or oratorial rather than philosophical." Thus, for Stephenson, the *Harmonices mundi libri quinque* were a "... valiant but misguided

[225]Gingerich 1970, p. 297.

[226]Gingerich 1991, pp. 60 and 63.

application of reason. … We judge, with the clarity [sic] of nearly four centuries' wisdom that Kepler was wrong."[227]

I am afraid that, with the "clarity" acquired through faith in the astronomic manuals of the twentieth century, Stephenson has, at this point, lost sight of the roots that Kepler placed in God for his truths. Without this, the truth invented by our imperial mathematician is downgraded, *of course*, to a "… mildly interesting substratum of coincidence[s]."[228] The severe God of Calvin cannot have assigned the same 'grace' to Kepler and to Stephenson.

The historian should be made wary by the event that Stephenson tried to present to us a Kepler who was hesitant about the meaning to assign to his Third Law. "… trying furiously [sic] to assimilate the new discovery into his harmonic speculations and physical theories, [Kepler] was not altogether certain whether the harmonic law was itself one of the Creator's archetypal principles … reveals his uncertainty whether it, too, is a small-eccentricity approximation."[229]

But how could our devout German astronomer have had any doubts, that the sesquialtera ratio 2:3, between the powers of planetary distances and times, might not have been established by God as a universal archetype? In a book where 2:3 was the consonant interval of the fifth between notes, did it perhaps not coincide with the music to which the planets were likewise made to dance?[230] And why, then, should our short-sighted imperial mathematician have worried that not all the numbers were perfectly correct, amid imperfect telescopes, corrections of every kind, boring approximate logarithmic calculations, carried out, undoubtedly, by sinners? Anyway, no trace can be found of the problem speculated about by Stephenson in the abundant relevant literature.

On the contrary, it is fairly easy to understand the reason for the efforts made by Stephenson: separating "… one of his most profound discoveries …" from the remaining dung, to use Kepler's language, which had become such as a result of the evolution of the sciences. But we all know, *pace* all these anachronistic historians, that from manure flowers spring up, but from a faith in reason, only bad philosophies with their relative a posteriori reconstructions.

As if this were not enough, after confessing to a "… relative ignorance of Renaissance musical theory …", our modern-day writer- astronomer compared

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[227]Stephenson 1994, pp. 8, 11, 249 and 251.

[228]Stephenson 1994, pp. 126–127.

[229]Stephenson 1994, p. 141.

[230]In the *Epitome astronomiae copernicanae*, the angle at which the Sun was observed from the Earth was 720 parts of 360°, that is to say, half a degree; this was the same whole number 720, the smallest, with which musical ratios were to be distributed into two octaves. 720 was also the same fraction of 360° at which the Moon could be seen. This allowed the copy of God on Earth, the measurer that is to say man, to observe a total eclipse of the Sun, and to invent astronomy. Kepler 1953, v. VII pp. 277–279. Kepler 1619 [1969], III p. 45–46; Kepler 1997, pp. 192–193. Cf. Koyré 1966, p. 295. In Caspar's edition , Kepler made reference to "Book III", Chap. VI, of the *Harmonices mundi* . . . , whereas "720" is found in Chap. VII. The printing error is remaining for a long time.

Kepler's distinction between "durus" and "mollis" to the subsequent major and minor tonalities. Thus he was able to discover, again, that our poor natural philosopher based his approach "… on a transitional and incomplete theory of the two types of harmony."[231]

We already have enough indications to stop and think that Stephenson would have not helped us to understand Kepler better, leaving him in his historical context. On the contrary, he would have selected for us what he preferred, in accordance with his own personal (limited?) knowledge of astronomy, physics, logic and music, as it is written today, also ignoring the equable temperament. Perhaps he did not fully realise either that he could speculate as much as he liked on whether Kepler had written "Book V" of the *Harmonices mundi* …, thus abandoning Ptolemy's edition, only after discovering the Third Law. But then the imperial astronomer would undoubtedly have imagined it consistent in every aspect with his harmony of the world, and not 'hard to assimilate' to the rest.[232] What else would have stopped Kepler from printing the Third Law separately? Does self-contradiction or otherwise, logically speaking, depend on the great difference between historians of the context and historians of *advertising*?

Rightly, also this English-speaking scholar noted that in general, Kepler's harmonic theory had been underestimated. He correctly traced its beginnings back to the letters written in 1599 to Edmund Bruce, Herwart and Mästlin.[233] Here, in fact, we can already see some of the ideas that are included, with adjustments, in the big book of 1619. But Stephenson presented it, as if he had believed that "… equally long strings with tensions in the proportion 3:4 would make the interval of a fourth, as between Saturn and Jupiter; with tensions in the proportion 4:8, they would make the interval of an octave, as between Jupiter and Mars; and so on."[234] If he had really written this, Kepler would have ignored one of the several defects for the ear propagated by the Pythagoreans. As we have seen above, and as is well known, this was the trap into which the musician Vincenzio Galilei and his son Galileo did not fall: the ratios for the tension of strings must increase with their squares, and not linearly.

In his letter to Bruce, instead, Kepler had sustained something different from what Stephenson attributes to him. The imperial astronomer had written to him because he wanted to get the opinion of Italians, and Bruce was then at Padua with Galileo Galilei. "Velim tamen ex aliquo excellenti Musico quibus abundat Italia, discere artificiosam et Geometricam tensionem totius clavichordij, aut si solo aurium judicio feruntur, quaero ex ipsis, an non alicubj in Organis et instrumentis duplex F, duplex A etc. fiat: Et an omnino idem sonet cantio ex G in D transposita. Velim ad jam subjectam formam tendj Instrumentum, et probarj, an aurium quoque judicio satisfaciat. Incipitur a Γ, ex qua D sic tenditur ut consonet (consonet, id est

[231] Stephenson 1994, pp. 13 and 120.

[232] Stephenson 1994, pp. 117 and 141.

[233] Kepler 1949, v. XIV, n. 128 pp. 7–16; n. 130, pp. 21–41; n. 132, pp. 43–59.

[234] Stephenson 1994, pp. 90–91.

sonum eundem teneat) cum 2/3 parte de Γ, C vero sic, ut cum 3/4 parte de Γ consonet, … Hae sunt voces indubiae. Quae sequuntur, dubitationem habent, cum bifariam possint tendj. …" ["However, I would like to learn from some excellent musician, of whom there are plenty in Italy, the fine, geometrical tension [tuning] of the whole clavichord, or if they are guided only by their ears; and I ask the same people if they do not calculate twice F [fa], twice A [la] etc. in organs, and in instruments in some places: [I would also like to know] whether, transposed from G [sol] to D [re], the melody sounds exactly the same. Then I would like the instrument to be stretched [tuned] in the form indicated below, and the question to be asked whether it satisfies also the judgement of the ears. Start from Γ [sol], from which D [re] is stretched [tuned] so that it is consonant (it is consonant, that is to say, it maintains the same sound) with 2/3 of the part of Γ, and thus also C [do], so that it is consonant with 3/4 of the part of Γ … These sounds are certain. Those that follow have an uncertainty when they can be stretched [tuned] on the double value."]

As musicians well know (and here we see that also Kepler knew), but perhaps not all astronomers, the Pythagorean scale does not allow transpositions. That is to say, in this scale, the intervals of the fifth, the fourth, … are not all equal to 2:3, 3:4, … and depend on the note from which you begin. This could be heard perfectly in organs and in harpsichords if they were tuned in the Pythagorean manner. "Sic aures non ablegantur in tensione ad octavas quintas quartas tertias, ubi decipj [sic! decepj] possunt, sed ad unisonum, quem citra errorem dijudicant." ["Thus in the tension [tuning], the ears are not relegated to octaves, fifths, fourths, thirds, in which they may be deceived, but to unison, which they judge without error."] To obtain a precise tuning, our German natural philosopher then described a theoretical instrument that took its inspiration from the cithara, called by him "directorium", with which it was possible to verify in unison the other instruments used in practice. All the numerical proportions referred here to the lengths, and he made no mention of the tension of the strings. For that, he would have had to deal with relative weights.

Stephenson's error was due not only to his admitted ignorance of the musical problems that afflicted people at the time, above all for fixed-tuning instruments like organs and harpsichords, on which it would not have been possible to modulate. He also suffered (his?) bad translation. In this case, the Latin *tensio*, which derives from *tendo*, is not to be translated into English as *tension*, but in general as *tuning*. After all, how could you stretch an organ pipe? This Bruce should therefore have read also Vincenzio Galilei and have addressed those Italians whose help the imperial astronomer sought in that period. At the end of his letter, through his English intermediary, Kepler asked Galileo Galilei a question about the local magnetic declination for the compass, referring to Mercator. It is thus clear that the letter would be shown by Bruce to the Italian astronomer. In actual fact, it ended up among Galileo's manuscripts kept in the Biblioteca Nazionale in Florence.[235] This, then, is likely to be the route by which our German natural philosopher may have

[235] Kepler 1949, v. XIV, pp. 13–15 and 459. Dickreiter 1973, pp. 76–77 and 130.

obtained an indication of the book *Dialogo della musica antica et moderna* of the Italian musician Vincenzio Galilei. On it, he subsequently meditated at the time of his mother's trial and his drawing-up the complete version of the *Harmonices mundi libri quinque*. But unfortunately, this same route would not be taken by Stephenson, who dedicated himself too much to astronomy, and too little to music, with the unhappy consequences that we see.

In the letter to Herwart, the question briefly cropped up again. "... exercitatus Organista posset solo aurium judicio de clavj judicare, vel nunquam alias audito illo Organo. Videor olim Stuccardiae aliquid hujuscemodj inaudijsse de duodecim Italorum tonis, et de geminatione quarundam atrarum clavium." ["... the expert organist might decide on the keys with the sole judgement of his ears, or otherwise, having listened to the organ, never. I think I heard something of this kind at Stuttgart, about the 12 notes of the Italians, and about the doubling of certain black keys."] On this subject , Kepler wrote that he would have been glad to ask Roland de Lassus, if he had still been alive, but this renowned Flemish polyphonist had died shortly before, in 1594, at Munich.[236] May he perhaps have heard that the Italians, including Vincenzio Galilei, tuned their organs and harpsichords by ear, dividing the octave into 12 equal intervals? But he probably had not yet read the book in which this musician had invented rules, which were not Pythagorean, to tune strings and wind instruments. At the time, Kepler instead had started to think about a way of explaining, by means of geometry, why, out of all the infinite possible proportions, music did not use more than seven. Here he wrote them as

$$\frac{1.1}{2}\ \frac{1.2}{3}\ \frac{1.3}{4}\ \frac{1.4}{5}\ \frac{1.5}{6}\ \frac{2.3}{5}\ \frac{3.5}{8}$$

Or, perhaps, our imperial mathematician had heard about Heinrich Loriti, known as Glareanus (1488–1563), who had theorised 12 musical modes, instead of the ecclesiastic 8? This Swiss theoretician was to be quoted in his subsequent letter to Mästlin. Here, he also proposed an analogy with the colours of the rainbow, again classified on the basis of their proportions, sustaining that the same could be done with tastes and smells.[237]

The same approach, of judging Kepler's musical theories by means of modern sciences, almost completely ignoring the historical behaviour of the imperial mathematician, was adopted also by Francis Warrain. However, this French scholar arrived at conclusions at variance with those of the two preceding severe critics. This study, relatively scrupulous in its attention to the *Harmonices mundi* ..., aimed to show that the results obtained by Kepler in that period not only should not be disputed, but could also incorporate the movements of the asteroids and the new planets discovered subsequently. Even the luminous frequencies of atoms were compared to the musical notes of the planets. Thus it is not so much a question

[236] Kepler 1949, v. XIV, pp. 29. Dickreiter 1973, p. 130.

[237] Kepler 1949, v. XIV, pp. 50–51. Dickreiter 1973, pp. 126–127.

of denying the harmonies, but rather of extending them to chromaticisms and enharmonic relationships.[238]

Equally well-disposed to follow Kepler in his metaphysical and musical arguments, though enhancing the historical events of his life, Alexander Koyré wrote about him. It is interesting that also this para-French philosopher and historian presented his efforts as aiming at the truth. And for Koyré, as for our imperial astronomer, this truth was to be reached in a relative independence from observational astronomy. For this reason, he should take a greater interest than others in the *Harmonices mundi* . . . , albeit, unfortunately, only in his "Book V": because this was the one which contained the Third Law. And yet, even Koyré ignored the event that Kepler had more faith in God than in human reason. Our para-French scholar even mistook him for a "faithful follower" of Pythagoras, because he left out his geometrical theory of music. In the end, this philosopher of science dropped his apparent tolerance, and sentenced that the tortuous route towards the truth, from the closed spheres of Kepler, would end up in the Infinite space of Newton. Here, "the thought of the divine Mathematician" would prove to be "completely devoid of any musical inspiration".[239] In Sect. 10.2, on the contrary, (naturally?), we shall see the musical model invented by the English scholar for that gravitational space so dear to the physicists of today.

Without the usual undisguised simplifications and aims, Gerald Holton sums up the complexity of Kepler quite well, were it not for the event that the reduced him excessively to the Pythagoreans, and completely eliminated music from his mathematical harmonies.[240] Arthur Koestler told us the novel of the life and works experienced by the imperial mathematician, trying to depict him as irrational, a fool, "... a mad spokesman of the Holy Spirit ...", a "... mad architect ...". He had decided to follow him along the course, full of shifts and second thoughts, of his discoveries, and through the unfathomable meanders of his explicit confessions. But our wandering Hungarian polyglot, essayist and writer concluded that his work was a "... Pythagorean synthesis of mysticism and science ...", confusing him with Fludd and the Rosicrucians. And yet, for him, Kepler succeeded (only in some of his works?) in separating astronomy from theology, to unite it with physics (that of Aristotle?). In short, he obtained, like a sleep-walker, those customary inedible results that he did not understand himself, which were destined to come to light with their (rational?) realisation only with Newton.

Koestler succeeded in the task of giving an effective summary of the *Astronomia nova*, with all its mistakes, which however led our sleep-walker towards the truth of the first two laws. But as regards the context for discovering the Third Law, he could only allude to "... long, patient endeavours ...".[241] However, that chatterbox of Kepler, who loved digressing on every subject, did not tell us them as such in

[238] Warrain 1942.

[239] Koyré 1966, p. 308.

[240] Holton 1993, pp. 3–23.

[241] Koestler 1991.

this case, whereas in the other he had considered it in the reader's interest to do so. May this therefore not have happened because the story had gone differently? Our Hungarian nomad, who wrote in English, was likewise ignorant of music, and therefore he could not understand that on the contrary, a "sesquialtera" ratio could be intuitive, and 'the object of a dream' in a book about harmony. Astronomy had led Kepler to discard a linear ratio between the times of revolution T and the distances R of planets from the Sun (T^1 proportional to R^1; 1:1, unison). They would become too fast, and Saturn, for example, would have to complete its revolution in just 9 years, since it is about 9 times farther from the Sun than the Earth. A double ratio (T^1 proportional to R^2; 1:2, octave) would not work, either, because the planets would move too slowly, and Saturn would have to take as long as 81 years to complete one revolution. The intermediate proportion between the two was the "sesquialtera" (T^2 proportional to R^3; 2:3, fifth), which became easy to imagine by analogy, and spontaneous when writing a book about musical intervals in astronomy, because it corresponded precisely to one of the most consonant ratios. At present, Saturn completes one revolution in about 30 years.

The musical arguments of our German natural philosopher have been of interest almost exclusively for historians of the relative theories and musicologists. Among them, Daniel Pickering Walker would appear to be the one most deeply struck. However, he suffers from the defect of judging Kepler's arguments by the ensuing justifications of consonances through harmonic tones. Therefore, as these would revitalise, by means of modern-day acoustics, some numbers of the Pythagoreans, the geometric theory of our Baroque character "… does not work very well …", above all for thirds and sixths. It is clear that Kepler derived his judgements (and prejudices) from a desire to submit music to (and discipline it by?) his philosophical-scientific convictions. We have already seen proof of this in the chapter about Vincenzio Galilei. Here we might add a couple of his observations about the Aristoxenians, who are considered "… opposed to our tradition …", and about the superparticular ratios of the Pythagoreans, seen as verified "… empirically on the monochord …". When, on the contrary, the best empiricists, thanks to their ear, were to be the former, who were also present in the Western tradition with Vincenzio Galilei.[242]

Told by Michael Dickreiter, the story of how much music Kepler must know, and how he took part in the musical life of his time, is extremely interesting. We must remember the importance given to music in the Lutheran liturgy. At school, he sang every day, and received lessons of musical theory. We have seen that our imperial mathematician quoted some of these hymns in his *Harmonices mundi* …. At Graz, he also would have encountered music in the Italian style, like that of Andrea Gabrieli (c. 1520–1586) and Orazio Vecchi (1550–1605). Nor should we forget Roland de Lassus and Adrian Willaert (c. 1490–1562). At the court in

[242]Walker 1967, pp. 238–239; Walker 1989a, pp. 72–73. Perhaps for the same reason, our English musicologist treated Vincenzio Galilei as a Pythagorean, while on the contrary, this Tuscan lutenist was rather on the side of Aristoxenus, as we have seen above in Sect. 6.7 of Part I.

Prague, our imperial astronomer may have met, among the most renowned artists, Philippe de Monte (1521–1603), Hans Leo Hassler (1564–1612) and Jacob Hassler (1569–1622). Here, they played various styles of music, from the kind with several choruses of the late Venetian Renaissance to the early accompanied monodies. The religious persecutor at whose hands Kepler suffered at Linz, Daniel Hitzler, took pains to correct also the hymn-books of that church. Would discussions take place between him and our heretic also about Luther's music, or what the correct hymns for a church should be? Kepler once wrote about a concert that he heard in the house of a noble.

Dickreiter concluded: "… wir bei der Untersuchung seiner Musiktheorie eine Kenntnis der Musik an der Wende vom 16. zum 17. Jahrhundert voraussetzen dürfen, die sowohl theoretischer Lektüre als auch in praktischer Übung erworben worden war." ["… together with the research on his musical theory, we may assume a knowledge of music about the turn of the sixteenth and seventeenth centuries,which he had acquired both in his theoretical readings and in practical exercises."][243] But our natural philosopher admitted that he did not practise the art of composing. "Artem vero componendi cantus, quae praxis est Musica, nequaquam profiteor." ["However, I do not in any way profess the art of harmonizing the singing, which is the practice of music."][244] Before returning to his Venice, Andrea Gabrieli had passed through Bavaria and Bohemia, where Kepler had lived. With his madrigals for the theatre, Orazio Vecchi must have been appreciated more highly at the court receptions attended by our German astronomer. Though his ideas of the solar system attracted him more towards the vocal polyphony of the period, it cannot be excluded that with a certain licence, he might permit himself also to enjoy some hidden Mediterranean melodies here and there.

Like every good musician and musicologist, Dickreiter noted the limitation of the choices made by Kepler for the ratios between the notes, compared with the transpositions to the different levels of the scale, and with modulations.

Without doubt, Kepler also described the equable termperament, which would make them possible without any difficulty, as we have already seen above. "Kepler ist damit der erste deutsche Musiktheoriker, der die gleichschwebende Stimmung exact beschreibt, soweit das mit den Mitteln der damaligen Mathematik möglich war." ["Thus Kepler is the first German theoretician of music who described the equable temperament with precision, as far as possible with the means of mathematics available in that period."] But one line above, our German-speaking musicologist had written that the ratio 18:17, taken as a semitone by Vincenzio Galilei, was only an approximation for this temperament.[245] Was it possible he did not know that Kepler should have calculated $\sqrt[12]{2}$, the exact value of the tempered

[243] Kepler 1955, v. XVII p. 52. Dickreiter 1973, pp. 123–138.

[244] Kepler 1619 [1969], III p. 85; 1997, p. 254. Dickreiter 1973, pp. 145–146.

[245] Dickreiter 1973, p. 158.

semitone, perfectly well?[246] Thus, he could not do it, because otherwise, he would have had to abandon his musical-astronomic construction based on polygons. However, for the imperial mathematician, sounds had become vibrations, and the string half as long, that is to say, tuned to the higher octave, produced twice as many.[247]

How the *Harmonices mundi* … was interpreted and evaluated by Marin Mersenne, John Wallis and Isaac Newton, we shall see in the following chapters. Athanasius Kircher refuted it in his *Musurgia Universalis*. Besides being Catholic, and sustaining the astronomical model of Tycho Brahe, the German Jesuit who had migrated to Rome found Kepler's harmony "… obscuram, & mysticis verborum involucris intricatam … […] … Sed hoc nequaquam prudenti astronomo satisfacere potest, cum nobis non constet utrum proportio motuum ex natura rei ita se habeat; cuius tamen diversitas observationum, prorsus contrarium ostendat, & proinde nihil certum & solidum ex hisce circa harmoniam coelorum concludi possit; …". ["… obscure and couched in mystical coverings of words ... . But this cannot satisfy a prudent astronomer at all, if there is no proof for us from the nature of the thing, whether the proportions of movements are exactly like that; however, the diversity of his observations shows exactly the opposite, and therefore from these nothing certain or solid can be concluded about the harmony of the heavens; …"].[248]

Less orthodox and conventional were the comments on the *Harmonices mundi* … made by Andreas Werckmeister (1645–1706), one of the first musicians and musicologists to sustain the validity of the equable temperament. Even if he did not always seem to understand all the aspects perfectly, especially the mathematical ones, he described it in the following terms. "Wie er dann an unterschiedlichen Orten das Temperament approbiert, und unter andern des Vincentii Galilaei Fleiß so er in Verfassung der Temperaturen angewendet sehr lobt." ["As he [Kepler] then approves of the temperament in various places, and among others the diligence of Vincenzio Galilei, so he expresses great praises in the composition applied to the temperaments."] But then, in his last (posthumous) theoretical work, he must have had doubts that in this way, he might have misunderstood Kepler. In reality, the imperial mathematician had tolerated the temperament of Vincenzio Galilei only for instruments with a fixed tuning, because he only considered the numbers of Pythagoras, Plato and Euclid to be truly harmonious. "… so saget er es sey ein höchst schädlich und verfürisch Ding wenn man durch die temperirten harmonischen Zahlen in rebus Musicis ein Judicium anstellen wolte das rechte Wesen des Gesanges dadurch zuerlangen." ["… Then he [Kepler] states that it is an extremely harmful and seductive [misleading] thing if by means of tempered

[246]The approximation of Vincenzio Galilei appears to be very good, because 18/17 equals 1.05882, compared with the root 1.05946 that is obtained from Table 8.3 of Zhu Zaiyu.

[247]Kepler 1940, v. VI, p. 388. Dickreiter 1973, p. 159.

[248]Kircher 1650, pp. 376 and 378. Dickreiter 1973, pp. 193–194.

harmonious numbers, one wanted to formulate a judgement about musical matters, to achieve in this way the correct essence of singing."][249]

It is therefore a nice paradox that Werckmeister had learnt how Vincenzio Galilei tempered his notes through our natural philosopher, who sustained something completely different. As in certain scientific questions, also in music Kepler had obtained the opposite effect to the one that he desired, in a world that was about to celebrate the splendour of the accompanied monodies, leaving aside his beloved polyphony *à la* Roland de Lassus.

In an admirable extract of fewer than ten pages, to which our modest character was reduced, the search for music would be vain. Rather, we would find "mechanicism", a profound philosophy for which the reader will judge by himself the degree of agreement with Kepler's *anima mundi*. Here we also find the sparkle of a beautiful pearl. In the Olympic contest disputed between Galileo and Kepler about who arrived first among modern scientists (in reality those of the 1960's), this general philosophical-scientific history in many volumes assigned the victor's palm to Galileo: "... Galileo occupa una posizione in un certo senso superiore a quella di Keplero." ["... Galileo occupies a position which in a certain sense is superior to that of Kepler."][250] The German's diversity had again been transformed into inferiority, with the aggravating circumstances of those intolerable nationalistic veins from which we particularly desire to stay at a good distance.

On the contrary, Silvia Tangherlini dedicated all the required space to underlining the Platonic and Pythagorean ideas present in the *Harmonices mundi* .... To find some of these roots also in the Renaissance of Marsilio Ficino,[251] this Italian scholar highlighted his effort to reconcile the neo-Platonism of Proclus with Christianity. Inhabiting in the Sun, depended on his "rational" God Kepler's decision to remain as far away as possible from any immanent form of the infinite. It was reduced, also by Kepler (hiding it from us), to the incommensurables of geometry, capable of avoiding numerical irrationals, as in the tradition of Euclid.[252] It is from these convictions that we can understand why the search for the truth in transcendence, away from sensible world, was to remain so often present and characteristic of Western sciences. Refusing the infinite for the universe left the Sun in the central position. Not using it in the ratios between notes made it possible to select those seven rational ones, excluding the equable temperament.

The fine illustrations reproduced in the popularising work written by Anna Maria Lombardi about Kepler do not exempt us from the thankless task of indicating errors, omissions and sloppiness. The complicated course towards the first two laws, and the results obtained in the field of optics are presented with interesting details ignored by us. But at a certain point in the text, the Third Law is enunciated in a nonsensical manner. In another table, certain ratios for notes are exchanged, with

[249] Dickreiter 1973, pp. 197–198.

[250] Geymonat 1970, v. II pp. 105–107 and 506–514.

[251] See Sect. 6.4 of Part I.

[252] Tangherlini 1974.

the addition of a debatable comment. Above all, the faith of our German astronomer in his variant of astrology is totally distorted, completely overlooking "Book IV" of the *Harmonices mundi* .... I will leave it to the reader to decide how serious this was, due more to the chief editor of the journal than to the authoress. A blur thus descends over the merits of this Italian graduate in physics and lover of the violin, in enhancing the essential function of music in the overall work of Kepler, which partly inspired the famous 3/2 of his last law. Unfortunately, also on this subject, she expresses anachronistic judgements, and ignores the equable temperament, perhaps because she is less sensitive to it, not playing the piano, but the violin. It is more serious that she cancelled the general polyphonic sense from the general structure of the *Harmonices mundi* ..., and condemned the work as a whole as "Attualmente ... non solo privo di senso, ma anche scorretto ..." ["At present ... not only without sense, but also incorrect ..."]. What is more, (perhaps she had not read Dickreiter?), she presents us with a Kepler "... rimasto isolato dalle novità musicali che si stavano sviluppando in Italia ..." ["... who was isolated from the musical novelties that were developing in Italy"]. Above, we have just seen the opposite, thus he knew them, and for clear reasons, he did not agree with them.[253]

To those who still believe in the exclusive resolving force of experiments, I would like to propose the following one. To verify whether the planets in their revolutions emit music that can be heard by our ears, and what this music is, the solar system would need to be filled with an adequate terrestrial air. Kepler wrote, "Jam soni in coelo nulli existunt nec tam turbulentus est motus ut ex attritu aurae coelestis eliciatur stridor." ["... No sounds exist in heaven, and the motion is not so turbulent that a whistling is produced by friction with the heavenly air."][254] The experiment would be difficult to perform, not so much because it would be a bit expensive, but because nobody knows how to divert military expenses for this purpose. In the era of the supercomputer, however, it might be possible to use it to simulate a solar system that revolved, not (unstably?) in an ethereal void, but in air. What notes would then be produced?

Not only colleagues and religious figures of several faiths have continued to persecute Kepler for centuries. Another war did so, 300 years later, harassing the person of Max Caspar. Under the bombing raids, he wrote the best biography of Kepler, while another Thirty-Years-War were finishing, the years that began in 1914, and were to come to an end (maybe) in 1945. Curious, isn't it? Was it Saturn again that was in opposition to Jupiter in Capricorn?

Recently, Kepler's hand-written comments on the *Dialogo della musica antica et moderna* printed by Vincenzio Galilei have been published. Among the most interesting observations, we can read the following. With the ratio "$\frac{\sqrt{72}}{9}$", the German astronomer could divide the Pythagorean tone 9:8 into two equal parts, even if he did not want to accept it. Sheet after sheet, he noted down the protagonists, ancient and modern, of the dispute. In the one between Zarlino and Vincenzio

[253] Lombardi 2000, *passim*.

[254] Kepler 1619 [1969], V p. 197; 1997, p. 423.

Galilei about the intervals of the third and the sixth taken from Ptolemy by the Venetian, he wrote, "Negat Galilaeus. Sto à Zarlino, ut in secundis teneam Galilaeum. … pugnant pater et filius ridiculè, utri Keplero nomen." ["[Vincenzio] Galilei denies it. I am on the side of Zarlino, as I consider Galilei the second. … father [Vincenzio] and son [Galileo] quarrel in a ridiculous manner, both the one and the other contend for the name with Kepler."] When he found Aristoxenus in the text, he generally criticised him.

As regards the main question of the choice between the ear and numbers, here Kepler expressed his side particularly clearly. "Auditus servus est ratio domina, incipit cognitio ab auditu, sed ratio consummat harmonices perceptionem, … […] … Pythagoras numeros spectavit, praeterijt itaque consonantias imperfectas: non satis attendit sensum, indulsit speculationi. Aristoxenus attendit sensum, qui cum esset crassus, nimio plus indulsit sensu, et cum latitudine nimia assumpsit speculationum principia. Ptolemaeus obtinuit sensum cum ratione conjunctum, at in Aristoxeno habet commune quaedam discordantia, sed non consentiunt … anima concordantibus cum Pythagoraeico vero contemplationes numerorum, … Ego rursum ad Aristoxeni sensus accedo, sed illos instruo sensus ratione aliter quam Ptolemaeus: Hic vult ratione super numeris de parte dextra e propinquo audiri, sensum eminus et a parte sinistra astipulari, ita ratio illi seducit sensus quia est erronea. Ego verò sensui rationem addo informatorem et quasi paedagogum in qua discat sua vi uti et rationem non super numerorum arcanis, sed super communibus axiomatibus dialecticis impossibilitatis." ["The hearing is the servant, reason is the mistress; knowledge starts from the hearing, but reason perfects the perception of harmony, … […] … Pythagoras examined numbers, and thus overlooked the imperfect consonances [third and sixth]: he did not observe the sense [of hearing] sufficiently, he took the liberty of speculating. Aristoxenus observed the sense [of hearing], but as this was unrefined, he took the liberty of listening too long, and assumed too many speculative principles in abundance. Ptolemy held sense fast together with reason, and yet he has in common with Aristoxenus some dissonances, but the considerations of numbers, together with the truth of the Pythagoreans, do not reconcile consonances with the soul. On the contrary, I am close to the senses of Aristoxenus, but I prepare the senses with a reason different from that of Ptolemy. On the right, he wants with his reason that the sense be listened very closely, more than the numbers, and also on the left, to fit in with them from far off; thus his reason pulls the senses to himself, because it is wrong. I, instead, add to the sense the reason that guides, a kind of educator, in which you learn to make use of its force: also, a reason, not over the arcana of numbers, but over the general axiomatic dialectics of impossibility."][255]

Let us stop here. We may observe that, with Galileo Galilei and Johannes Kepler, the journey of European natural philosophers towards transcendent (religious in part) and dualistic scientific truths, expressed by mathematics, was now well under way. They were starting to lose Latin as a common language, to speak their various

[255] Kepler 2009, pp. 21, 24, 28–29. Cf. Kepler 1619.

national languages (all alphabetic and linear, with that dualistic characteristic of being able to use small or capital letters): Galileo did this most. But wouldn't those local, contingent terms, subject to their relative history, have cast doubts on the great claims of constructing laws outside space and time, with a consequently universal and eternal validity? Without doubt, Kepler still thought that Latin should continue to be the perfect language, not only for Christianity, but also for sciences, as participants in the truths transmitted by God to the souls of men. How, then, were they to solve this problem? The following natural philosophers were to decide differently, inventing a special symbolism for mathematics, made independent of national languages and therefore suitable for that purpose.[256]

> They castrated the Poet, so that he would not be lewd,
> and after taking the testicles away, the life remained.
> Beware, Pythagoras, they say you think too much:
> they leave the life and first destroy the mind.
> JohannesKepler

> You also know that the sciences
> that I love and cultivate most
> are those of higher mathematics
> thanks to which I subtract from time
> and thanks to which I steal from fame
> the right and the task
> of producing new discoveries;
> for when in my tables
> I descry the novelties
> of the centuries that will come,
> I take from time the privilege
> of narrating what I have said.
> Calderon de la Barca, *Life is a dream*, I 612–623.

[256]For different points of view, see Gozza 2000.

Chapter 9
Beyond Latin, French, English and German: The Invention of Symbolism

Le silence éternel de ces espaces infinis m'effraie.
Blaise Pascal, *Pensées,* [233].

9.1 From Marin Mersenne to Blaise Pascal

In 1628, Kepler had written that his adversary, Robert Fludd, had entered into a debate also with the Frenchman, Marin Mersenne (1588–1648), who had attacked him. In 1623, between the German and the Englishman, Mersenne had taken side with the first in his *Questiones … in Genesim… [Investigations … on Genesis …]*, "holding tight to my hair" (that of the astronomer). Now, therefore, Kepler expected this occultist doctor, in defending himself, to damage him.[1] Among the notable characters taking part in the scientific discussions of the seventeenth century, Mersenne was the one who, together with the famous imperial mathematician, had dedicated most attention to music, and to harmony in general. For this reason, unlike Kepler, he was to achieve even more success in this field, leaving his name associated with the first law of future acoustics, the one from which we started our history.

He gave his most important book the title of *Harmonie Universelle, Contenant La Theorie Et La Pratique De La Musique [Universal Harmony, including musical theory and practice]*.[2] In this, some parts seemed to be taken literally from the astronomer's *Harmonices Mundi Libri Quinque*, such as the analogy between the major third, masculine achievements, war and generation with respect to the

[1] Kepler 1983, v. XI-1, pp. 471–472. Baumgardt 1951, pp. 181–182. Lenoble 1971, pp. 103–109 and 367–370. Amman 1967, pp. 210–219. Crombie 1981, p. 320.

[2] Mersenne 1636. He dealt with harmony in several other books, written and published from 1623 to 1648. Lenoble 1971, pp. XII–XXV.

T.M. Tonietti, *And Yet It Is Heard*, Science Networks. Historical Studies 47,
DOI 10.1007/978-3-0348-0675-6_3, © Springer Basel 2014

feminine minor third, or the modes simplified into two categories.[3] But this French friar, who belonged to the Order of the Minims, founded by S. Francesco da Paola (1416–1507), did not accept the geometrical explanation elaborated in order to exclude the number seven from the generation of consonant ratios.

If the discriminating criterion was to be incommensurability, then 7 to 2 and 7 to 1 would not be "… irrationels, & incommensurables, …" ["… irrational and incommensurable, …"]. "… puisque tous les battemens d'air que font les Consonances ou les Dissonances sont commensurables; car il n'y a nul nombre de mouvements ou de battements d'air, qui ne soient commensurables à tous autres nombres de mouvements." ["… because all the strokes of the air that form consonances or dissonances are commensurable, seeing that no number of movements or strokes of the air exists that is not commensurable with all the other numbers of movements."] Mersenne would have tolerated Kepler if the latter had been content with constructing analogies. But he did not approve of the demonstration. He, too, (at last) used fractions; he added the numbers 1, 2, 3, … and compared them together to obtain the desired consonances. "… il ne s'ensuite pourtant pas que 7 comparé à 6, & à l'unité, ou à quelqu'autre nombre, ne soit Consonance, quoy qu'il vienne de l'addition de 6 & 1, de 3 & 4, & de 2 & 5; …" ["… it does not follow therefore that 7 compared with 6 or with one, or with any other number, is not consonant, seeing that it comes from adding 6 to 1, 3 to 4, or 2 to 5; …"].

The French scholar was not satisfied with the explanation supplied by Plato and his followers: that those numbers were impressed from birth in men's soul, and could thus be aroused by the corresponding sounds. "… car il faudroit premierement monstrer que la raison sesquisixiesme, sesquiseptiesme, & les autres qui ne font point d'accords, ne sont point dans l'ame, …" ["… because it would be necessary, first of all, to show that the ratio of the sesquisixth [7 to 6], and the sesquiseventh [8 to 7] and the others that do not form chords, are not present at all in the soul …"]. Nor would invoking the ear or air strokes have solved the problem. Why should the ratios of dissonances offend it, and not the others? "… car les battemens de la Tierce & de la Sexte mineures commencent à deplaire, & approchent bien pres des Dissonances, car elles ont plus de battemens d'air qui ne s'unissent point, que de ceux qui s'unissent, …" ["… because the strokes of the minor third and the minor sixth start to become unpleasant, and come very close to dissonances, seeing that they have more air strokes that do not unite than those that unite, …"].

However, Mersenne complied with the seven traditional consonances, like Kepler, although he had not found any stringent reasons to do so. Seeing "… que la

[3]Mersenne 1636, [tr. 4] pp. 188–190. This long essay was composed of different Treatises on various topics, each numbered separately in the pages. I have assigned a number to the Treatises, for convenience in referring to them, in accordance with the order in which they were printed; details are in the Bibliography.

longue coustume puisse introduire l'usage de la sesquisixiesme & sesquiseptiesme … puisque la pratique y est conforme … que les concerts sont principalement composez des dites Consonances … finalement que les 7 chordes qui font ces 7 intervalles consonans, font trembler les autres chordes plus fort que nulles autres, puisque leurs tremblemens se rencontrent plus souvent, & consequemment que les dites chordes estans touchees frappent plus souvent les chordes qui n'ont pas esté touchees, …". Seeing ["… that a long-standing usage can introduce the use of the sesquisixth and the sequiseventh … because practice conforms to it [to the seven consonances] … and that concertos are mainly composed of these consonances … and that lastly, the seven strings that form these seven consonant intervals make the other strings vibrate more than all the rest, because their vibrations meet more frequently, and consequently these strings, when touched, strike those that are not touched more often …"].

And yet, having rejected Kepler's geometrical explanation, the French natural philosopher declared that he was not satisfied with the practical and experimental explanations, either. "… dautant que si le plaisir de la Musique commence par la consideration de l'esprit, qui est capable de contempler toutes sortes de raisons, il faudroit dire pourquoy les intervalles dissonans luy deplaisent dans les sons, puis qu'ils ne luy deplaisent pas dans les lignes, ny dans les figures; …" ["… seeing that if the pleasure of music starts from the consideration of the spirit, which is capable of contemplating all sorts of reasons, it is necessary to explain why dissonant intervals are displeasing to it in sounds, when they are not displeasing in lines and in figures; …"]. But to him the pleasure of music appeared to be thoroughly natural, affecting both the ignorant in spirit and the learned. Then Mersenne fell back on the "… chatoüillement des esprits animaux & des nerfs, …" ["… titillation of animal spirits and nerves …"].[4]

Our Christian brother had rejected the idolatry of the number 7 as a universal reason, effective also in music. In the dualism between heaven and earth perpetually in conflict, it would not be possible to reconcile, in that number 7, the 3 of the former and the 4 of the latter. Furthermore, he seemed to suggest that consonances were the fruit of a practical convention. Jean des Murs (1291–1351), a renowned theoretician and precentor of Notre Dame at Paris, had sustained that also the ratios 9 to 4, 9 to 2 and 9 to 1 were consonances. Thus there could be more than seven of them, or they could be reduced to the first two, unisonand the octave. In this way, Mersenne opened up spaces towards usage, the ear, and the practice of concertos.

However, after criticising and undermining the other justifications, our French scholar did not hesitate to propose one of his own, because it was (unconditionally?) true, also for him, that certain ratios between sounds were pleasant, and others less so. Indeed, he wanted to attribute a significance to this characteristic property of music. In the case of food and smells, men showed different personal tastes, but everybody would find the octave and the fifth pleasant. "… les sons approchent plus pres de l'esprit & de l'intellectuelle, que l'objet des autres sens. … les sons peuvent

[4]Mersenne 1636, [tr. 4] pp. 85–88.

apporter plus de lumiere à la Philosophie que nulle autre qualité; c'est pourquoy la science de la Musique ne doit pas estre negligee, quoy que les chants & les concerts fussent entierement abolis & defendus; car ils ne sont pas la principale fin de la Musique, comme croyent les Praticiens, qui mesprisent, ou ignorent la raison." ["… sounds come close to the spirit and the intellect more than the object of the other senses … sounds can throw light on philosophy more than any other quality; this is the reason why the science of music must not be neglected, even if songs and concertos were to be completely abolished and prohibited, because they are not the main purpose of music, as practical experts who despise and ignore the reason for it believe."]

Thus Mersenne had faith in an absolute reason for sounds, which he sought in the realm of the heavenly spirit, and not the earthly one, because hearing is liable to make mistakes. In the end, therefore, he too remained anchored to ratios between whole numbers. And he struggled to explain why the interval based on the ratio 5 to 7 was less pleasant than the minor sixth, 5 to 8. He continued to fall back on the theory of the strokes that coincided in the ear, which, despite appearances, would favour the second interval for him as the more consonant one. Two sounds whose ratio is 5 to 7 "… s'unissent toujours ensemble à chaque 7 coup du plus aigu, & que les 2 sons de la Sexte mineure ne s'unissent qu'à chaque 8 coup, …" ["… always unite together every 7 strokes of the more acute one, whereas the two sounds of the minor sixth [5 to 8] only unite every 8 strokes …"], clearly. But the intervals were not to be separated from their complements, and had to be taken together with them to form the octave. Therefore, "… 5, 7, 10 ne s'unissent pas si viste ensemble que ces 3 autres 4, 5, 8, qui surpassent les precedens de 2 degrez d'union dans le son plus aigu, & de 2 autres degres dans le son du milieu, & d'un degré dans le premier: …" ["… 5, 7, 10 do not unite together so readily as these others, 4, 5, 8, which are two levels higher than the foregoing in the highest sound, again two levels higher in the middle sound, and one level in the lowest one: …"].

Even the number 7 could now be readmitted, but (following the model of Aristotle) it was used to construct an analogy between the 7 consonances and 7 colours, yellow, red, purple, green, blue, black and white. However, the Minim Friar excluded incommensurables from his *Harmonie Universelle*.[5]

By means of this universal harmony, Mersenne intended to represent every quantitative relationship between natural phenomena, seeing that it would be rational and stable for him. He dedicated several of his writings to the problem; some included the term "harmony" in the title, even if they were written in French, or also in Latin. This word seemed to be the most appropriate one for him, not so much in view of its Greek etymology, but above all because he was deeply convinced that sounds were the key to the universe. "A sçavoir si l'on peut representer la quadrature du cercle, la duplication du cube, & toutes les choses du monde par le moyen des Sons." ["In order to discover whether it is possible to represent the quadrature of the circle, the duplication of the cube, and all the things in the world by means of

[5]Mersenne 1636, [tr. 4] pp. 82–90.

sounds."] A loud sound would be capable of great violence: it caused a person to lose his mind, it ruined wine in the cellars, it killed babies in their mother's wombs, it broke panes in the windows of houses.

This French scholar preferred to argue by describing concrete experiments. It was sufficient, then, to take strings and make them vibrate at the desired ratios, e.g. one as long as the diameter and the other as long as the circumference, and the heard sounds would have represented precisely the problem of π 3.14159. "... mais si l'on oyt ces Sons ensemble, ils font des Dissonances qui seront d'autant plus mauvaises que les chordes, ou les lignes que les Sons representent sont plus irrationelles." ["... but if these sounds are heard together, they create dissonances that will be so much the worse, the more the strings or the lines represented by the sounds are irrational."] In this way, Mersenne thought it was possible to reproduce all geometry and arithmetic. In this way, he remained inside the general pattern of ratios, which dominated these traditional subjects. He undoubtedly knew that it was possible to divide the tone into 2 equal parts, and the octave into 12 semitones, obtaining different intervals from those generated by means of the harmonic and arithmetic ratios. The "plus doux" ["sweeter"] consonances were those that derived from the arithmetic ratios.

"... l'on peut representer tout ce qui est au monde, & consequemment toutes les sciences par le moyen des Sons, car puis que toutes choses consistent en poids, en nombre & en mesure, & que les Sons representent ces trois proprietez, ...". "D'où il s'ensuit que le parfait Musicien peut inventer des dictions, & une langue parfaite, qui signifie naturellement les choses, & qu'il peut enseigner les sciences sans user d'autre langage que de celuy d'un Luth, ... Et si quelqu'un avoit l'oreille assez bonne & assez sçavant, il pourroit discerner & recognoistre les proportions de toutes sortes de lignes par le moyen des Sons, & consequemment il pourroit expliquer toutes les propositions de la Geometrie en ioüant de tel instrument qu'il voudroit, ou en chantant, ...". ["... it is possible to represent everything that exists in the world, and consequently all the sciences, by means of sounds, because everything consists of weights, numbers and measurements, and sounds represent these three properties, ...". "It follows from this that the perfect musician can invent expressions and a perfect language which will indicate things naturally, and he can teach sciences without making use of any other language than that of the lute ... And if anyone had such a precise, expert ear, he could distinguish and recognize the ratios of every sort of line by means of sounds, and consequently he could explain all the propositions of geometry, by playing the instrument that he desired, or by singing, ..."].

Unfortunately, such precise hearing did not exist, and the perfect musician had not yet appeared on earth. However, considering the vibrations of strings, sounds also involved ratios like the one between the diagonal and the side of a square. Therefore, sounds proved to be more powerful than arithmetic, and were equal to geometry. As a result of all this, "... l'on peut establir une nouvelle science qui

se nomme Psophologie, …" ["… it is possible to establish a new science called Sophology [wise discourse], …"].[6]

Father Marin reasoned and argued in music using whole numbers. In their values and their terminology, these might seem to be the same ones used in the variants of the ancient Pythagorean-Ptolemaic tradition proposed by Zarlino and Kepler. And yet the Frenchman's pages were changing their meaning. Numbers no longer appeared to be the abstract, disembodied ones of Plato, but served to measure things that existed in the world, and parts of the Earth that could be observed. "… le son se fait par les battemens de l'air, ou par les allees & venuës, ou tours & retours de la chorde, … les plus longues font moins de retours, & battent moins de fois l'air que les plus courtes, … des sorte que la raison de la multitude des retours que font les chordes, est inverse de leurs longueurs, …, la chorde de 100 pieds de long ne bat qu'une seule fois l'air, pendant que celle d'un pied de long le bat cent fois. … C'est pourquoy il faut marquer les Consonances suivant le nombre des battemens de l'air qui produisent les sons, & non pas selon la longueur des chordes, laquelle n'est pas la cause formelle des sons, mais la cause efficiente & externe." ["… sound is produced by the strokes of the air, or by the turns and the returns or the movements to and fro of the string … the longer ones do less turns, and strike the air fewer times than the shorter ones … with the result that the ratio of the number of turns of the strings is the inverse of their lengths … a string 100 feet long only strikes the air once, whereas the string that is one foot long strikes it 100 times … This is why the consonances must be indicated in accordance with the number of air strokes that sounds produce, and not in accordance with the length of the strings, which is not the formal cause of the sounds, but the external efficient cause."][7]

Thus Mersenne gave the numbers of the sounds in ascending order, increasing in pitch, for example, by a fifth and by an octave 2, 3, 4. The ratios of the lengths were their inverses in descending order, 1/2, 1/3, 1/4 or (maintaining the proportions) 6, 4, 3, which diminished. He preferred the simplicity of the first numbers, placed in an arithmetic succession, compared with the second group of the harmonic succession. But what did he mean by the "turns and the returns or the movements to and fro of the string"? "Il faut donc remarquer que l'aigu du Son ne vient pas du mouvement plus viste des corps ou de l'air, mais de la seule frequence ou vitesse des retours ou reflexions du dit air, ou des corps qui le battent & qui le divisent. C'est peut-estre pourquoy l'on dit que l'objet de la Musique est le *nombre sonore*, parceque le Son est d'autant plus aigu que l'air est battu plus de fois, & que le nombre de ces battemens n'est autre chose que le grave & l'aigu, & l'oreille ne peut juger du ton qu'elle oyt, si elle n'a esté battuë autant de fois de l'air, comme il a esté battu de la chorde ou des autres corps, de sorte qu'on peut dire que l'action de l'ouye n'est autre chose que le desnombrement des battemens de l'air, soit que l'ame les conte sans que nous l'appercevions, ou qu'elle sente le nombre qui la touche: car Platon croid qu'elle est un nombre harmonique; …". ["It should therefore be noted that the

[6]Mersenne 1636, [tr. 1] pp. 42–44.

[7]Mersenne 1636, [tr. 4] pp. 97–99.

acute element of sound does not derive from the faster movement of bodies or of the air, but only from the frequency, or speed, of returns, or reflections of the said air, or of the bodies that strike it and divide it. For this reason, perhaps, the object of music is said to be the *sonorous number*, because the sound is higher, the more times the air is struck, and the number of these strokes is no different for the deep and the high sound; the ear cannot judge the tone that it hears unless it is struck the same number of times by the air, as it has been struck by the string or by other bodies. As a result, the action of hearing can be said to be nothing other than numbering the strokes of the air, whether the soul counts them without our perceiving them, or whether it hears the number that touches it: because Plato believes that it is a harmonic number; . . . "].[8]

In the king's enormous stables, the scholar of sounds had stretched out some extremely long cables, and had succeeded in perceiving that they oscillated up and down, while emitting particularly deep sounds. Perhaps he had even succeeded in counting the strokes, what we, nowadays, would call vibrations.[9]

We have already seen these *battements* in Maurolico and Benedetti under their Latin names of *ictus* and *percussiones*.[10] But now the French Friar associated the blows and the strokes with the movement of the string that generated them, and perhaps succeeded skilfully in counting some of those associated with the deeper sounds. He had transformed them into sensible observations, and in this way, his eyes could count, in the wavy oscillation of the string, what his ears perceived only as the pitch of sounds, which arrived at them by means of the induced strokes of the air.

The inverse relationship between the pitch of the sound and the length of the string was no longer asymmetric; not knowing so far how to specify the former any better in an independent manner, it had assumed as its own the numbers of the latter. Thus, the relationship of inverse proportionality had been taken as a definition, with the aim, if possible, of being able to classify sounds by means of the length of strings.

Mersenne, however, had invented another property: the longer the strings, the less fast they could oscillate upwards and downwards. This monk who studied sound sometimes used to compare it to the movement of water, disturbed by stones thrown into it, or by a body immersed in it for washing. But he also mentioned the idea of Democritus and Epicurus that sound was made of the movements of tiny invisible atoms emitted by bodies, which arrived at the ears. In any case, he described their propagation as a *continuum*.[11] Brought into the foreground by the Arabic tradition, analogously with the waves of the sea, and also those taken into

[8] Mersenne 1636, [tr. 1] pp. 23–24. This was probably the first time that the term "frequency" had been used for the pitch of a sound. We shall see in Sect. 10.1 the idea of an unconscious counting in the soul, taken up by Leibniz.

[9] Mersenne 1636, [tr. 1] "Seconde Observation" and p. 23; [tr. 4] p. 139. Crombie 1981, p. 319.

[10] See Part I, Sects. 6.5 and 6.6.

[11] Mersenne 1636, [tr. 1] pp. 4–10.

consideration by Benedetti,[12] air waves were transferred, taking their origin from the undulatory movements of the string. Here, they could be observed directly in the cases of the deeper sounds. The traditional discrete theory of separate strokes transported to the ear now found in the *Harmonie Universelle* physical support and a continuous substratum. Transferred on to the Earth, the Pythagorean model of explanation was losing some of its more characteristic aspects. And yet one of these was maintained, also by our French scholar, as it had resisted the geometry of Kepler.

Let us read, then, how Brother Mersenne considered the equable temperament of Aristoxenus, and in particular Vincenzio Galilei who had reproposed it. He admitted that with "11 moyennes proportionelles" ["11 proportional means"] the octave could easily be divided into 12 equal semitones. "… je sçay que l'oreille n'est pas capable d'appercevoir la difference des Consonances qui viennent de cette division d'avec celles dont j'ay parlé. … mais j'ay mieux aimé suivre la justesse & la facilité des nombres & des raisons ordinaires recevës par tous les anciens, & preferer l'intelligence au sentiment qui n'est pas capable d'election, ny de jugement. … quoy que l'oreille ne soit pas capable d'en appercevoir la precision." ["… I know that the ear is not capable of perceiving the difference of consonances that come from this division with those that I have spoken of … but I have liked to follow the correctness and ease of [whole] numbers and of the ordinary ratios received from all the ancients, preferring intelligence to sentiment, which is not capable of choosing or of judging … seeing that the ear is not capable of hearing their precision."][13]

With respect to the Aristoxenians, he aimed "… à faire voir la justesse des intervalles, & à conserver les systemes du canon Harmonique; quoy qu'il soit permis à un chacun de suivre Aristoxene , …" ["… to present the correctness of the intervals, and to maintain the systems of the harmonic canon; insofar as anybody is allowed to follow Aristoxenus, …"]. He doubted whether the ancient Greek musician had ever calculated the 11 proportional means needed to divide the octave, by using numbers, into 12 equal parts. Here, he now did so (Table 9.1).

But his aim was that of repeating the traditional Pythagorean-Euclidean-Ptolemaic commonplace that incommensurable ratios, also in music, could not be expressed by means of (whole) numbers. If musical instruments could undoubtedly be tuned in accordance with the manner of Aristoxenus, with intervals that were all equal, this depended on the ear. These ratios would be "… si pres de ceux que j'ay expliqué par nombres, que l'oreille n'en peut quasi remarquer la difference, …" ["… so close to those that I have explained by means of [whole] numbers, that the ear can hardly notice the difference, …"].[14] It should be underlined

[12]The analogy with waves in water would also be found in a page of Greek attributed to the Stoics, and in Boethius, but these were outside the Greek traditions for music which passed into Arabic culture; cf. Burnett 1991, p. 56. Here, Part I, Sects. 6.2 and 6.6.

[13]Mersenne 1636, [tr. 4] p.112.

[14]Mersenne 1636, [tr. 4] pp. 115, 132. In any case, he gave numerical rules about how to divide the neck of the lute in the equable temperament; [tr. 1] "Preface Generale" [v–vii]. He had still

		Demitons égaux, I	Demitons inégaux. II
1	C	100, 000	100, 000.
			Demiton majeur
2	♯	105946	106666 $\frac{2}{3}$
			moyen
3	B	112246	112500
			majeur
4	A	118921	120, 000.
			mineur
5	xg	125993	125000.
			majeur
6	G	133481	133333 $\frac{1}{3}$
			majeur
7	xf	141422	140947 $\frac{5}{9}$
			moyen
8	F	149830	150, 000
			majeur
9	E	158741	160000
			mineur
10	xd	168179	166666 $\frac{2}{3}$
			majeur
11	D	178172	177777 $\frac{7}{9}$
			moyen
12	xc	188771	187500
			majeur.
13	C	200,000.	200, 000.

Table 9.1 How Mersenne divided the octave into 12 equal semitones (Mersenne 1636, p. 132)

that the '*hardly*' weakens the force of the argument considerably. Indeed, as the sciences started to oscillate from the field of exact, eternal, universal laws into that of approximate, contingent models, it could perfectly well be overturned in favour of the opposite hypothesis. Seeing that the ear could *hardly* perceive the difference, or better, it could do so if it wanted to perceived it, why not adopt the

maintained the 'Pythagorean' essence of his initial writings about music, although in other aspects he had introduced modifications. Cf. Lenoble 1971, pp. 527–529.

equable temperament, which musicians appreciated, because it granted them greater freedom of expression? But perhaps this had become the problem.

And sure brother Marin had written, a few pages before, "… ce n'est que la profonde connoissance de la Musique en diminuë le plaisir, & que le contentement que l'esprit reçoit de la speculation des raisons Harmoniques l'occupe tellement, qu'il n'y laisse point de place pour le plaisir sensible & corporel; car il semble que la capacité que nous avons d'estre touchez & affectez des voluptez sensibles, se diminuë à proportion que l'esprit s'addonne aux plaisirs intellectuels, & que les actions de l'entendement chassent & aneantissent peu à peu celles des sentimens, qui sont comme assoupis & endormis en ceux qui sont mors aux voluptez passageres, dont la mort est en grande estime envers Dieu, …" ["… it is that a deepened knowledge of music diminishes its pleasure, and the satisfaction received from the spirit by speculating about harmonic ratios occupies it so much that no space remains for sensible or corporeal pleasures; because it seems that the capacity that we show for being affected and touched by sensible pleasures gradually diminishes proportionately as the spirit dedicates itself to intellectual pleasures, and the actions of the intellect drive out and gradually annihilate those of the feelings, which are as if drowsy and anaesthetised in those who are dead to passing pleasures, which death is highly prized by God, …"]. "… cette esperience de Musique, qui amolist, & enerve le courage, & qui émousse la pointe de l'esprit des jeunes gens, devroit estre bannie des Republiques, comme toutes les autres choses qui corrompent les bonnes moeurs, …". ["… this experience of music, which softens and debilitates courage, and blunts the spirit of young men, should be banished by States, like all the things that corrupt good customs, …"].[15] Here, the Minim friar had undoubtedly taken his inspiration from Plato's *Republic*.

Even from the fall of heavy bodies, our Frenchman knew how to obtain musical harmony. "Determiner de quels lieux les poids doivent tomber pour faire telles Proportions Harmoniques, & tels Accords, ou Discords que l'on voudra, lors qu'ils se rencontrent vis à vis les uns des autres. Si la doctrine d'Aristote est veritable, à sçavoir que le son est dautant plus aigu qu'il se fait par un mouvement plus viste, ou si les reflexions, & boüillons de l'air sont dautant plus frequens que les mobiles se meuvent plus viste, l'on peut donner les lieux d'où ils doivent tomber pour faire de bons accords: …". ["Determine from what places bodies should fall to produce certain harmonic ratios and certain consonances or dissonances as desires, when they meet each other. If Aristotle's doctrine is true, that is to say a sound is the more acute, the faster the movement by which it is created, or if the reflections and the agitations of the air are the more frequent, the faster mobile objects move, the places can be fixed from which weights should fall to produce good consonances: …".]

In these pages, he was able in calculating the distances covered by weights falling for 10″, 20″, 30″ and so on. In this way, he succeeded in obtaining numbers that stood to one another in the desired musical ratios.[16] For this reason, he took up

[15]Mersenne 1636, [tr. 4] p. 118. [tr. 1] "Preface generale" [v] and [xi]. Lenoble 1971, pp. 522–531.
[16]Mersenne 1636, [tr. 4] pp. 133–137.

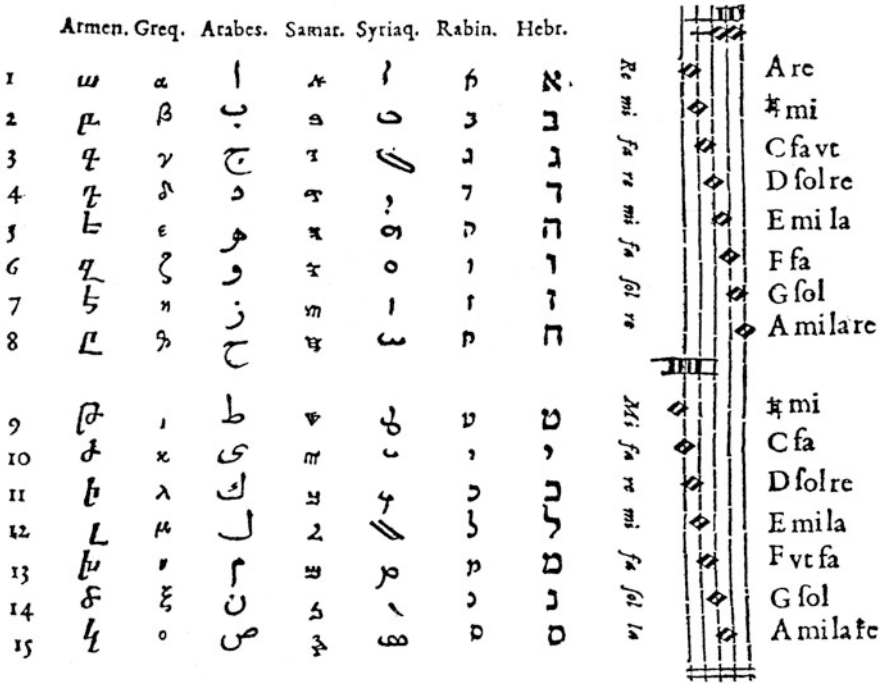

Fig. 9.1 How Mersenne represented musical notes by means of the letters of the different cultures (Mersenne, ibid., p. 253)

the theory of Galileo Galilei, which he quoted and discussed in great detail.[17] In general, the event that Mersenne was closer to Galileo than to Kepler made his *Harmonie* characteristic, as we shall subsequently see better.

Among the diatonic, chromatic and enharmonic genres, or the different ecclesiastical modes, Mersenne seemed to admit that the preference given to one or the other derived from custom and teaching: it was not at all "natural". He compared the different notations for music, which used the letters of the respective alphabets, including even the Arabs (Fig. 9.1).

He wrote that Jews, Arabs, Greeks and all the other nations could conform to our way of singing and writing every kind of song.[18]

As regards Kepler, Mersenne briefly took up the relationships between astrological aspects, movements of the stars and consonances in music, but only to move away from them. For him, notes were obtained best and first from the linear monochord; whereas it would not be possible in reality to make the circular one, used in the astronomer's *Harmonices Mundi* . . . , vibrate, as they did not know how

[17]Mersenne 1636, [tr. 1] "Livre Second".

[18]Mersenne 1636, [tr. 4] pp. 251–253.

to stretch it. Furthermore, as the sounds were made by movements of air, "… la consideration des figures de Kepler n'est nullement necessaire pour l'intelligence de la Musique." ["… in order to understand music, it is not at all necessary to take into consideration Kepler's figures."] The Frenchman did not seem to trust him, as a result of the relatively precise results achieved in astronomy. He sustained that the movements and distances of stars were not sufficiently well known to demonstrate anything, in spite of the "… lunettes d'approche …" [telescope], "… de sorte que Kepler a eu sujet de reformer les mesures & les distances qu'il donne aux Planettes, & consequemment leurs Consonances ou leurs Dissonances: …" ["… with the result that Kepler had ground to recalculate the measurements and distances that he attributes to the planets, and consequently their consonances and dissonances: …"].[19] In its 'modernity', as we saw in the previous chapter, his insinuation about the German scholar was to cause him to gain some consensus with the philosophers of 'progress' and 'mechanicism'.

In reality, this Minim friar had assumed an all-too-prudent position as regards Copernicanism, reconcilable with the Church of Rome. We can align him with Copernicus, Andreas Osiander (1498–1552) and many Jesuits. Sure, he sustained that there was not sufficient proof as regards the Earth in movement, but that it could be assumed as a plausible mathematical hypothesis, useful for calculations. It was clear for him that the ecclesiastic hierarchies had every right to protect the minds of their faithful, above all from the attacks brought by Giordano Bruno (1548–1600) against Christianity.

As Galileo Galilei was one of his favourite interlocutors, even if the scholar from Tuscany never condescended to answer his letters, he contemplated for a while standing up for him in writing. But, as was foreseeable, he did not do anything. On one essential point, their ideas were very different. Whereas Galileo claimed to be able to demonstrate his truths with certain proofs, Mersenne limited himself to the appearances of phenomena, even if they could be known by means of experiments. Reinterpreted and, where necessary, corrected in this way, father Marin attempted to promote the works of the Italian natural philosopher in France. The truest truth, also in earthly questions, was to remain reserved to God, for the devout friar. Human beings would have to be content with searching for those approximations that were within their reach. Thus he repeated the observations of Galileo, sometimes with different results, and also conducted some on vibrating strings.

"… je suis bien esloigné de vouloir demonstrer ce que je prouve par l'experience, qui sera suivie de tous ceux qui la feront, … afin que l'on ne croye pas que j'use tousjours de la diction *demonstrer* ou *demonstration* dans une sens Mathematique; ce que ceux-là concluront aysément qui sçavent la difficulté qui se rencontre à demonstrer aucune chose dans la Physique, dans la quelle il est tre-difficile de poser

[19]Mersenne 1636, [tr. 5] pp. 27–31. Sometimes, however, Mersenne used Kepler's measurements; Mersenne 1636, [tr. 1] pp. 104–105. Once, in 1627, he had appreciated the music of the heavenly spheres; now he rejected not only that of Fludd, but also that of Kepler. Cf. Lenoble 1971, pp. 367–370.

d'autres maximes plus avantageuses que les experiences bien reglées & bien faites, qui monstrent perpetuellement que chaque tour ou retour de la chorde tenduë, & arrestée par les deux bouts, come est celle d'une Viole, ou d'un autre instrument, dure autant l'un que l'autre; ...". ["... I am very far from wishing to demonstrate what I put to the test by means of an experiment ... so that people will not think I have always used the term *to prove* or *proof* in a mathematical sense; this will easily be concluded by those who know what difficulty is encountered to demonstrate something in physics, with which it is extremely difficult to ensure any other more advantageous maxims than experiments well regulated and well conducted, which always show that the movements to and fro of the string, taut and fixed at its two extremities, like that of a viola or another instrument, last each as long as the other; ..."]. While our more modest natural philosopher did not claim to achieve absolute knowledge like Kepler, Galileo, Descartes or Francis Bacon (1560–1626), he did not let himself catch, as a result, by scepticism: by means of natural experiments, some *Verité des sciences [Truth of the Sciences]* would be possible.[20]

Having thus insured himself against the accusations of heresy, that were always impending, and having found in his philosophy some truths that were reconcilable with his religious faith, it comes as no surprise to us that the *Harmonie Universelle* was expounded by means of axioms, propositions and corollaries, with some geometrical reasoning *à la* Euclid. There was still little that was debatable about the Alexandrian Greek. If the experimental system of the noble English philosopher and Lord Chancellor was more often theorised in words than put into practice in person, what can we say about the direct observations performed by the Frenchman?

He would have liked to know whether sound was refracted on passing from air to water, like light. For the latter, he reported "... la vraye proportion des refractions aux inclinations ..." ["... the true proportion of refractions with respect to inclinations ..."] found by "... un des plus excellens esprits de ce siecle ..." ["... one of the most excellent spirits of this century ..."]: $\frac{\sin i}{\sin r}$ (i angle of incidence, r angle of refraction). The reason why Mersenne did not mention explicitly the name of Descartes may have been because the inventor of this famous relationship (one of the two; the other one was Willebrord Snell, 1581–1626) had tried at the beginning to keep it secret. As the French churchman rectified the refraction tables of others using a table of his own, this meant that he had carried out experiments with light. But he did not give any numbers for sound, and limited himself to suggesting advice, for those who anyway wanted to create a *Dioptrique* themselves, so that also the distance between the ears would be taken into consideration. By playing with pieces of glass and crystals, men had learnt to guide rays of light where they wanted. But they were not capable of doing the same thing with sound. This was believed to have been reserved only to the angels, who controlled the vibrations of air.[21]

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[20]Mersenne 1636, [tr. 1], pp. 76, 167, 209–212; "Preface Generale" [i]; "Première Observation", "Seconde Observation". Lenoble 1971, pp. 190–199, 413, 555–560 and passim. Crombie 1981.

[21]Mersenne 1636, [tr. 1] pp. 63–67. Mersenne criticised Maurolico (1611) for considering i proportional to r. Shea 1994, pp. 156–164.

He must have perceived just with his own ears that a bell immersed in water emitted a sound that was a major tenth deeper ($do-la$ in the deeper octave). Brother Marin deduced from this that the speed of vibrations in air and in water stood in the same proportion of 5 to 2. This led him to conclude that the rarefaction of air stood in the proportion 125 to 8 with respect to that of water, because the question regarded solid bodies. He then passed on to consider other liquids, such as brandy, a vegetable oil, wine and milk, relating an experiment carried out by Francis Bacon for relative densities. Still with the idea of studying the properties of bodies by means of sounds, he shot musket-balls through different means and in them dropped weights of different shapes, recording the various times that they took to fall to the ground.[22]

Listening to some of the king's famous singers, who included an excellent bass and an excellent sopranist, he discussed whether deep or acute voices were more pleasant, but he would have preferred to avoid depending on the passing humour, which was where he always ended up. Mersenne contradicted Aristotle, who had argued in favour of deep sounds, preferring the acute voices of children and sopranists. As they vibrated more often, the acute sounds were more continuous, and also closer to the Being and to the Light than deep ones, which, on the contrary, would slide away towards nothingness and darkness. But in "... Physique ou science des choses naturelles ..." ["... physics, or the science of natural things, ..."] as demonstrations "hardly" exist, he ended up by granting us that it was possible to prefer deep sounds, and everyone was free to believe whatever he wanted.[23]

The most interesting observations of our French natural philosopher regarded the movements of bodies, in particular vibrating strings. Having rejected indivisibles, on account of the paradoxes of Zeno from Elea (who he did not mention), he preferred to believe in movement made possible by the infinite points of the *continuum*. He was so keen on his experiments that he discussed them scrupulously in every detail, so that he would not arrive at mistaken conclusions. For example, he dedicated pages to the tension of strings, discussing their conditions at length, and where they broke, in order to study their sound effects precisely.[24]

There is little doubt that in his "Livre Second des Mouvemens de toutes sortes de corps" ["Second book of movements of all kinds of bodies"], he did not compare his own direct observations, transformed into lists of numbers, with those of the books written by Galileo Galilei. However, our good friar did not always trust the Italian scholar. And he re-proposed the subject that was congenial for him, of the relationship between experiments and physical theory. "... l'experience n'est pas capable d'engendrer une science, & qu'il ne se faut pas trop fier au seul raisonnement, puis qu'il ne respond pas toujours à la verité des apparences, dont il s'eloigne bien souvent: ...". ["... an experiment is not capable of generating a science, and we must not put too much faith in reasoning by itself, because it does

[22]Mersenne 1636, [tr. 1] pp. 67–71.

[23]Mersenne 1636, [tr. 1] pp. 71–73.

[24]Mersenne 1636, [tr. 1] pp. 74, 77–84.

not always correspond to the truth of appearances, from which it very often moves far away: ...”].[25] To measure time, he dedicated his attention to clocks, even if, like Galileo, he made use of pulse beats.[26]

“La raison du nombre des retours de toutes sortes de chordes est inverse de leurs longueurs. ... comme monstre l’experience ...”. [“The ratio of the number of returns made by every kind of string is the inverse of their lengths. ... as the experience shows ...”]. Mersenne argued by describing experiments and drawing the geometrical figures of the oscillating movements. The observed results were transferred to lute strings. Sometimes he compared the model to a bow shooting arrows. He did not conceal the difficulty of certain observations that could even become impossible to perform. “Mais les experiences sont si difficiles qu’à moins d’une chorde de mille pieds on ne peut s’en asseurer; ... de sorte qu’il est tousjours necessaire que la raison supplée quelque chose dans les experiences, qui seules ne peuvent servir de principes pour les sciences, qui desirent une parfaite justesse, que les sens ne peuvent remarquer: ...”. [“But the experiences are so difficult that, without a string one thousand feet [300 mt.] long, it is impossible to be certain, ... with the result that it is always necessary for reason to supply something in the experiences, which cannot serve as principles by themselves in sciences, as these require a perfect precision that senses cannot recognize ...”].

And yet he devoted himself to calculating the vibrations of a lute string, sustaining that at the 132nd vibration which was still audible, it still moved by one ten thousand millionth of a line (one line = one twelfth of an inch). The string continued to vibrate, however, even after it could no longer be heard, and at the 1584th return (oscillation, vibration), it would be still moved by an infinitesimal fraction of a line, which he wrote as $\frac{1}{10^{60}}$ (where he printed sixty 0’s). He concluded with the corollary that everything in the world vibrated without our being able to perceive it. “... de sorte que tous les corps du monde peuvent faire une perpetuelle harmonie, quoy que null ne l’entende, & que nous avons sujet de nous humilier dans nostre ignorance, à la quelle nous ne pouvons remedier jusques à ce qu’il plaise à Dieu de nous delivrer de l’obligation que nous avons à la stupidité des sens.” [“... so that, all the bodies in the world may create an eternal harmony, even though nobody hears it, and that we have material to humiliate us in our ignorance, to which we cannot find a remedy until God pleases to free us from the obligation that we have to the stupidity of our senses.”][27]

Mersenne took as a reference point the note G [sol] “ton de Chappelle”, emitted by an open organ pipe 4 ft long, with an opening of 2, almost the lowest to which his voice could go down. The lute string that reproduced it returned 84 times a minute according to [his frequency]. But in order that the string “... ne tremble pas trop viste, & qu’elle donne loisir de conter ses retours, ...” [“... would not oscillate too fast, and would give the opportunity to count its returns, ...”] he specified its

[25] Mersenne 1636, [tr. 1] pp. 84–156, p. 112.

[26] Mersenne 1636, [tr. 1] pp. 135–137, “Preface Generale” [ii], 22, 167 and passim.

[27] Mersenne 1636, [tr. 1] pp. 158, 162, 168. Cf. Bailhache 1993, Sect. 2.2.

length (17 and a half feet), width (made of sheep guts like those of rackets) and tension. "… la quelle revient seulement deux fois dans le temps d'une seconde, lors qu'elle est tendüe avec une demie livre, quatre fois estant tendüe de deux livres, & huit fois estant tendüe de huit livres: …". ["… which returns only twice a second when it is stretched with a half-pound weight, four times when it is stretched with two pounds, and eight times with eight pounds: …"]. Thus also Mersenne, like Vincenzio Galilei, who he did not quote here, violated the Pythagorean dogmas, and in order to multiply the frequency by two, to arrive at the higher octaved, he attached to the string weights that were not twice as great, but four times as great. "D'où il faut conclure que les tremblemens sont en raison sous-doublée des poids, …" ["It must be concluded from this that the vibrations stand to the weights in the ratio of the square root, …"]. If the eye had been capable of counting the oscillations of the string, they would have been too few for the ear to perceive them as sounds. On the contrary, if the ear had started to hear sounds, the vibrations would have been too fast and too frequent for the eye to be able to count them.[28]

Changing the kind of sonorous body, like cylinders and other figures, the proportions found by means of experience for strings lost their validity. He experimented with sheets of iron, full or empty cubes, and anvils struck with hammers. Was he perhaps trying to verify the legend attributed to Pythagoras? It was denied in no uncertain terms. "D'ailleurs ceux qui parlent de la maniere dont Pythagore a inventé les Consonances se trompent lourdement, & tesmoignent que s'ils disent vray qu'il n'avoit pas une bonne oreille, car ils disent qu'il print des marteaux, dont il avoit ouy fraper sur l'enclume, qu'il pesa, & dont il trouva les poids en mesme raison que celles des Consonances; […] ce qui est premierement faux, comme l'on experimente sur l'enclume: secondement, si l'on attache ces poids à des chordes egales en longueur, & en grosseur, ils ne feront pas les Consonances qui sont contenuës par ces nombres, comme Ficin croit […], car j'ay monstré ailleurs que les poids doivent pour le moins estre en raison doublee des termes qui contiennent les Consonances." ["On the other hand, those who speak of the way in which Pythagoras invented consonances are completely wrong, and confirm, if they were telling the truth, that he did not have a good ear: because they say that he took the hammers which he had heard striking the anvil, weighed them, and found that their weights were in the same ratios as the consonances […]. This is first of all false, as we can experiment on an anvil; secondly, if we attach these weights to strings of equal length and breadth, they will not form the consonances contained in these numbers, as Ficino believes […], because I have shown elsewhere [tr. 1] that the weights must stand at least in a double [square] ratio with the terms containing the consonances."] For solids, the ratio of the octave would become 1 to 8, instead of the 1 to 2 ratio of strings. Our French scholar even calculated his own personal music of the seven planets and the Earth, suspending them on eight strings of equal breadth and length, to imagine their sounds. The weight of the Earth was thought to

[28]Mersenne 1636, [tr. 1] pp. 169–170. In this connection, however, he had referred to Vincenzio Galilei in two previous books. Cohen 1984, p. 183.

have raised the note by more than XLI octaves.[29] He indicated possible correlations between the higher planets and deep voices, and the lower ones with acute voices, but he denied that astrology had any influence on them.[30]

As in the tradition that we have seen in Boethius, Maurolico and Benedetti,[31] also Mersenne calculated the number of *commas* contained in major and minor tones. However, like Benedetti, he only used the comma 81 to 80 of Didymus, concluding, like the Italian, at the end of long multiplications, that 10 *commas* were more than one major tone, and 9 were more than one minor tone. Our French friar had not been very satisfied with how they had printed his treatises, because the pages had been numbered separately, sometimes getting the numbers wrong, and making various other mistakes. Perhaps he had not had the opportunity to correct the proofs; on the other hand, he would have liked to make some additions or to modify the text, as we can see from the comments that he left in the margin of an extant copy of his own. But here, with the *commas*, he was the one who wrongly corrected a number that was right, while on the contrary, he left another mistake. He overlooked that the minor tone had become, in the printed version, less than 8 *commas*, when he had just demonstrated that it was less than 9. On the contrary, he corrected the right number "10" of the *commas* that were more than the major tone, writing "9" in the margin.[32]

Mersenne had mentioned one of Maurolico's affirmations regarding the refraction of light, criticising it.[33] He might have read this in one of the few books already printed for the abbot of Messina.[34] If he had come to know of it, bypassing the censure of the Jesuits (but this was unlikely), we do not know whether he would have appreciated the elegant proof, without all those long calculations invented by Maurolico for Pythagorean *commas*, which were far more problematic then those of Didymus.[35] In actual fact, our Parisian friar liked calculating. He filled up his treatises with numbers which were sometimes very large. From his experiments, he obtained measurements, and based on these, he elaborated other considerations to describe the phenomena under examination. Within models guided by proportions, these became ratios that were sometimes written also as fractions of numbers.

Not only that; thinking of music as made up of several separate notes, Mersenne would have liked to give an exhaustive description of it, listing all the possible melodies composed of 1, 2, 3, 4, … 22, … 64 notes. Already with 22, he calculated a nice 22-figure number, 1124000727777607680000. With 64, then, it increased to the 90-figure number:

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[29]Mersenne 1636, [tr. 1] pp. 175, 179, 180, 185–187; [tr. 4] p. 216. [tr. 8] pp. 25–26.

[30]Mersenne 1636, [tr. 3] p. 9.

[31]Part I, Sects. 6.2, 6.5, 6.6.

[32]Mersenne 1636, [tr. 4] pp. 123–126.

[33]Mersenne 1636, [tr.1] p. 65. See above, note 21.

[34]Maurolico 1611. Moscheo 1988, p. 341.

[35]Tonietti 2006b.

221284059310647795878786453858545533220443327118855467387637279113594747033
600000000000000

Not satisfied with this achievement, he listed one by one the 720 "chants" [songs: $2 \times 3 \times 4 \times 5 \times 6 = 6!$] of *ut, re, mi, fa, sol, la*, which he even transferred on to a stave (Fig. 9.2). With these, he patiently and tenaciously filled up seven pages of the book.

He had found the combinations of 2, 3, 4, … 22, … 64 notes, thus succeeding in calculating 2×1, $3 \times 2 \times 1$, $4 \times 3 \times 2 \times 1$, … and so on, that is to say, what we nowadays call the factorial of the numbers 2!, 3!, 4!, … 22!, … 64![36] In this way, all the possible melodies would be created and ordered by means of numbers. Inversely, a number would correspond to a certain melody, which could consequently be codified and encoded.

Our natural philosopher had already tried to do with the letters of the alphabet what he had done with the notes. "Si l'on pouvoit inventer une langue dont les dictions eussent leur signification naturelle, de sorte que tous les hommes entendissent la pensee des autres à la seule prononciation sans en avoir appris la signification, comme ils entendent que l'on se réjoüit lorsque l'on rit, & que l'on est triste quand on pleure, cette langue seroit la meilleure de toutes les possibles: car elle feroit la mesme impression sur tous les auditeurs, que feroient les pensees de l'esprit si elles se pouvoient immediatement communiquer entre les hommes comme entre les Anges." ["If it were possible to invent a language whose expressions had their natural meanings, such that all men could understand the thought of others simply by means of the pronunciation, without having learnt the meaning, as they understand that we are happy when we laugh, and we are sad when we cry; such a language would be the best of all possible languages: because it would have the same effect on all the listeners, who would think the thoughts of the spirit if they could be communicated to each other immediately, as [happens] among the angels."].

The project provided for the listing of combinations of letters and sounds of the various languages. "Expliquer combien il y a de dictions poßibles, & prononçables, soit que l'on use de l'Alphabet, & des lettres Françoises, ou des Greques, Hebraiques, Arabiques, Chinoises, & & consequemment donner toutes les langues poßibles." ["Explain how many possible pronounceable expressions exist, whether we use the French, Greek, Hebrew, Arabic or Chinese alphabet and letters, and consequently give all the possible languages."] As this also contemplated the repetition of letters, this time Mersenne calculated the powers of 19 consonants; thus he obtained, among other things, the combinations with the repetition of 19 elements: 19^{19}. But he also performed other calculations with 10 vowels and so on. It is curious that he thought that also Chinese used letters.[37]

[36] Mersenne 1636, [tr. 3] pp. 107–148.

[37] Mersenne 1636, [tr. 3] pp. 65–72. Cf. Lenoble 1971, pp. 514–521

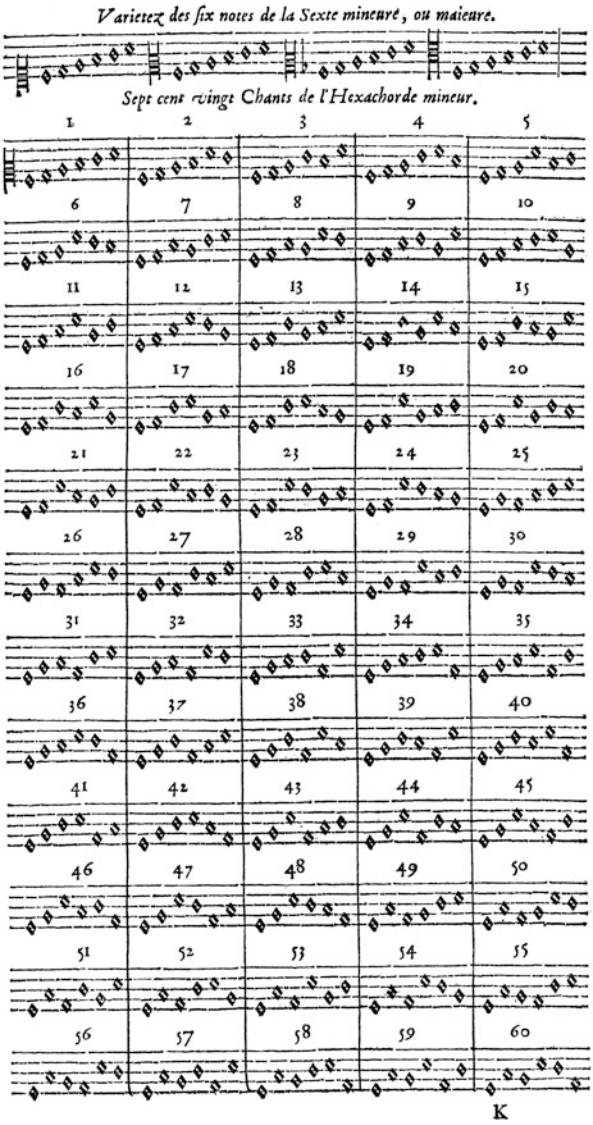

Fig. 9.2 Mersenne listed the possible combinations without any repetition of six musical notes (Mersenne, ibid., p. 117)

Also in the case of words, therefore, father Marin had ended up by creating a code for their translation into numbers. For example, a verse of Psalm 72 became, for him,

168949, 613179, 50536012, 45447, 4087, 7687, 3803197, 190494, 849029
Quam bonus Israel Deus his qui recto sunt corde.

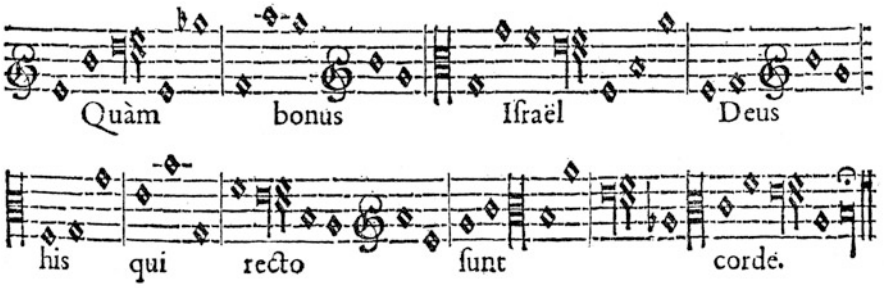

Fig. 9.3 Bad harmony obtained by Mersenne with the numbers for a Psalm (Mersenne, ibid., p. 139)

But if the same numbers had served to encode notes, and have those words sung to the corresponding melodies, he sustained that a bad harmony would be obtained[38] (Fig. 9.3).

For composing in any case, he offered rules, transcribed into numbers. For father Marin, numbers performed a function of purification. "… l'entendement qui considere les nombres intellectuels dans leur pureté, descend à la matiere qui les accompagne, & qui les rend sensibles, a fin de l'élever en quelque façon à un estre intellectuel & raisonnant, & de la dégager de l'alteration, de la mutabilité, & de la corruption." ["… the intelligence that considers intellectual numbers in their purity, descends into the matter that accompanies them and makes them sensible, in order to lift them up somehow to an intellectual, reasoning existence, and to free them from alteration, from changeability and from corruption."][39]

Like Vincenzio Galilei, who however he did not always quote, brother Mersenne argued in favour of monodies, which he preferred to singing several parts together. He sustained this, referring to the capacity for variety revealed by the former style. "… la distinction est l'un des principes du plaisir, comme elle l'est de la science, & que la confusion est le principe de la tristesse, & de l'ignorance." ["… distinction is one of the principles of pleasure, as it is of science, whereas confusion is the principle of sadness and ignorance."] However, he admitted beauty also for polyphony. It should be remembered that the latter music had been the kind imagined by Kepler for the harmony of the stars. Considering the pros and the cons, he concluded that the question remained problematic.[40]

He denied, however, that the ancient Greeks, of whom little was known, could be taken as judges of the question. "Ceux qui croyent que la terre se meut, & que le firmament se repose, se servent de la comparaison de la Basse, la quelle estant faite par les plus grands corps, est semblable au dit firmament, comme le Dessus

[38]Mersenne 1636, [tr. 3] p. 139.

[39]Mersenne 1636, [tr. 4] pp. 245–251, 278–282 and 374.

[40]Mersenne 1636, [tr. 4] pp. 200–201.

est semblable à la terre, dautant que la Basse tient ferme pendant que le Dessus fait plusieurs sortes de mouvemens." ["Those who believe that the Earth moves, and that the firmament is at rest, make use of the comparison with the bass: being it made for larger bodies, it is similar to the above-mentioned firmament as the sopranist is similar to the Earth, with the result that the bass remains still while the sopranist moves in many ways."][41] Even if he did not mention him by name, the French natural philosopher was clearly referring to Kepler here.[42]

The good friar compared dissonances to sins. "... je viens à la Pratique des Dissonances, ..., car elles font quasi le mesme effet parmy les accords que les vices parmy les vertus, d'où les Theologiens concluent que Dieu ne permet point les pechez, & ne reprouve personne, que pour faire paroistre sa grace & sa bonté avec plus d'esclat & de splendeur dans les vertus, [...] a fin que les gens de bien tirent tous les avantages des peches qui se font, que les Musiciens tirent des discords, a fin d'imiter Dieu qui tire les grands biens des grands maux." ["... I come to the practice of dissonances, ..., because among concordances they create almost the same effect as vices among virtues, which is the reason why theologians conclude that God does not allow any sins, and does not rebuke anyone, except in order to make his grace and goodness appear more clearly and magnificently among the virtues [...] so that upright people may take full advantage of sins that are committed, as musicians do of discordances, to imitate God, who draws great good from a great evil."][43]

In the chapters of the treatise dedicated to composition, the person most highly considered by Mersenne was the court musician François Eustache Du Caurroy (1549–1609). He was very famous, and had mainly composed motets, *chansons*, psalms and masses in the traditional polyphonic style, but also instrumental music. Among the theoreticians, Zarlino, above all, figured prominently. In the style of the accompanied monody offered by the Florentine Camerata de Bardi, Girolamo Mei (1519–1594), Giulio Caccini (1550–1618) and Giacomo [Jacopo] Peri (1561–1633) were to be found here and there. But, more than as innovators in musical style, they were considered to be capable in the art of singing, and were compared to the French cantors "... qui les surpassent en mignardise, mais non in vigueur." ["... who are superior to them in elegance, but not in vigour."] And yet Caccini, besides singing and playing the lute, had composed one of the first musical dramas, *Euridice*, and Maria de' Medici had invited him to Paris. Peri, on the other hand, the rival of the preceding musician, as well as singing was also the author of a *Dafne* and another *Euridice*. It is legitimate to suspect that Mersenne did not like this new musical style, and preferred religious polyphony. Vincenzio Galilei was listed together with others like Francesco Salinas (1513–1590) and Franchino Gaffurio (1451–1522). In

[41] Mersenne 1636, [tr. 4] pp. 204, 209.

[42] Part II, Sect. 8.3.

[43] Mersenne 1636, [tr. 4] p. 293. Compare with Leibniz, who will make the same comparison between dissonances and sins, in Sect. 10.1.

any case, the Minim father was not only interested in theory, and his own theory was even better suited to polyphonic compositions, which were poor in modulations.[44]

The dispute between the Florentine musician and Zarlino was traced back to the opposition between art and nature. The latter sustained that it was possible to sing spontaneously "true" musical intervals (those of Ptolemy) imitating nature perfectly, whereas the former put art and artist-artisans before everything else, asserting that singers had to learn the "true" intervals from instruments. From the difficulty in teaching cantors certain notes, Mersenne recognized that also vocal intervals were the fruit of art, but he criticised both the antagonists. "… il semble que ses 2 personnages n'ont pas eu assez de connoissance; autrement Galilée n'eut pas nié que l'on puisse trouver 3 ou 4 Quartes ou Quintes de suite avec une Tierce majeure, ou mineure dans aucun genre que dans l'incité d'Aristoxene, …, & Zarlin n'eust pas pris tant de peine à expliquer le Synton de Ptolomée, dans le quel il manque plusieurs degrez, s'il eust eu connoissance des claviers que je propose …" ["… it seems that these two characters did not have sufficient knowledge. Otherwise, [Vincenzio] Galilei would not have denied that three or four fourths or fifths could be found one after another with a major or minor third in any genre except in the *incited* one of Aristoxenus, …, and Zarlino would not have taken such trouble to explain Ptolemy's Syntonon, in which many degrees are missing, if he had known the keyboard instruments that I propose …"]. The churchman from Paris concluded impartially that art was superior to nature when it added perfection to it, without denying, however, that the latter preceded the former in many cases.[45]

Apart from the 1,000-year-old dispute (and in the case it should be needed), it was clear that Mersenne knew the book "… Dialogues … de l'ancienne & de la nouvelle Musique …" [*Dialogue on ancient and modern music*] written by Vincenzio Galilei. Here, those non-Pythagorean positions had already been defended, with other ratios for weights and pipes, which were now subsequently proposed also in *Harmonie Universelle*.

Though well aware of these new ratios, as a reference point to tune the other instruments, as regards the theory, also Mersenne remained faithful to the monochord, for which he gave the same figure as Ptolemy (Fig. 2.5). He even (strangely?) accepted the criticism of the famous Alexandrian Greek against the unreliability of flutes. For practical questions, he proposed the spinet: a keyboard instrument of various forms with a fixed tuning system, invented in the sixteenth century, which looked like a small harpsichord. Though the French Minim friar reasoned above all on the basis of Ptolemy's monochord, which for this reason he called "harmonic" or "canonical", and appurtenances with the usual Pythagorean variants, by now he could not, however, ignore the division of the octave into equal parts in the style of Aristoxenus. Leaving aside the geometrical proportional means, he relied above all on the precision of numbers (Table 9.2).

[44]Mersenne 1636, [tr. 4], pp. 290, 357–358, 143.

[45]Mersenne 1636, [tr. 4] pp. 7–9.

I	II	III	IV	V
a	100,000.	c.	100,000.	n
xg	105946	♯	105945	m
G	112246	b	112245	l
xf	118921	A	118920	k
F	125993	xg	125992	i
E	133481	G	133480	h
xd	141422	xf	141421	g
D	149830	F	149829	f
xc	158741	E	158740	e
C	168179	xd	168178	d
♯	178172	D	178171	c
b	188771	xc	188770	b
A	200,000.	C	200,000.	

Table 9.2 How Mersenne tuned a keyboard in the equable manner (Mersenne, ibid., p. 38)

He had obtained these numbers by approximating the extraction of the appropriate irrational square and cubic roots. The table was double, with one column of larger values and the other one of smaller values of exact irrational numbers, to indicate the approximations.

He calculated the differences between the "correct" intervals of the "harmonic" division and the tempered equal intervals, to conclude that they were not sensible to the ear. But he claimed that he had done with numbers, obtaining a greater precision and a procedure that was easy to repeat, what artisans and players had practised ever since the times of Aristoxenus. It was perfectly easy to compose also with equal intervals. "… & consequemment la composition en sera beaucoup plus aysée, & plus agreable, & mille choses seront permises que plusieurs croyent estre deffenduës: …" ["… consequently, composing will become much easier and more pleasant; thousands of things will be permitted that many believe to be prohibited: …"]. And yet this music remained linked to the senses, whereas following the intellect led to the other "harmonic monochord".[46]

To prepare his arguments regarding musical instruments, Mersenne took up his experiments on vibrating strings again. If it should not be possible to measure the times of vibrations, or to count their number, he referred to the oscillations of the pendulum: a string attached at one extremity, with a weight attached to the other, set in motion. He continued to measure the time by means of pulse beats, but also, vice versa, he proposed that doctors should count the pulse strokes of their patients using pendulums of different lengths, which would thus oscillate more or less rapidly. He

[46]Mersenne 1636, [tr.5] pp. 38 and 41.

knew that if the pendulum of a certain length (4 ft) counted 1 beat in 1 s, in order to retard its beat to every 2 s, it would not be sufficient to double its length, but it was necessary to raise it to its square (16 ft).[47]

As related by Vincenzo Viviani,[48] Galileo Galilei had observed that the oscillations of the pendulum took place in the same time (isochronism), following their rhythm with that of music. Now, vice versa, Mersenne modelled the sonorous vibrations of strings, hypothesising that the ratios between the movements of a string, fixed at its two extremities and suitable taut, followed those of a string attached at only one extremity. The circle had been completed: leaving the name of "harmonic motion" for the pendulum in future manuals of physics, and the name of "harmonic succession" for the succession of inverses of an arithmetic succession.

When explaining how to tune instruments, our French scholar often also added how to do so with equal intervals, which he called the "temperament", referring regularly to Aristoxenus. Thus, as the makers of instruments like lutes, violas and others practically tempered them, he considered them (offensively?) to be unconscious followers of Aristoxenus. "Il y en a ancore plusieurs qui croyent que cette division d'Aristoxene doit estre preferée à toutes les autres, ce que Vincent Galilée s'est efforcé de prouver en faveur de ses amis Aristoxeniens, par ce que ce Systeme est le plus aysé de tous, & que le jugement des sons depend entierement de l'ouye, . . . " ["There are still several who believe that this division of Aristoxenus is to be preferred to all the others: this is what Vincenzio Galilei made every effort to prove on behalf of his Aristoxenian friends, because this system is the easiest of all, and the judgement of sounds depends wholly on the hearing, . . . "].

Father Marin, however, rejected the judgement formulated by the Florentine lute player and composer. ". . . il confesse en faveur de la verité que la Quinte Pythagorique est plus agreable que l'Aristoxenique, & que la nature n'a pas esgard à nos commoditez; de sorte qu'il ne s'ensuit pas que le Systeme d'Aristoxene, dans le quel la Quinte contient 7 douziesmes de l'Octave, soit plus parfait que celuy, dans le quel elle est juste. [. . .] il n'est pas veritable que la division d'Aristoxene soit plus aysée que celle qui suit la justesse des intervalles que j'ay expliqué en plusieurs lieux: [. . .] celuy là est le plus aysé à chanter, & le plus naturel qui suit les nombres Harmoniques, . . . " [". . . he testifies in favour of the truth that the Pythagorean fifth is more pleasant than the Aristoxenian one, and that nature does not care about our convenience: thus it does not follow that the system of Aristoxenus, in which the fifth contains seven twelfths of the octave, is more perfect than the one in which it is just [. . .] it is not true that the division of Aristoxenus is more pleasant than the one which follows the justness of intervals as explained by me in several places [. . .]; this is the easiest one to sing, and the most natural one, which follows harmonic [whole] numbers, . . . "]. This did not in the least stop luthiers from merrily

[47] Mersenne 1636, [tr. 5] pp. 45- - 45[bis]-46[bis]; the book presents page numbers that are repeated, and some are missing.

[48] Part II, Sect. 8.2.

continuing to follow the equable temperament when they distributed the frets along the necks of their instruments.[49]

In spite of the fact that he always ended up in the glory of his heavenly numerical ratios, father Mersenne did not refrain from displaying a great quantity of practical know-how: how to construct well, and play even better, the musical instruments in use in his period, as well as the relative music. It remains, now, for us to see how he tuned wind instruments, those instruments that had been so often disregarded by the Greek theoretical tradition, for which Vincenzio Galilei had already given rules a few decades earlier, not to mention the Chinese. With these, our Parisian friar performed multiple experiences and various demonstrations regarding the correct ratios for pipes, but in the end, he arrived essentially at the same results obtained by the Florentine musician and by the ancient Orientals. Sure enough he denied that it was possible to construct organs with pipes either all equal in their length, or all equal in their breadth, because they would not produce notes of the desired frequency.

"... par exemple, que celuy qui est double en longueur ne fait pas l'Octave, comme plusieurs croyent, il faut encore icy consulter l'experience, ... [...] Puis que l'experience nous a fait voir que les tuyaux doivent estre de differentes longueurs & grosseurs, pour faire tous les sons de l'Orgue, il faut joindre ces deux dimensions ensemble, afin d'avoir des sons qui soient proportionnez tant en leur aigu, qu'en leur douceur, & en leur harmonie. [...] le tuyaux font justement l'intervalle que l'on veut, si leurs hauteurs & leurs largeurs ont mesme raison que ledit intervalle: ..." ["... for example, that the [pipe] that is twice as long does not produce the octave, as many believe; here again, it is necessary to refer to experiences, [...] Because experiences have shown us that the pipes must be of different lengths and breadths in order to produce the sounds of the organ; these two dimensions need to be united together to obtain sounds that are proportional both in their pitch and in their sweetness and harmony. [...] The pipes produce the desired interval with precision if their lengths and their breadths have the same ratio as the said interval ..."]. That is to say, to obtain the bass octave 2 to 1, it is necessary to take a pipe that is both twice as long and twice as broad. Seeing that in the calculation of the circular section, 2 appears squared in the formula πr^2, when it is multiplied by the length, we obtain 8. 8 to 1, then, becomes the desired ratio between the volumes of pipes tuned to the octave. For the fifth, the proportion 3 to 2 of strings is transformed into 27 to 8 with pipes; for the fourth 4 to 3, into 64 to 27 and so on.[50] Organs, too, deserved to be tuned, in the end, in accordance with the equable temperament: in this way, the intervals could be moved along a keyboard of 13 keys to the octave, without having to change the ratios.[51]

Even if he never lost an opportunity to move away from those sensual ears that were so much appreciated by musicians, so as to take refuge in his transcendent

[49]Mersenne 1636, [tr.5] pp. 48–49, 60–61.

[50]Mersenne 1636, [tr. 5] pp. 331–335.

[51]Mersenne 1636, [tr. 5] pp. 341–344.

truths of numbers, all the same the French friar left us some precious testimonies about the (new?) sounds that he had succeeded in hearing. "... la chorde frappée, & sonnée à vuide fait du moins cinq sons differens en mesme temps, dont le premier est le son naturel de la chorde, qui sert de fondement aux autres, ... les autres sont si foibles qu'il n'y a que les meilleures oreilles qui les entendent aysément. Or il faut choisir un grand silence pour les appercevoir, [...] Or ces sons suivent la raison de ces nombres 1, 2, 3, 4, 5, car l'on entend quatre sons differens du naturel, dont le premier est à l'Octave en haut, le second à la Douziesme, le 3 à la Quinziesme, & le 4 à la Dix-septiesme majeure ..." ["... the string plucked and played emptily makes at least five different sounds at the same time, the first of which is the natural sound of the string which serves as a basis for the others, ... the others are so weak that only the best ears can hear them easily. Now it is necessary to choose a great silence to perceive them, [...] Now these sounds follow the ratios of the numbers 1, 2, 3, 4, 5, because four sounds different from [*do*] natural can be heard, the first of which is the octave higher [2 to 1, *do*], the second is the twelfth [3 to 1, *sol*], the third is the fifteenth [4 to 1, *do*] and the fourth is the major seventeenth [5 to 1, *mi*] ..."]. The fifth sound above the fundamental one, perhaps Mersenne heard it badly, because he took it for a twentieth [*la*], when it should have been a nineteenth [6 a 1, *sol*].

With commendable honesty, father Marin referred that the phenomenon seemed to admit that Aristotle was right: the Greek philosopher had sustained, perhaps alluding to a similar experience, that a deep sound then contained the acute one: he was referring to the strings, where the longest one contained the shortest one. But now the French scholar wanted to reason in terms of vibrations of the air: so how could a certain number of such frequencies also contain the number that was twice as many, three times as many, and so on? Because, in his opinion, the string could only make one movement, he firmly denied "... que la moitié de la chorde le bat deux fois tandis que la chorde entiere le bat une fois, & qu'en mesme temps la 3, 4 & 5 partie le battent 3, 4 & 5 fois, ce qui est contre l'experience, ..." ["... that half the string strikes it [the air] twice, while the whole string strikes it once, and in the same time, one third, one quarter and one fifth of the string strike it 3, 4, and 5 times; this is contrary to experience, ..."].

Thus he imagined that it was the air that was divided, variously, generating the sounds of increasing pitch. For this model of his, he would have been ready to consider even the atoms of Democritus. He even tried to link the phenomenon with the notes generated by glasses full of water, where the movements could be observed, while hearing at the same time two or three sounds. Transporting "... la Physique aux actions humaines, ..." ["... physics into human actions, ..."], he consoled himself, thinking that just as the note would be more harmonious, the more different sounds he succeeded in producing, so the actions would be more appreciated by God, the more they were animated by good motivations. Thus he listed those for which "... l'on jeusne pour macerer le corps, ..." ["... one fasts in order to mortify the body, ..."]. He must have had some doubts, however, if in recording the same phenomenon for the lowest notes of the voice in his last treatise, he also hypothesised that vibrating bodies like the larynx "... ne le battent pas,

ou n'en sont pas battus uniformement, mais qu'une partie le bat tellement qu'il tremble plus viste vis à vis de cette partie, que vis à vis des autres: …" ["… do not strike it [the air], or are not struck, uniformly, but that one part strikes it in such a way that it vibrates more rapidly with respect to this part than with respect to the others: …"]. He again took up the glass, where the vibrations might be thought to be propagated on the water, dividing up into tiny eddies; he added in the margin a hand-written comment on Galileo Galilei, who wrote that he had observed a similar phenomenon.[52]

Even if scholars of sound were to admit generally in the following centuries that Aristotle was correct, describing the different kinds of vibration of the string as generators of these series of notes, subsequently called higher harmonics, we must include this important page of Mersenne in their history.[53] But isn't it a bit strange that the intuition of this friar, so capable of imagining strings in motion, even when it was impossible to see it, and so bold in hypothesising it by extension or by analogy, now came to a stop before a variant of the phenomenon not among the most uncommon? Was this attitude influenced by his conflictual relationship of love and hate with Aristotle? However he also considered the phenomenon of "batemens" [beats], which is created when two sounds have very similar frequencies, to use it in the tuning.[54]

This French churchman dedicated several corollaries of his propositions to God, as the supreme judge of men and nature. He thought it was easier to draw moral teachings from mathematics than from the natural stories of Pliny or Plutarch, because in his opinion the former would be indestructible and unquestionable. Thus he developed an analogy between God and Euclid's geometrical point, which was defined as without parts. Without this, quantities would be reduced to nothing; quantities were thought to contain an infinite number of such points. "… d'ou il s'ensuit que les infinis sont plus grands les uns que les autres, en raison donnée, de sorte qu'ils sont comparables en autant des sortes que les finis, c'est pourquoy les propositions fondées sur l'incommensurabilité demeurent en leur entier." ["… from this it follows that infinities are larger than one another in given ratios, such that they can be compared in many types, like finite things; this is the reason why propositions founded on incommensurability remain in their entirety."]

As an infinitely luminous point, God would emanate creatures as straight rays, which bend as a result of human malice. However, he also used the traditional divine model, which recurred also in Kepler, of a spherical surface with its centre everywhere, and its circumference nowhere. In his constant assessment of the pros and the cons, in his continuing oscillation between experimenting with the senses and elevating himself into a spiritual transcendence, it is not easy to understand in this case where the pendulum would stop. "… le mouvement droit est quasi inutile au monde en comparaison du circulaire: …" ["… straight movement is

[52]Mersenne 1636, [tr. 5] pp. 208–211; [tr. 8] pp. 15–16. See Part II, Sect. 8.2.

[53]See below, Part II Sects. 11.2 and 12.2.

[54]Mersenne 1636, [tr. 5] pp. 362–363, [tr. 8] p. 20.

almost useless in the world compared with the circular one: …"], and yet at infinity [in God?] "… la ligne droit infinie, & son mouvement droit convient avec le mouvement circulaire infiny." ["… the infinite straight line and its straight movement agree with the infinite circular movement."][55] At the dawn of projective geometry, and in close contact with Girard Desargues (1591–1661) who was elaborating it, may he have been sensing intuitively that a circumference with an infinite radius would become a straight line?

Not only are all gods not equal, but neither are all Christians. Even among the faithful of the Roman Catholic church, considerable differences can be detected. The order of the Minims, one of the mendicant orders, like the Franciscans, received and practised charity. Thus the religion of brother Marin was not an abstract, contemplative piety, but was dedicated to good works. The harmony that he imagined was consequently distant from the Platonising idea of an ideal beauty, and should demonstrate that it was useful for everything and for everybody. "… l'experience assujettie aux sens, & justifiée en toutes sortes de manieres confirme grandement la verité des sciences contre l'opinion de ceux qui croyent qu'elles ne sont autre chose que des fantaisies de l'esprit humain, …" ["… experience subject to the senses, and justified in all kinds of ways, greatly confirms the truth of sciences, in contrast with the opinion of those who believe that they are nothing other than fantasies of the human spirit, …"]. Thus heavenly ideas could take form in mechanics and earthly matter without any danger of losing their purity.[56]

The list compiled by Mersenne, where his treatises of universal harmony should have made themselves useful, was rich, long, sometimes bizarre and even disturbing. He offered it not only to philosophers interested in physics, but also to architects, friar preachers and theologians. His studies would serve to gain a better understanding of Aristotle and also of the *Bible*. If all the light desired could be concentrated by a parabolic mirror at only one burning point, the same thing had to happen with the body of Christ in the consecrated host. By counting the syllables, it was possible to measure sermons in church, and speeches in court, increasing their effectiveness by means of the prudent use of tones in pronouncing them. Then it would be possible to make the desire for God's presence so strong in the faithful as to make them desire death in order to obtain it.

Taking his inspiration from the founder of the Minims, Saint Francesco da Paola, our churchman suggested that preachers should explain the divine infinity by means of those "minimal" elements like the point and the number 1, which were equally capable of generating infinity. But if he dared to discuss about infinity, he ended up in paradoxes and contradictions, like those of Galileo Galilei. "… voyons-nous que les Mathematiques considerent seulement la quantité finie, qui borne la portée de l'esprit humain, sans qu'il puisse penetrer, ny mesme considerer l'infinie, sans s'embarasser en mille contradictions, dont il ne peut se desgager, non plus que l'oyseau pris à la glus ou au ré, & au filet. C'est pourquoy plusieurs nient qu'il

[55]Mersenne 1636, [tr. 6] pp. 77–79.

[56]Mersenne 1636, [tr. 7] p. 1.

y puisse avoir autre infiny que Dieu, qui surpasse toute sorte de contradiction, & consequemment que le monde n'a peû estre de toute eternité, & que l'esprit crée, quoy que bienheureux dans le Ciel, ne pourra comprendre l'infinité de Dieu, & qu'autrement il s'ensuivroit qu'il seroit luy mesme infiny …" ["… we see that mathematics considers only the finite quantity which limits the range of the human spirit, since it cannot either penetrate or contemplate infinity as well without becoming confused in a thousand contradictions, from which it does not succeed in freeing itself, any better than a bird trapped with lime or in a net. For this reason, many deny that any infinity can exist that is different from God, who can overcome every kind of contradiction; consequently, the world cannot have existed for all eternity, and the created spirit, however happy it may be in heaven, will not be able to understand the infinity of God, otherwise it would follow that it would be infinity himself …"].

And yet in the world, small circumferences could be found with the same number of parts as larger ones, limited segments of a straight line with the same number of parts as unlimited ones, since they precisely are all infinitely divisible. "… d'ou l'on peut conclure que tout ce qui est, porte le caractere divin, qui tesmoigne l'infinité absoluë du Createur, & qui monstre que la jurisdiction de l'esprit humain est entre ces deux sortes d'infiny, sans qu'il puisse l'estendre d'un costé ny d'autre, & qu'il a de merveilleux sujets de s'humilier lors qu'il considere son peu de lumiere, & son ignorance, …" ["… from which we may conclude that everything that exists bears the divine character, which testifies to the absolute infinity of the Creator, and shows that the jurisdiction over the human spirit is situated between these two types of infinity, without the possibility of extending it either on one side or on the other, and that there are wonderful subjects to humble oneself when one considers his limited light and his ignorance, …"].[57]

The God of Mersenne was a God both of science and of grace, who had filled our soul, as Plato had said, with harmony. In conclusion, like the Jesuit Clavius, whose works and whose Gregorian calendar he quoted, also father Mersenne was convinced that mathematics elevated the soul towards heaven. So much so that he would even have been ready to save certain atheists, provided that they followed reason and the Word: among these, not only Socrates, but even Heraclitus.

The Chinese and other nations, becoming converted to God, would bring a testimony "… que il n'y a point d'hommes si sçavans au monde que ceux qui ont la vraye Religion, & qu'ils seront contrains d'apprendre de nous les plus excellens secrets de la Geometrie, …, de la Musique, & de toutes les autres sciences, …, quand ils verront une science si universelle & si profonde dans l'Eglise de Dieu, …, au lieu de nous maudire comme font les Mussulmans, la force de la verité leur fera dire *Quàm pulchra tabernacula tua Iacob*, … Et certes il est raisonnable que ceux qui adorent plus purement la souveraine raison, en participent davantage, & que ceux qui l'ont receuë venant en ce monde, & qui la croyent fermement, tesmoignent au reste du monde qu'ils sont enfans de lumiere, dont ils espandront les rayons par

[57] Mersenne 1636, [tr. 7] pp. 17, 19.

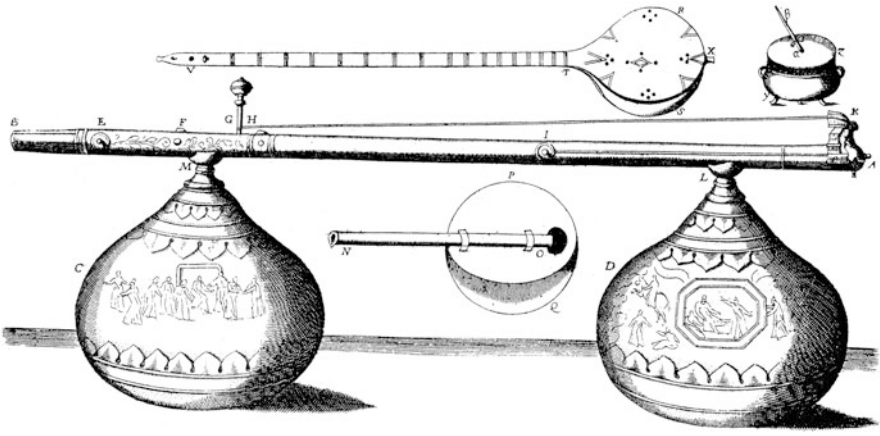

Fig. 9.4 Oriental musical instruments according to Mersenne (Mersenne, ibid., p. 228)

tout, lors que les infideles n'y mettront plus d'empeschement." What would these Chinese testify? ["… that there are no men in the world so wise as those who possess the true religion, and that [they] will be forced to learn from us the most excellent secrets of geometry, …, of music and of all the other sciences, …, when they will see such a universal and such a deep science in the Church of God, …, instead of cursing us, as the Muslims do, the force of the truth will make them say *How beautiful your tabernacles are, Jacob*, … And it is undoubtedly reasonable that those who adore the sovereign reason in the most correct way participate best in it; those who received it coming into the world, and believe it firmly, testify that they are children of light, the rays of which they will spread everywhere, when the infidels do not create any more obstacles."]

Our French friar included in his repertoire also Turkish, Indian and Chinese musical instruments, maybe creating a certain confusion among them (Fig. 9.4). The instrument attributed by him to China probably came from India. Thus, universal harmony and universal sciences became useful to propagate true religion.[58]

We do not know how many co-brothers father Mersenne had succeeded in convincing with his analogies between the geometrical point and God, light and the angels or Christ, or the straight line and virtue. These comparisons certainly seem to be staggering to us today, even if he tried to back them up with some precise designs about the geometrical properties of light rays. But other applications of harmony, on the other hand, should shock us, I hope. Father Marin thought not only of saving the souls of the unbelievers, but he also took an interest in their bodies.

He recalled, with some legitimate technical doubts, Archimedes who was capable of burning the ships of Marcellus, and how Vitruvius controlled war machines by listening to the notes emitted by the relative stretched ropes. Our French natural

[58]Mersenne 1636, [tr. 7] pp. 23, 26; [tr. 5] pp. 227–228. Sachs 1996, p. 264.

philosopher thus passed on to "… les utilitez de l'Harmonie pour les Ingenieurs, pour la Milice, pour les Canons, & pour les gens de Guerre: où l'on void les portées & les calibres du Canon." ["… the uses of harmony for engineers, for the army, for cannons and for people at war: where we can see the range and the calibre of cannons."]

In this way, he taught soldiers to fire cannon-shots with the "… force de l'Harmonie." ["… force of Harmony"]. He measured calibres and weapon ranges. He listened to the whistle of bullets. With his pulse-beat, he controlled distances and speeds of the shots fired. He gave the proportions of gunpowder: 4 parts of nitre, 1 of sulphur, 1 of carbon. He compared the trajectory of the ball at zero elevation with the parabolic trajectory at 45° to obtain the longest range. He projected a system of communications based on the speed of sound, measured as 230 'toises' per second, trying to demonstrate that the king would receive news from all of France in less than an hour, and it would take less than a day to arrive at the antipodes. Apart from technical details, the order of magnitude would seem to be correct, even if the number is too large.[59]

To explain the utility of harmony in morals and in politics, Mersenne referred to Plato and Ptolemy, who, faced with the "… volupté inexplicable …" ["… inexplicable pleasure …"] instilled by music in the soul of listeners, attributed to Apollo the citharist "… plus de force sur eux que n'en a Bacchus ou Venus." ["… more force over them than Bacchus or Venus exert."] As regards Kepler, the French friar referred that political digression inspired by Bodin about arithmetic, geometric and harmonic means, seen here as models for oligarchic, aristocratic and monarchic governments. But to make them "more real" and "better grounded", he referred to his own experiences, where "… les plus grandes chordes qui se meuvent le moins, font trembler les moindres, comme les plus grands des Republiques font remuer le peuple par leur seule parole; …" ["… the longer strings which move less make the shorter ones vibrate, as the largest states make the people tremble just by means of their words; …"]. Thus, for him, deep voices, which move less, represented the higher powers, arriving at God.

It is curious that, in spite of the dispute from which we started, of all people this French scholar borrowed from Fludd, (who he did not quote) the figure of universal harmony, with exactly the same distribution of the notes, the four elements, and the stars. A hidden, transcendent God (the same one?) made a hand come out of the clouds, to tune the monochord (Fig. 9.5).[60]

Nor did he omit advice for judges and lawyers, in the cases of believers against unskilled organ makers, or in the choice of the best musicians. At the time, one of those instruments might cost even more than 20,000 lire, whereas a music teacher earned 20,000 lire a year.[61]

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[59] Mersenne 1636, [tr. 7] pp. 26–46. Taking a 'toise' to be 1.7 m long, this would give a speed of 391 m/s, which is higher than the 331 m/s measured today. Cf. Lenoble 1971, pp. 485–486.

[60] Mersenne 1636, [tr. 7] p. 49. Cf. Lenoble 1971, pp. 524–530 and 549–552. Ammann 1967, p. 23.

[61] Mersenne 1636, [tr. 7] pp. 47–50.

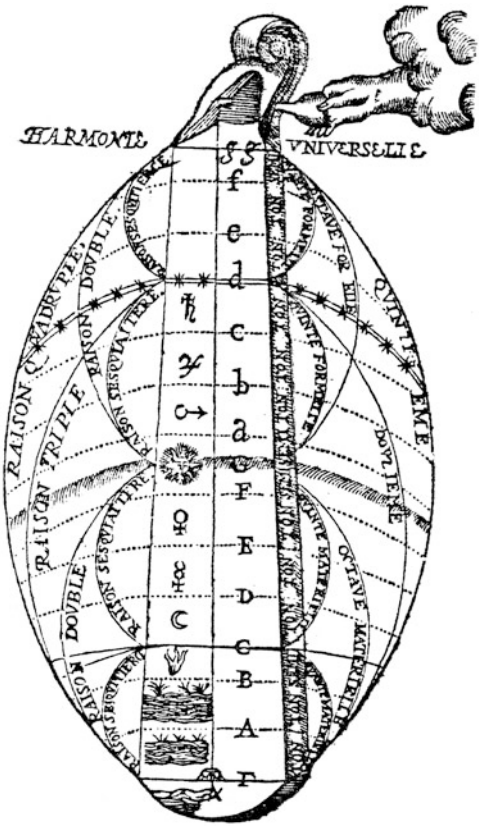

Fig. 9.5 Mersenne borrowed from Fludd the model of the divine tuner of the universe (Mersenne, ibid., p. 49)

"Puisque nos Traitez Harmoniques dépendent de la connoissance des raisons & des proportions, lesquelles sont comme la forme, & l'ame de toutes les Mathematiques, …" ["Because our harmonic treatises depend on knowledge of proportions and ratios, which are like the form and the soul of all mathematics …"], father Marin gave the rules to carry out the necessary calculations with them. These were the same rules followed in the previous books of the fifteenth and sixteenth centuries, e.g. of Maurolico. We would look in vain for the novelty of fractions. Despite all his interest in searching for a universal, perfect, unified language, our French scholar used the same pattern with the large "X" as Luca Pacioli, in order to calculate that taking away a fifth from the octave left a fourth, instead of a more practical division between fractions.[62]

And yet he must have read something written by the Arabs. We know that he had received the *Book of the benefit about the knowledge of melodies* by al-Saydawi. But what, then, could hinder its use, when clear practical advantages

[62]Mersenne 1636, [tr. 7] pp. 50–57.

would be obtained? The theory of Mersenne remained linked with Greek orthodox tradition, in spite of the event that his ear was indicating other routes with his experiments. We can understand this, when he repeated the classic commonplace "… qu'il y a des raisons *irrationelles*, c'est à dire, qui ne peuvent s'exprimer par nombres, …" ["… that *irrational* ratios exist, that is to say, ratios that cannot be expressed by means of numbers, …"]. But wasn't providing the numbers of irrationals, exactly what he was doing when he calculated the ratios of the equable temperament? We sometimes read curious contradictions which perhaps only his God could understand. In the case of Arabic algebra, the novelties also had to suffer the defect of originating from infidels. Religious prejudices were worsened by the negative and limited information that arrived from the East. However, this was filtered by the partial interests shown in the possible messengers, who were generally either missionaries or merchants. We shall see that in time, also the most important extra-European novelties would succeed in filtering through, even if they underwent some deformations. The worst of these, of course, was to be the loss of all memory about their place of origin.

How better could he finish his treatise on the utility of harmony, than by calculating and tabulating the ranges of cannons, depending on the degrees of the elevation angles?[63] Among a miscellany of physical problems regarding the fall of heavy bodies, water and air in motion, sounds and equable tuning for organs and harpsichords, even in the pages clearly consigned last of all to the printer, emerged "De l'usage des sons pour la guerre." ["On the use of sounds in warfare"]. By hanging cymbals on plants, and burying jars, the tunnels dug out by the enemy to place mines would be discovered through sounds. In the end, he bustled about certain whole numbers, obtaining, among other things, prime numbers, nowadays known as Mersenne's numbers, of the form $2^p - 1$ with p a prime number. Recently, using computers, a new one has been calculated, composed of a few million figures. All the treatises concluded with a short combination of Hebrew letters. In the dedication at the beginning of the "Treatise of the voice and chants", the 10 *sefirot* of the cabbalistic tradition, associated with the 10 strings of the harp, would serve so that the Biblical King David could restore divine harmony, which had been upset by demons.[64]

For sound, Mersenne did not use either the term "onde", or "vague" [wave], but "cercles de l'air" [circles in the air] or "… cercles semblables à ceux qui se font sur l'eau …" ["… circles similar to those that are created in the water …"]. As its best illustration, he gave the following. "… l'on peut comparer les petit flots d'air qui frappent les languettes, aux flots de la mer & des rivieres, ou aux sauts & boüillons que fait l'eau, quand elle rencontre des cailloux, ou d'autres obstacles, car

[63] See Part I, Sect. 5.3. Mersenne 1636, [tr. 7] pp. 54, 26, 64–68. Cf. Lenoble 1971, pp. 474–475, 554–555 and 588.

[64] Mersenne 1636, [tr. 8] pp. 23–24, 26–27. He copied the last verse of Psalm 150 in Hebrew: "Let everything that has breath praise the Lord [Yah]. Praise the Lord [Yah]". Thanks are due to Giovanni Mazzini for the translation. [tr. 3]. Cf. Lenoble 1971, pp. 96–109

les flots ont coustume d'estre en plus grande multitude, lors que les empeschemens sont moindres, mais lesdits flots sont plus petits en recompense: de sorte que leur frequence est d'autant moindre qu'ils sont plus grands, . . . " ["... the little air waves that strike the tongues [of the organ] can be compared to the waves of the sea and of rivers, or to the springs and eddies that water forms when it comes up against rocks or other obstacles, because the waves are usually more numerous if the obstacles are smaller, but in compensation then these waves are smaller: so that, the larger they are, their frequency is smaller . . . "]. Though an audible harmony arrived to us from the air waves, it would not have been possible to obtain the same thing from the waves of the sea, which were not so well known, and not so regular. "... la cognoissance ne nous en est pas moins cachée, que la raison du flux & reflux de la mer: . . . " ["... knowledge of them is no less hidden to us than the reason for the ebb and flow of the tides: . . . "].[65]

While Kepler preferred to argue using geometry, Mersenne, on the contrary, liked calculating with numbers, which he listed and combined continuously. Mersenne invented harmony on Earth, whereas Kepler had situated it in Heaven. The former had chosen as his favourite interlocutor the son Galileo Galilei, whereas the latter, in music, followed the father, Vincenzio Galilei. Both of them, however, moved away from the Florentine musician, in spite of the event that, in readmitting the sense of hearing as a reliable judge, he had determined a revival not only in music, but also in studies on real sound. For the scientific community that was being formed, the fruits of his change were to be reaped by Mersenne. Vincenzio Galilei, generally speaking, was to disappear from the imaginary of natural philosophers and future physicists, and is remembered (with difficulty?) as the father of Galileo for his experimental method. Whereas Mersenne, with his surveys of styles of composition and of musical instruments, was to maintain a place even in histories of music, and not only in the histories of the sciences, for the first laws of the future acoustics.[66]

This different destiny was a result of the intense network of personal and epistolary relationships woven by the Parisian scholar. He was capable of getting famous characters like Gilles Personne Roberval, Girard Desargues, (Pierre de Fermat), Etienne Pascal and Blaise Pascal (together with René Descartes) to attend the meetings of his association, known as the "Academia Parisiensis", held in the convent of Place Royale. All of them were quoted and variously exploited for his *Harmonie Universelle*.[67]

Mersenne published a whole treatise by Roberval (1602–1675) entitled "Traité de Mecanique des Poids Soustenus par des Puissances sur les Plans Inclinez à l'Horizon ["Treatise on the mechanics of weights sustained with forces on planes inclined with respect to the horizon."]

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[65]Mersenne 1636, [tr. 1] "Abregé de la Musique speculative" V and p. 23; [tr. 5] pp. 361–362.

[66]Mersenne 1636, [tr. 4], [tr. 5] and [tr. 6]. For example, Stillman Drake did not say anything about Vincenzio Galilei's new rules for pipes; Drake 1981.

[67]Mersenne 1636, [tr. 2], [tr. 7] pp. 71–74, [tr. 4] pp. 331ff., [tr. 5] pp. 308ff. Lenoble 1971, pp. 581–594. Cf. Dear 2000.

In the "Livre Sixiesme de l'Art de Bien Chanter" ["Sixth book on the art of singing well"], Mersenne had copied "... une methode fort aysée, laquelle Monsieur des Argues excellent Geometre, a dressé en des termes, qu'il a reconnu propres pour l'insinuer dans l'esprit des enfans, ..." ["... a very easy method which Mr. Desargues, an excellent geometrician, has presented in terms that are recognized to be suitable to be insinuated into the minds of children, ..."]. Thus we discover that even the founder of projective geometry had taken an interest in music.[68] It is only thanks to this Minim brother and other scholars that nowadays in general we can still read the results obtained by Desargues, including the theorems that go under his name. He himself was not too fond of writing, or publishing, but preferred to tinker with machines, graphics, projective drawings and stone-cutting, often quarrelling with the relative artisans. More of a military engineer and architect than a natural philosopher, he did not use symbolisms, or rather, he referred to concrete objects, and reasoned by means of geometrical images. The straight line was called a "palm" by him, but then it became a "tree" in certain projective cases. "Harmonic" was the name given to a set of four points on the straight line, when the numerical ratios of the distances measured from the first point were those of a harmonic succession. In this way, the numerical ratios for musical notes entered into the geometry of conic sections, now seen by Desargues as the theatre of properties that were maintained by means of projections and sections.[69] However, all these new inventions of his were to be obscured by those of Descartes, who was to lead mathematical sciences to evolve in a completely different direction. Desargues was to be rediscovered only at the beginning of the nineteenth century by the French school of projective geometry.[70]

Father Marin reported a complete argument of Fermat (1601–1665) on the fall of bodies.[71] But here our interest in him stems above all from the event that he, too, left us some pages about music. This senator and jurist from Toulouse commented on the relative Greek theory taken from Manuele Bryennius (fourteenth century), correcting and recalculating the divisions of the tetrachord 4 to 3. In the Byzantine scholar, Fermat had found an argument that was mistaken as a result of the usual prohibition of dividing the tone into two equal parts. It was not like dividing 9 by 2. But for this reason, he too did not seem to admit the use of irrational numbers. He also considered incomplete a list of 15 ways of dividing the tetrachord. Not only did he offer others, but he sought a general result that was unknown to the ancients. "... pulcherrimum problema subjungeremus, si per otium liceret: Nempe datâ qualibet proportione superparticulari invenire quot modis in tres proportiones superparticulares dividi possit, aut generalius, quot modis in datum proportionum superparticularium numerum dividi possit, verbi gratiâ, quot modis proportio sesquioctava in decem proportiones superparticulares dividi possit." ["...

[68] Mersenne 1636, [tr. 4] p. 331f.

[69] Fano & Terracini 1957, pp. 356–358.

[70] Taton 1981.

[71] Mersenne 1636, [tr. 7] pp. 61–64.

we would submit a fine problem, if tranquillity allowed us to do so: that is to say, given any superparticular ratio [n + 1 to n], find in how many ways it can be divided into three superparticular ratios [n + 1 to n = p + 1 to p, q + 1 to q, r + 1 to r; using fractions, $\frac{n+1}{n} = \frac{p+1}{p} \times \frac{q+1}{q} \times \frac{r+1}{r}$], or more generally, in how many ways it can be divided into a given number of superparticular ratios, for example, in how many ways the sesquioctave ratio [9 to 8] can be divided into ten superparticular ratios."][72]

That "generalius" ["more general"] attitude of his, in a page that might not seem to be very interesting (to other historians), underlines, on the contrary, how unjust the judgement that Descartes expressed of him was: "Monsieur Fermat est Gascon, moi non." ["Mr. Fermat is a braggart, I am not."] The jurist from the provincial town in Gascony was not, then, as our philosopher of the method would have us believe, a braggart devoid of any real art, who had a few lucky strikes. These two French rivals, who had dealt with similar subjects, in the same period and in the same area, had used methods that were so different that their results, while similar and comparable, had led to implacable strife and insoluble disputes, which were protracted by their followers.

With the idea of restoring and completing the treatises of Pappus, Apollonius, Diophantus or Archimedes, Fermat used the symbolic algebra introduced by the jurist François Viète (1540–1603) for his algebraic calculations and his proofs. Thus, our scholar from Toulouse created equations between curved lines seen as loci of points, and calculated their maximums, minimums, tangents and underlying areas. In spite of the event that he dealt with, and solved, many of the problems that were to lead to the fame of natural philosophers like Descartes, Leibniz and Newton, with their analytical geometries and their infinitesimal calculus, he, on the contrary, hardly left any trace in the immediate evolution of mathematical sciences. Why? Fermat did not desire to publish anything; he only wrote letters to Mersenne or a few others; some of his writings came out in an anonymous form; he sometimes left elliptic notes in the margins of his beloved Greek books. He still expressed himself in Latin. The "Last theorem" that is famous today under his name, came from his copy of the *Arithmetic* by Diophantus. It was proved only a few years ago: $a^n = b^n + c^n$ does not admit solutions with a, b, c that are whole numbers if $n > 2$. He clearly had a weakness for whole numbers, as we can understand also from his comment on music. He had still remained too closely attached to the Greek geometrical tradition in which he nourished a greater trust, despite his large-scale use of algebraic symbolism, using Latin letters, like Viète, for equations. But for an unknown quantity, he used the notation of the symbol A, and for a second one E, that is to say, capital vowels, and in the place of known numbers, he now put B, C, D, ..., the consonants. For them, nowadays, we all use symbols like x, y, ..., a, b, c, ..., that is to say, we follow the notation introduced by Descartes. Eventually, the name of Fermat was to be used above all for the principle

[72] Fermat 1891, pp. 394–410.

of optics which justified, better than others, the formula of Descartes-Snell for the refraction of light: nature follows the easiest, fastest course.[73]

Blaise Pascal (1623–1662) was not interested in music, but he deserves to be considered here, because he helps us to have a better understanding of the direction towards transcendent symbolism that the scientific community of the period was taking, agitated as it was by contrasts of a profoundly religious nature. Fermat himself was a convinced Catholic, who was to be called to settle disputes between Catholics and Huguenots. On the contrary, his father Etienne Pascal (1588–1651), amid his institutional engagements with the King's taxes and his inclination towards mechanics, actually had something to do with music. After all, he consorted with Marin Mersenne. Actually, the Minim friar even dedicated to him his "Traité de l'Orgue" ["Treatise on the organ"], praising his perfect knowledge of the subject. Did Etienne know the instrument only as "… une des plus admirables machines Pneumatiques qui furent jamais inventées." ["… one of the most extraordinary pneumatic machines ever invented"], or could he also play it?[74]

His son, who was a particularly precocious little genius, followed the geometrical style that he had learnt from Desargues, and developed his theory of conic sections, obtaining original results. Like the engineer, however, Blaise Pascal did not disdain practical, and concrete occupations. Thus, in order to help his father in his calculations for the administration of taxes, he invented a machine with gears for arithmetic operations, which he constructed and sold personally: the first calculating machine, numbered today among the ancestors of the computer. He then projected and implemented the first form of collective public transport at Paris, using horse-drawn carriages, "Les carrosses à cinq sols", for which the fare was 5 cents; it came into operation in the year of his death. He also got his brother-in-law to repeat the experiments of Evangelista Torricelli (1608–1647), regarding the weight of the atmosphere and the presumed vacuum that remained in a glass tube, at the top of a mountain in central France; he then went on to carry out various experiments on liquids, finally defining the notion of pressure.

In order to solve a problem posed by a friend of his who was a gambler, the young Blaise calculated the probabilities of fortuitous events. For this purpose, he arranged in a triangle the numbers resulting from the combinations, with the repetition of two letters two by two, three by three, and so on, which were subsequently called binomial coefficients. One of our Chinese scholars had adopted a similar distribution some centuries earlier.[75] Like Fermat, Torricelli and others, this brilliant French brain displayed the ability to cope with the geometrical problems of the moment in an original manner: calculating tangents, areas, centres of gravity, lengths of ancient and new curves. In the wake of the points and infinite straight lines discussed by Desargues, the traditional Greek prohibition of infinity had disappeared from his methods, though he did not like, or use, algebra. All the same, some historians have

[73] Mahoney 1981.

[74] Mersenne 1636, [tr. 5] p. [308bis]. Lenoble 1971, pp. 436–437.

[75] See Part I, Sect. 3.4 and Fig. 3.10.

advanced the idea that with more time at his disposal, he could have invented the infinitesimal calculus of Leibniz and Newton.[76]

As compensation for a cruel destiny, because he passed away before the age of 40, after being tormented by continual illnesses, history was to attribute Pascal's theorem to him, as well as Pascal's triangle, and Pascal's snail (studied by his father); more recently, the hecto Pascal has become the unit of measurement for atmospheric pressure used in weather forecasts, substituting Torricelli's millimetres of mercury. In reality, all these inventions had been plagued by controversies from the very beginning, because other scholars in different countries had elaborated the same inventions in the same period. In his environment, his main antagonist appeared to have been Descartes. While they were in Paris, Christiaan Huygens (1629–1695) and Wilhelm Leibniz (1646–1716) took an interest in his manuscripts.[77]

In general, historians of mathematics and sciences stop here, ignoring the other large part of his life and his works. We, on the contrary, will deal with it, because, in his case, like that of the other protagonists, it was an integral part of the evolution of sciences. He had called his theorem for conic sections "Mysterium hexagrammaticum" [hexagram of mystery], because it regarded the hexagon inscribed in a conic section. He hid his identity behind pseudonyms and anagrams. With him, and around him, he felt the presence of the Invisible. He was a Catholic, but also the son of a father who was a follower of a famous sceptic like Michel de Montaigne (1532–1592); in various circumstances, such as his father's illness and his own recurring health problems, he entered into contact with some professional churchmen. After his father's death, one night he experienced a mystical crisis: in contact with the holy relic of a thorn borne by Jesus Christ, his niece miraculously recovered from an eye disease. Consequently, he decided to change his way of life, and to dedicate himself to Christianity, forsaking all the vanities and the pride generated by his scientific knowledge. However, it was not to prove easy to divide either his work or his life into two distinct parts. Another night, to distract his attention from all the pain caused by his illnesses, he went back to studying a geometrical curve like the "roulette" [subsequently called a cycloid]: then, as if by magic, his suffering suddenly ceased. Did God want to make him understand that also mathematical studies were edifying?

And yet, among Catholics, he engaged in defending the heretical followers of Cornelius Jansen (1585–1638). While Mersenne and Descartes had received their scientific, cultural and religious formation at the Jesuit College of La Flèche, Blaise Pascal, on the other hand, grew up in the orbit of the Abbey of Port-Royal, where the readings were those of Saint Augustine (354–430), which were different from the orthodox ones established by the Church of Rome. In defence of them, and above all of Antoine Arnauld (1612–1694), who had been expelled from the Paris University of the Sorbonne for heresy, Pascal junior circulated a polemical booklet, *ça va sans*

[76] Boyer 1990, p. 422.

[77] Lenoble 1971, pp. 430–435. Taton 1981a. See below, Sects. 9.3 and 10.1.

dire under a pseudonym, in the form of letters: *Les Provinciales [The provincials]*, which the King's Council ordered to be burnt by the hangman.

Apart from the discussions of whether divine grace was indispensable to free us from our irremediably corrupt nature, and whether this grace was dispensed only to a few elect (Was it sufficient? How effective would it be?) as the Jansenists together with Pascal believed, *The provincials* directly and openly attacked the all-powerful Jesuits. For us historians of the sciences, the most interesting accusation, brought against them with extraordinary ability and style by our converted mathematician touched by the grace of God, regarded the truth.

Pascal was convinced that "... faith, and the tradition that maintains it, ... was always one and the same in all times and all places; ...". On the contrary, several Jesuits, in their books distributed across half of Europe, had invented a (new?) doctrine which was widely accepted: probable truths. Our intransigent mathematician criticised them for going so far as to "... make people believe that something false is true." But Saint Augustine of Hippo had written: "... 'who would dare to say that the truth must remain disarmed against falsehood, or that enemies of the faith will be permitted to disconcert the faithful with strong words ...?' ". And in actual fact, the Jesuits succeeded in having their adversaries found guilty, only by slandering them and attributing false positions to them. They had even theorised, in their long theological manuals, that it was legitimate to lie for reasons that were to be considered valid and of a higher priority. They behaved in the same way, not only against the Jansenists, but also against a Capuchin friar, and a famous Italian natural philosopher.

Pascal's position here was solidly based on Catholic pillars such as Saint Augustine and Saint Thomas Aquinas, to distinguish the truths of the faith from those of reason and of fact. The *Bible* had undoubtedly always been subject to various interpretations, "... whereas the testimony of the senses is one and the same. [...] 'It is necessary – says Saint Thomas in his *Summa theologica* I pars, quaest. 68, art. 1- to observe two things, following Saint Augustine: firstly, that Scripture always has a true sense; and secondly, that as it is liable to be interpreted in different ways, when we find one which reason proves to be undoubtedly false, we must not obstinately insist that this is the natural meaning of it, but we should search for another meaning which is in agreement with it'. [...] And thus it was in vain that [you Jesuits] obtained the decree from Rome against Galileo, condemning his opinion about the movement of the Earth. That decree will not be sufficient to prove that the Earth stands still; and if there were evident observations proving that it is the Earth that goes round, then all the men put together would not be able to stop it from going round, nor could they stop themselves going round with it." It may be noted that this young French natural philosopher did not defend the movement of the Earth in itself, but rather, like Mersenne, he invoked a method based on the senses to arrive at an experimental truth, which was not evident at the time.[78]

[78] Pascal 1988, pp. 58, 110, 140, 154–159, 174–176.

Also the other attacks against the Jesuits regarded questions that are not without interest today, and not only to gain a better understanding of the evolution of the sciences. Everything ended up with the undoubted ability of certain people wearing a cassock to bend the meaning of words at their pleasure in every possible, and impossible, sense. Should we condemn this ability from the heights of our morality, or should we admire their technical intelligence, and their surprisingly modern approach? The reader will judge for himself, on the basis of his own values, bearing in mind, however, that Pascal himself was no less skilful in his choice of witnesses in favour of, or against, following the art of interpreting words in agreement with his own hypothesis. Viscount Chateaubriand (1768–1848) was, in his turn, to define him as "an ingenious slanderer".

Thus the Jesuits succeeded in excusing usury by calling it "gratitude" towards the money-lender. They absolved assassins, if they killed to defend their honour and their estate. Even a priest would be *forced* to kill a slanderer *etiam aliquando debet occidere* [must sometimes even kill]. In this way, they were preparing the way to justify anyone who took such drastic action to eliminate the Jansenists and the other presumed enemies of the Catholic Church. However, even our pious, polemical mathematician failed to give a clear-cut, absolute meaning to the commandment "Thou shalt not kill", in spite of all his research into the truth. Also for him, therefore, it was possible to kill criminals: it was sufficient to do it in the name of God. Then it would no longer be any man who was acting, committing a crime, but God himself. The only judge of life and death would be using us as the instruments of his justice.

In the Order of the Minims, chastity must have been considered with greater severity than in the Society of Jesus. Consequently, in the confession manuals, it was easy to tolerate those young ladies *sans-gêne* with a somewhat limited degree of virtue, who must have abounded in the courts and the churches of Paris. " '... the girl possesses her virginity just as she possesses her own body; she can do what she thinks best with it, except for provoking death or the mutilation of her members.' " The casuistry of sins was adapted to the circumstances. In this way, the Jesuits could accomplish their mission, all over the world, from Paris to China, moulding moral principles, holy writings and uninhibited behaviour, in order to obtain the maximum advantage. Did a good end justify any means? Were they the inventors of relativism? In the seventeenth century, not only were new sciences being offered, but also a new Catholic morality, which Blaise Pascal clearly did not like at all. According to him, they had succeeded in hiding the essence of Christianity. "... this is the rule that they followed in the Indias and in China, where they even allowed Christians to commit idolatry, thanks to their subtle invention of hiding an image of Jesus Christ under their clothes, and teaching them to refer mentally to this image the public worship that they rendered to the idol Chacimchoan and to Keum-fucum [Confucius]: ...". It is a pity that the Chinese sciences did not surface in the disputes of the other religious orders regarding the behaviour of the Jesuits in China, because

we would have been glad to read Pascal's comments on the subject.[79] Anyway, we know that the Jesuits' tactics in China were more subtle, making a direct use of the sciences, and in the end, the double morality revealed its two-edged character: finally, they nonchalantly dedicated themselves to the noble art of war (in the name of God?). The Jesuits in China probably devoted too little attention to mathematical details, such as the triangle of binomial coefficients, and consequently, as Pascal was unaware of it, he invented his own version independently.

In his *Pensées [Thoughts]*, the Jesuits were to move away from the centre of the picture, but that little problem of the truth remained. Here, the dispute was now, rather, with Montaigne. Without running any risk of exaggerating, we may say the Blaise Pascal above all took up, and commented on the *Essais [Essays]* of the sceptic-humanist who had lived a century before. On earth, Montaigne was right, the redeemed mathematician seems to tell us, and therefore he was left with only one pathway to arrive at the truth: God. His god was the one who became flesh in Christ, was prophesied in the *Bible*, and above all was capable of performing miracles. The one of the holy thorn, who healed his niece in the Abbey of Port-Royal.

On the contrary, the sciences were vanity, incapable of bringing man out of his ignorance. Descartes said: "…: 'Cela se fait par figure et mouvement', car cela est vrai. Mais de dire quelles et composer la machine, cela est ridicule, car cela est inutile et incertain et pénible. Et quand cela serait vrai, nous n'estimons pas que toute la philosophie vaille une heure de peine." ["…: 'this happens by figure and by movement', because it is true. But to say what they would be, and to put the machine together, that is ridiculous, because it is useless, uncertain and distressing. And even if it were true, we do not think that all philosophy is worth an hour's exertion."] The model of Descartes was rejected: "Écrire contre ceux qui approfondissent trop les sciences: Descartes". "Descartes inutile et incertain." "Nous n'avons ni le vrai ni le bien." ["Writing against those who examine the sciences in too great detail, Descartes". "Descartes useless and uncertain." "We do not have either the truth, or the good."][80]

Thought [6] regards the truth of the Christian religion, like thought [813]. Between these, the others created continuous variations. The last thought [816] recalls the errors of the Jesuits. "Ainsi les Jésuites ou font embrasser les erreurs ou font jurer qu'on les a embrassées, et font tomber ou dans l'erreur ou dans le parjure, et pourissent ou l'esprit ou le coeur." ["Thus the Jesuits either make people embrace errors, or make them swear that they have embraced them, and make them fall prey either to error or to perjury, and corrupt either the spirit or the heart."] Among the

[79] Pascal 1988, pp. 57–58, passim. Today, in 2014, at least in some part of the world, those who exchange sex for money would no longer be called prostitutes and customers, but escorts and politicians. The problem of public morality is solved by words: Jesuit ability.

[80] Pascal 2004, pp. 29, 67, 69, 365, 335, … passim.

many, we will choose a few. One of them [450] deals with Pyrrhonism.[81] "Chaque chose est ici vraie en partie, fausse en partie. La vérité essentielle n'est point ainsi, elle est toujour pure et toute vraie. Ce mélange la déshonore et l'anéantit. Rien n'est purement vrai, et ainsi rien n'est vrai en l'entendant du pur vrai. [...] De ne point tuer? Non, car les désordres seraient horribles et les méchants tueraient tous les bons. De tuer? Non, car cela detruit la nature. Nous n'avons ni vrai ni bien qu'en partie, et mêlé de mal et de faux." ["Here, everything is partly true, and partly false. The essential truth is not like that, it is always pure, and wholly true. This mixture dishonours the truth, and destroys it. Nothing is purely true, and thus nothing is true, meaning by this purely true. [...] Never kill in any case? No, because the disorder would be terrible, and the bad people would kill all the good people. Kill? No, because this would destroy nature. We do not possess either the truth, or good, which is partly mixed with evil and falsehood."]

"Nous connaisons la vérité non seulement par la raison, mais encore par le coeur. [...] Le coeur sent qu'il y a trois dimensions dans l'espace et que les nombres sont infinis, et la raison démontre ensuite qu'il n'y a point deux nombres carrés dont l'un soit double de l'autre. Les principes se sentent, les propositions se concluent, et le tout avec certitude, quoique par différentes voies." ["We know the truth not only by means of reason, but also by means of the heart. [...] The heart feels that there are three dimensions in space, and that numbers are infinite, but then reason demonstrates that no two square numbers exist, of which one is twice the other.[82] Principles are felt, propositions are proved, all with certainty, albeit by different routes."] "La volonté est un des principaux organes de la créance. Non qu'elle forme la créance, mais parce que les choses sont vraies ou fausses selon la face par où on les regarde." ["The will is one of the main organs of trust. Not that it creates trust, but because things are true or false, depending on the aspect from which they are considered."] "Mais est-il probable que la probabilité assure? Différence entre repos et sûreté de conscience. Rien ne donne l'assurance que la vérité. Rien ne donne le repos que la recherche sincère de la vérité." ["But is it likely that probability will reassure? Difference between tranquillity and certainty of conscience. Nothing gives certainty except the truth. Nothing gives tranquillity except the sincere search for the truth."][83]

A curious variation on the saying *In vino veritas* is found in thought [72]. "Trop et trop peu de vin. Ne lui en donnez pas, il ne peut trouver la vérité. Donnez-lui en trop, de même." ["Too much wine and too little. Don't give him any and he will not be able to find the truth. Give him too much, and the result is the same."] This was a joke about the habit of making a gift of wine to the judges of a trial, to convince them

[81] The Greek philosopher Pyrrho of Elis, who died in 288 B.C., harboured doubts about everything, and taught that the truth could never be found. Consequently, some people called scepticism the Pyrrhonism, and the word remained common for a certain time.

[82] If it is not true that $n^2 = 2m^2$, that is to say, $\frac{n}{m} = \sqrt{2}$ with n and m whole numbers, then $\sqrt{2}$ is irrational.

[83] Pascal 2004, pp. 4–5, 576–579, 758, 344–345, 84–85, 360–361, 390–391.

to take sides with the donor. "Car la vérité ne s'altère que par le changement des hommes." ["Because the truth only changes when men change."] "L'homme n'est donc que déguisement, que mensonge et hypocrisie, et en soi-même et à l'egard des autres. Il ne veut donc pas qu'on lui dise la vérité. Il évite de la dire aux autres." ["Man is nothing other than disguise, falsehood and hypocrisy, both in himself and towards others. Therefore he does not want to be told the truth. He avoids telling it to others."] "On se fait une idole de la vérité même, car la vérité hors de la charité n'est pas Dieu et est son image et une idole, qu'il ne faut point aimer ni adorer. Et encore moins faut-il aimer ou adorer son contraire, qui est le mensonge." ["The truth is turned into an idol, because truth without charity is not God, but his image and idol, which we should not love or worship. Even less should we love or worship its opposite, the lie."] *Et ainsi de suite*.[84]

Pascal's references to the truth are too frequent and constant not to raise historical and epistemological problems. Truly as a result, his devotion, body and soul, to the Christian religion should not be reduced to the mystic crisis of one night, but should rather be connected with his search for truth(s): those of the scientific kind no longer satisfied him.

His search for laws was parallel and similar to this. Should human laws be based on natural laws? "Certainement ils les soutiendraient opiniâtrement si la témérité du hasard, qui a semé les lois humaines, en avait rencontré au moins une qui fût universelle. Mais la plaisanterie est telle que le caprice des hommes s'est si bien diversifié qu'il n'y en a point. [...] Il y a sans doute des lois naturelles, mais cette belle raison corrompue a tout corrompu: *Nihil amplius nostrum est, quod nostrum dicimus artis est. Ex senatusconsultis et plebiscitis crimina exercentur. Ut olim vitiis, sic nunc legibus laboramus.* [...] Rien n'est si fautif que ces lois qui redressent les fautes; qui leur obéit parce qu'elles sont justes, obéit à la justice qu'il imagine, mais non pas à l'essence de la loi. [...] C'est pourquoi le plus sage des législateurs disait que pour le bien des hommes il faut souvent les piper; et un autre bon politique: *Cum veritatem qua liberetur ignoret, expedit quod fallatur.*" ["Undoubtedly they would tenaciously support them if the disrespect of chance, which has disseminated human laws, had hit on at least one that was universal. But the funny thing is that man's fancy has diversified so widely that there is not even one. [...] Without doubt natural laws exist, but this fine corrupted reason has corrupted everything: *Nothing is ours any longer, what we say is ours is artificial. Crimes are committed on the basis of decrees of the Senate and regulations of the people. As we used one time to suffer as a result of vices, so now we suffer as a result of laws.* [...] Nothing is so faulty as these laws that correct errors; anyone who obeys them because they are just, obeys an imaginary justice, but not the essence of the law. [...] For this reason, the wisest of legislators said that, for men's good, it is often necessary to hoodwink them; and another good politician: *when he ignores the truth that would free him, it is appropriate that he should be deceived.*"][85]

[84]Pascal 2004, pp. 34–35, 781, 236–237, 740–741, 718–719, ..., passim.

[85]Pascal 2004, pp. 52–57, 791–793.

In sequence, Pascal had read the Latin sayings of Cicero, Seneca, Tacitus, (Plato) and Saint Augustine in Montaigne. Let us therefore give honour where honour is due to this sage, so modern in his approach, who shows us what great progress peoples had made from the time of Cicero up to his period, and from his period to our great age. "Il est croyable qu'il y a des loix naturelles, comme il se voit ès autres creatures; mais en nous elles sont perdues, cette belle raison humaine s'ingerant par tout de maistriser et commander, brouillant et confondant le visage des choses selon sa vanité et inconstance." ["It is believable that there are natural laws, as we can see in other creatures; but in us, these are lost, since this fine human reason meddles in everything, with the aim of domineering and commanding, clouding and confusing the aspect of things, in accordance with its vanity and its inconstancy."] The only true law, the Law, the most ancient of all of them, the one which was subsequently copied by the others, the most perfect and rigorous, the most stable in time and space, was called by this Frenchman, illuminated by grace, the law of Moses, as written in the *Bible*.[86]

In his struggle against scepticism rather than against the angel, our pious mathematician contrasted reason with the heart: "Le coeur a ses raisons, que la raison ne connaît point" ["The heart has its reasons that reason has no knowledge of"]; he contrasted beasts with angels, dogmatism with Pyrrhonism, because a neutral person would be the perfect Pyrrhonist, the body with the soul, matter with the spirit and Jews with Christians, who were respectively carnal and spiritual. "Double loi, doubles Tables de la Loi, double temple, double captivité." ["Double Law, double Tables of the Law, double Temple, double captivity."] It was necessary to teach man "… sa double capacité de recevoir et de perdre la grâce à cause du double péril où il est toujours exposé, de désespoir ou d'orgueil." ["… his double capacity to receive and to lose God's grace, due to the double danger to which he is always exposed, that of despair and that of pride."] "Cette duplicité de l'homme est si visible qu'il y en a qui ont pensé que nous avions deux âmes." ["This dualism of man is so visible that there are some [the Manicheans] who have thought that we have two souls."] Pascal's most famous dichotomy has remained the "Différence entre l'esprit de géométrie et l'esprit de finesse." ["Difference between the spirit of geometry and the spirit of subtlety."] "Finesse" is often interpreted as "intuition". The expert of geometry reasons on the basis of fixed principles which give a tangible certainty, but which are far away from common use. The subtle (intuitive) person, on the contrary, would start from common practice and succeeds in proceeding without any effort, because he possesses a particularly clear sight, which would allow him not to make mistakes. Thus the former would give systematic demonstrations obtained from principles. The latter would see things immediately, at a glance, without reasoning on them. We can easily imagine that our young Parisian, pervaded with his extremely rare *esprit de finesse*, had melted it at times with the geometrical field.[87]

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[86]Montaigne 1969, II p. 246; 2005, p. 771. Pascal 2004, p. 792. Pascal 2004, pp. 568–571 and 581.

[87]Pascal 2004, pp. 85, 532–533, 93, 99, 171, 233, 266–269, 406–407, 512–517.

As regards the dualism between truth and error, enough has already been said. The truth itself was dualistic for him, without any fear that it might become contradictory. Thus some would be heretics "… ne pouvant concevoir le rapport de deux vérités opposées et croyant que l'aveu de l'une enferme l'exclusion de l'autre, …" ["… not succeeding in conceiving of the relationship of two opposite truths, and believing that the acceptance of the one implies the exclusion of the other, …"]. For example, Arian heretics admit the humanity of Christ, but exclude his divinity. The mixture of good and evil already existed in Montaigne, who, between the two upper and lower extremities, pulled Pascal towards the medium. "Il se peut faire qu'il y ait de vraies démonstrations, mais cela n'est pas certain. Ainsi cela ne montre autre chose sinon qu'il n'est pas certain que tout soit incertain. À la gloire du Pyrrhonisme." ["It may happen that there are true demonstrations, but this is not certain. This simply does not demonstrate other thing different from that: it is not certain that everything is uncertain. To the glory of Pyrrhonism."] From the provincial sceptic, our pious Parisian had learnt that evil was easier because it is infinite, whereas there is only one good. And yet excellence in evil is as rare as excellence in good.[88]

From a recurrent dualism, it would be possible to rise up only to transcendence. "… la vérité n'est pas de notre portée ni de notre gibier, qu'elle ne demeure pas en terre, qu'elle est domestique du ciel, qu'elle loge dans le sein de Dieu et que l'on ne la peut connaître qu'à mesure qu'il lui plaît de la révéler. Apprenons donc de la vérité incréée et incarnée notre véritable nature." ["… truth is not within our reach, nor is it game for our hunting; it does not dwell on Earth, it resides in Heaven; it lodges in the bosom of God, and can only be known to the extent that He pleases to reveal it to us. Let us thus learn our true nature from the uncreated, incarnate truth."] Even here, he found hints in Montaigne, but in the end "… l'homme par la grâce est rendu comme semblable à Dieu et participant de sa divinité, et que sans la grâce il est cencé semblable aux bêtes brutes." ["… by grace man is made similar to God, and participates in his divine nature, but without grace, he may be considered similar to brutish beasts."]

"La dernière démarche de la raison est de reconnaître qu'il y a une infinité de choses qui la surpassent. Elle n'est que faible si elle ne va jusqu'à connaître cela. Que si le choses naturelles la surpassent, que dira-t-on des surnaturelles?" […] "La seule religion contre la nature, contre le sens commun, contre nos plaisirs, est la seule qui ait toujours été." ["The last step of reason is to recognize that an infinity of things exist that go beyond it. It is simply weak if it does not succeed in recognizing this. If natural things transcend it, what can we say of the supernatural?" […] "The only religion against nature, against common sense, against our pleasures, is the only one that has always existed."] Probably referring to Descartes, he wrote thought [692]. "Quand un homme serait persuadé que les proportions des nombres sont des vérités immatérielles, éternelles et dépendantes d'une primière vérité en qui elles subsistent et qu'on appelle Dieu, je ne le trouverais pas beaucoup avancé pour son

[88] Pascal 2004, pp. 452–453, 893, 349, 894, 350–351, 355, 896.

salut. Le Dieu des chrétiens ne consiste pas en un Dieu simplement auteur des vérités géométriques et de l'ordre des éléments: . . . ". ["Even if a man were persuaded that the proportions of numbers are immaterial, eternal truths that depend on an initial truth in which they subsist, which is called God, I would not consider him to be too far forward towards salvation. The God of the Christians does not consist of a God who is only the author of geometrical truths, and of order among the elements: . . . "].[89]

Above all, the infinite transcended reason in Pascal. "Tout ce qui est incompréhensible ne laisse pas d'être. Le nombre infini, un espace infini égale au fini." ["Things that are incomprehensible do not cease to exist. An infinite number, or an infinite space equal to the finite."] He repeated the famous image of ". . . une sphère infinie, dont le centre est partout, la circonférence nulle part." [". . . an infinite sphere, whose centre is everywhere and whose circumference nowhere."] But, while this was referred to God in the medieval tradition, now Pascal curiously wrote, rather, about the incommensurability of nature compared with man. "Qu'est-ce qu'un homme, dans l'infini? [. . .] Un néant à l'égard de l'infini, un tout à l'égard du néant, un milieu entre rien et tout. [. . .] . . . on comprende que la nature ayant gravé son image et celle de son auteur dans toutes choses, elles tiennent presque toutes de sa double infinité. C'est ainsi que nous voyons que toutes les sciences sont infinies en l'étendue de leurs recherches. Car qui doute que la géométrie, par exemple, a une infinité d'infinités de propositions à exposer?" ["What is man, amid the infinite? [. . .] A nothing, compared with the infinite, which is everything compared with nothing, in the middle between nothing and everything. [. . .] . . . we can understand that, having nature impressed its image and that of its creator on all things, they almost all participate in its double infinity. This is the reason why we see that all the sciences are infinite in the extension of their research. For who doubts that geometry, for example, has an infinity of infinities of propositions to expound?"]

Now our young Frenchman found infinities everywhere: in the large as in the small, in arithmetic as in geometry. "Et il me semble que qui aurait compris les derniers principes des choses pourrait aussi arriver jusqu'à connaître l'infini. L'un dépend de l'autre et l'un conduit à l'autre. Ces extrémités se touchent et se réunissent à force de s'être éloignées, et se retrouvent en Dieu, et en Dieu seulement. [. . .] Notre raison est toujours déçue par l'inconstance des apparences, rien ne peut fixer le fini entre les deux infinis qui l'enferment et le fuient." ["It seems to me that whoever had understood the ultimate principles of things could also arrive at an understanding of the infinite. One depends on the other, and one leads to the other [the infinite in largeness and the infinite in smallness]. These extremes touch themselves by force of moving away from each other, and meet in God, only in God. [. . .] Our reason is always frustrated by the inconstancy of appearances; nothing can fix the finite between two infinities which enclose it and flee from it."][90]

[89] Pascal 2004, pp. 100–101, 104–105, 150–151, 232–233, 564–565, 960.

[90] Pascal 2004, pp. 132–133, 158–159, 160–167.

"La nature recommence toujours les mêmes choses, les ans, les jours, les heures; les espaces de même; et les nombres sont bout à bout, à la suite l'un de l'autre. Ainsi se fait une espèce d'infini et d'éternel. Ce n'est pas qu'il y ait rien de tout cela qui soit infini et éternel, mais ces êtres terminés se multiplient infiniment. Ainsi il n'y a, ce me semble, que le nombre qui les multiplie qui soit infini." ["Nature always starts the same things again, years, days, hours; spaces do the same; and numbers follow each other head-to-head one after the other. Thus a sort of infinity and eternity is created. It is not that does not exist anything of all those things that is infinite or eternal, but these limited beings are infinitely multiplied. Therefore there is only the number that multiplies them, it seems to me, that is infinite."] "Nous connaissons qu'il y a un infini, et ignorons sa nature. … il est faux qu'il soit pair, il est faux qu'il soit impair, car en ajoutant l'unité il ne change point de nature. Cependant c'est un nombre, et tout nombre est pair ou impair (il est vrai que cela s'entend de tout nombre fini). Ainsi on peut bien connaître qu'il y a un Dieu sans savoir ce qu'il est." ["We know that there is an infinite, but we ignore its nature … it is false that it is even, as it is false that it is odd, because by adding one unit, it does not change its nature. However, it is a number, and every number is either even or odd (this is true, we think, for every finite [whole] number). Thus it is perfectly possible to know that a God exists without knowing what He is."]

Beyond the comparison with God, however, Descartes refused to consider the infinite: "Therefore we shall never get involved in disputes about the infinite, since it would be ridiculous if we, who are finite, tried to establish something about it, and in this way to suppose it to be finite, in an attempt to understand it." Thus Blaise Pascal was moving away both from the Greek-medieval tradition and from Descartes, and at the same time, drawing closer to Leibniz and to infinitesimal calculus. As is known, about God our good mathematician would even have bet, staking on his existence. However, he refrained from calculating numerically here the probability of winning, as he had discussed with Fermat for dice. Anyway, this was another argument where he made a connection between the sciences and his religion.[91] To unbelievers, our French heretic offered another model of the infinite God. "Je vous veux donc faire voir une chose infinie et indivisible. C'est un point se mouvant partout d'une vitesse infinie. Car il est un en tous lieux, et est tout entier en chaque endroit." ["I want to show you, then, something infinite and indivisible. It is a point that moves at an infinite speed everywhere. Because it is in all places and it is whole in every recess."][92] Our French mathematician was starting to perceive the problem of the infinite, the presence of which he felt everywhere, though he could not solve it, nor did he want to. It reminded him all too clearly of God. It was possible to talk about it, though it continued

[91] Pascal's reasoning recalls that of lotteries, where by betting a small sum, or hardly anything, it is possible to win a lot. In the case of God, one would win Paradise for eternity. However, as in the case of lotteries, also Pascal's God becomes, in reality for common mortals (the non-elect), a tax: the most hateful one, because it is imposed on poverty and hope.

[92] Pascal 2004, pp. 418–419, 522–523, 955, 524–533.

to remain outside the sphere of mathematical investigation. In Sect. 4.2 of Part I, the *jaina* Indians had shown a different attitude. Inspired by the vast numbers of their staggering calculations, they had classified them as "numerable, almost non-numerable, absolutely non-numerable, innumerably non-numerable, almost infinite, absolutely infinite or infinitely infinite". That is to say, these mathematicians had dealt with their infinities, bringing them into line with other numbers, without excluding them as if they were untouchable.

When he arrived at thought [233], Blaise Pascal put his seal on the scientific revolution of the seventeenth century in one sentence: "Le silence éternel de ces espaces infinis m'effraie." ["The eternal silence of these infinite spaces frightens me."] Not having heard the music in the universe, he resisted his fear by turning to God.[93] What did this young inventor of certain mathematical mysteries and heretical Christian truths write about music? He compared men to the common church organs. However, he also distinguished between them, because musical instruments, thanks to the harmonies of their consonances, would be consistent, whereas living beings often would change and vary easily. Also this last thought [88] came from Montaigne. "La faiblesse de l'homme est la cause de tant de beautés qu'on établit, comme de ne savoir bien jouer du luth n'est un mal qu'à cause de notre faiblesse." ["Man's weakness is the cause of many created beauties, just as not being able to play the lute well is not an evil save for our weakness."] "Trop de plaisir incommode, trop de consonances déplaisent dans la musique et trop de bienfaits irritent." ["Too much pleasure is a bother, in music too many consonances are disagreeable, and too many benefits irritate."] He always varied Montaigne's version, including music. Instead of thinking of elevated topics, and of God, Pascal rebuked whoever dedicated himself "… à danser, à jouer du luth, à chanter, à faire des vers, … , à se battre, …" ["… to dancing, lute-playing, singing, writing poetry, …, fighting [a duel], …"]. It is significant that among the *Thoughts* there was one [632] for which another character of Port-Royal was responsible. Here he gave a warning against the theatre and feelings of love that it was capable of provoking. "Tous les grands divertissements sont dangereux pour la vie chrétienne." ["All great amusements are dangerous for the Christian life."] We may also imagine that music was also subjected to criticism.[94]

China returned in thought [665]. Denying a presumed insuperable antiquity of the *Bible*, someone had sustained that other non-Jewish men had existed before Adam. Blaise Pascal had personally heard that the chronologies of the Chinese went back for tens of thousands of years. But he did not believe this, and answered those who said that China would blot out his ideas about the Christian story of salvation: " 'La Chine obscurcit, mais il y a clarté à trouver. Cherchez-la' … Il faut donc voir cela en détail, il faut mettre papiers sur table." [" 'China makes things obscure, but there is clarity to be found. Look for it.' … This needs to be examined

[93]Pascal 2004, pp. 172–173.

[94]Pascal 2004, pp. 49, 790, 74–75, 166–167, 835, 402–403, 466–467. Montaigne 1969, II p. 7, III p. 155; Montaigne 2005, pp. 430, 1252.

in detail, the cards need to be put down on the table."][95] Unfortunately, no further information about that faraway country was to arrive from our young philosopher. After all, how could it? With his poor health, he would never have survived the long, dangerous journey. Who else could have given him more precise information? Only those Jesuits whose uninhibited, malleable ideas about Christianity, sins, the 'probability' of events and behaviour he was verifying and attacking. Pascal would clearly not have trusted reports about China from them. Long before reading how forcefully this famous Frenchman accused the members of the Society of Jesus of his time, even I described how changeable and opportunistic the behaviour of those missionaries was in China and in Rome.[96] Whether referring to mathematics, or astronomy, or whatever, we can only imagine the great interest with which our French Jansenist would have received news about China. Unfortunately, he knew very well why "transparency" about that Oriental country was still very distant. Would he condemn the country because it was not Christian, and therefore needed to be converted? Would he have recognized their cultural diversity? If he had been confronted with Chinese mathematics, and their conceptions of the energetic *qi*, or the rest of their natural philosophy, would Pascal have found new reasons to deny that it was possible, by scientific means, to arrive at some eternal, universal truth? Undoubtedly, his God would remain the best refuge from an Earth like this, capable of generating only doubts and sins.

Perhaps Pascal ignored that his beloved Jesuits had even started casting cannons in order to succeed in spreading the truths of Christianity, astronomy and mathematical sciences in China. And yet, for the victory of the truth, making war was considered to be legitimate, also by our converted mathematician. "Et n'est-il pas visible que, comme c'est un crime de troubler la paix où la vérité règne, c'est aussi un crime de demeurer en paix quand on détruit la vérité? [...] La vérité est donc la première règle et la dernière fin des choses." ["Is it not therefore clear that, as it is a crime to disturb the peace where truth reigns, it is equally a crime to remain in peace when the truth is destroyed? [...] Truth is thus the first rule and the last purpose of things."] Defence of the truth, in the Church against the Jesuits, justified war, just as maintaining the safety of one's goods in States: "... et la guerre qui le peut défendre devient et juste et nécessaire; ..." ["... and the war that can defend them becomes both just and necessary; ..."].

In the "Twelfth letter" of the *Provincials*, truth and violence were considered to be "... impotent as regards one another." Violence could not suppress the truth, just as truth could not succeed in stopping violence. With the difference, the renowned Parisian sustained, that violence would be limited by God until the truth triumphed, which, on the contrary, would remain eternal. Unfortunately, the union of force with justice was not successful "... parce que la force a contredit la justice et a dit qu'elle était injuste, et a dit que c'était elle qui était juste. Et ainsi, ne pouvant faire que ce qui est juste fût fort, on a fait que ce qui est fort fût juste." ["... because force

[95] Pascal 2004, pp. 506–509, 902, 948.

[96] See Part II, Sect. 8.2.

challenged justice, saying that it was unjust, and sustained that force itself were just. And therefore, not succeeding in making what is just strong, what was strong was made just."] "La force est la reine du monde, et non pas l'opinion. Mais l'opinion est celle qui use de la force. C'est la force qui fait l'opinion." ["Force is the queen of the world, and not opinion. But opinion is what makes use of force. It is force that makes opinion."] "Faut-il tuer pour empêcher qu'il n'y ait des méchants? C'est en faire deux au lieu d'un. *Vince in bono malum*, saint Augustin." ["Is it necessary to kill in order to prevent criminals from existing? This means creating two instead of one. *Overcome evil with good*, Saint Augustine."] "La plus cruelle guerre que Dieu puisse faire aux hommes en cette vie est de les laisser sans cette guerre qu'il est venue apporter. *Je suis venu apporter la guerre*, dit-il; et pour instrument de cette guerre: *Je suis venu apporter le fer et le feu*. Avant lui le monde vivait dans cette fausse paix." ["The cruellest war that God can make on man in this life is to leave them without this war that He came to bring. *I came to bring war*, he says; and as the instrument of this war: *I came to bring the sword and fire*. Before Him, the world lived in this false peace."][97]

Even in the quiet Paris of Pascal, it cannot have been difficult for news of the various European wars to arrive. We may add to the one already seen with Kepler another one between France and Spain, without forgetting the Turks. By the way, Muhammad was opposed by the young Christian to the Redeemer. "Différence entre Jésus-Christ et Mahomet. Mahomet non prédit. Jésus-Christ prédit. Mahomet en tuant. Jésus-Christ en faisant tuer les siens. Mahomet en défendant de lire, les Apôtres en ordonnant de lire." ["Difference between Jesus Christ and Muhammad. Muhammad was not prophesied. Jesus Christ was prophesied. Muhammad by killing. Jesus Christ by letting his own be killed. Muhammad by stopping people from reading, the Apostles by ordering people to read."] We can easily understand that Pascal defended the *Gospels* and criticised the *Koran*, because he believed in the miracles of Jesus and in the prophecies of the *Bible*. Thus Christ was divine for him, whereas Muhammad was a man who had proclaimed himself a prophet. Likewise, we should not wonder that he did not appreciate the morals of many other religions, because they were considered to be lacking in "… preuves qui peuvent m'arrêter. Et qu'ainsi j'aurais refusé également et la religion de Mahomet et celle de la Chine, et celle des anciens Romains et celle des Égyptiens, par cette seule raison, que l'une n'ayant point plus marques de vérité que l'autre, …" ["… proofs that can bring myself round. And thus I would have refused in the same way the religion of Muhammad and that of China, that of the ancient Romans and that of the Egyptians, for this single reason, that one does not display any more signs of truth than another, …"].[98] But how could a similar intelligence, lively as it was, trust a source that attributed to Muhammad in the *Koran* the prohibition of reading? Thanks to the Koranic schools, those who had studied Arabic were probably more numerous than his Parisians who could read the Latin of the *Bible*.

[97]Pascal 2004, pp. 742–745, 799, 80–81, 364–365, 416–417, 716–717.

[98]Pascal 2004, pp. 178–179, 578–581.

Without music, and unable to put up with the vice of his world, Pascal found refuge in the Jansenist heresy. Descartes, on the contrary, had followed a path from which the younger Blaise had often moved away, as we have seen. And yet, from his *Thoughts*, one gets the impression that the Christian scholar dedicated a constant attention not only to Montaigne, but also to the philosopher of the method. Readings in this sense are revealed to us by Pascal's observations about "doubting", about heat, about sensations, about the circulation of the blood, about the value of thought, about the body of Christ in the Eucharist, as well as when he took up the image of philosophy as a tree, or that of pagan morals as founded on the sand.[99]

As well as the welcoming convent of Mersenne, Descartes and Pascal had unfortunately shared a hostile cultural context, which showed no tolerance for free research, against which the Parisian Academy proved to be an insufficient defence. To complete the idea that a reader of the third millennium should have of the dangerous climate to which these two Frenchmen were subjected, together with a consideration of the famous Jesuit College of La Flèche, one may also read *El Héroe [The hero]* by the Jesuit Baltasar Gracián (1601–1658). Here, in 1637 a year which we shall immediately encounter with Descartes, as it is the same year of the *Discourse on Method*, the famous Spanish writer and philosopher listed the gifts, not only the military ones, that were necessary for heroes. Among the first of these supremely necessary qualities, he included the art of dissimulating, hiding and disguising. He suggested which achievements determined applause. "Rico vive de aplauso un histrión, y perece de crédito. [...] porque lo belicoso tiene más de plausible que lo pacífico. Entre los jueces se entresacan los justicieros a inmortales, porque la justicia sin crueldad siempre fue más acepta al vulgo que la piedad remisa."["A charlatan rich in applause lives and dies on credit. [...] because the bellicose obtain more applause than the pacific. Among judges, those who condemn to death are preferred as immortal, because justice without cruelty has always been better accepted by the people than hesitant piety."][100]

The Spanish Jesuit must have followed some of this advice himself because, with the exception of one case, he did not publish under his own name, or with the necessary authorisations. But perhaps he did not succeed in harmonising them together, undergoing the criticism and the condemnations inflicted by the generals of his religious order. In the end, he attempted to leave the Society of Jesus, without succeeding, because death arrived first. In the words of Pascal, and in the writings of the other Jesuits, reputation came before everything else, and it had to be safeguarded by any means beyond all imagination. One case contemplated was that of a clergyman who, overcome by concupiscence, had then killed the woman with whom he had sexual intercourse. The danger of dishonour, listed in the casuistry of the order, undoubtedly thinking more of the honour of the prelate than that of the woman, would have authorised him to commit this terrible gesture.[101] Bearing this

[99] Pascal 2004, pp. 99, 427, 459, 465, 493, 944, 679, 999, 435, 923, 685, 1002.

[100] Gracián 2008, pp. 88–89.

[101] Pascal 2004, p. 1012.

in mind, saving all disproof, I feel authorised to harbour doubts about the writer's death that arrived just the right moment.

> … el Engaño a la entrada del mundo y el Desengaño a la salida, la mentira al principio, la verdad al fin.
> [Life meets "deception entering into the world and disillusion going out of it, the lie at the beginning, the truth at the end."]
>
> Baltasar Gracián

> [Descartes] started from what he knew clearly, with the aim of finding the cause of what he saw.
>
> Fontenelle

9.2 René Descartes, Isaac Beeckman and John Wallis

With René Descartes (1596–1650), we can return to music, and surprisingly, but not excessively so, the context of his youthful work, *Compendium Musicae [Compendium of music]* is again another war. "… this writing has been put together all at one go, only for you [Isaac Beeckman (1588–1637)] by a man left inactive, surrounded by the ignorance typical of the military environment, and yet still free, that is to say, taken up by thoughts and activities that have nothing to do with that environment. Held at Breda, in Brabant, January 1st, 1618."[102]

Thus the movements of the young Descartes were set in the same Europe, tormented by tragic religious, political and military events, as the one where Kepler roamed around. Yet in spite of everything, our French natural philosopher succeeded in isolating himself in order to study and to write. His '*poêle*', that is to say, his heated room, remained famous, where he conceived his *La Méthode [Method]* and prepared his mathematical research in 1619–20. Previously, he too had punctually written his own book about musical theory, destined to be published only in the year of his death.

Instead of imagining the harmony of the world, or of the universe, our French philosopher attributed to music "… finis ut delectet, variosque in nobis moveat affectus;" ["… the aim of pleasing and of arousing different emotions in us;"]. He explored its various conditions, among which he immediately excluded the "geometrical" ratios "… the parts … are incommensurable and therefore, I think, they cannot be perceived perfectly in their reciprocal ratio, …". We must therefore expect him, too, to compile his own variant on the Pythagorean-Ptolemaic tradition. He chose his whole numbers, in accordance with the variety permitted, and the facility with which the hearing could perceive them. For rhythm, the Cartesian ratios were 2:1 and 3:1.

He considered the strings of the lute, and heard that those tuned to the octave and the fifth above vibrated also by resonance, without touching them. "… existimo

[102] Descartes 1966, X p. 141. Cf. Descartes 1979, p. 127.

nullum sonum audiri quin huius octava acutior auribus quodammodo videatur resonare." ["... I believe that no sound is heard unless its higher octave seems to the ears to vibrate in a certain way."] He deduced that, as "... one sound is to another as one string is to another one, ...", the longest strings, which contained the shorter ones, emitted the deeper sounds, which included the more acute ones. Then he divided the deep string in accordance with the arithmetic of whole numbers into two, three, four, five or six parts, obtaining the consonances of the octave, the fifth, the twelfth, the thirds, the fourth and the sixths. He thus made again Zarlino's scale, stopping at the division into six parts "... because the ear, due to its weakness, is not capable of distinguishing greater differences between sounds without an effort."

Every now and then, Descartes repeated that the ratios had to be commensurable, and therefore expressed by means of whole numbers. His favourite procedure consisted of dividing successively by two, thus obtaining that the octave contained all the intervals admitted by him. "... quos quidem ex divisione ditoni bifariam, ut ditonus ex divisione quintae oriri possem probare." ["... which, I could demonstrate, originated from the division of the ditone [5:4] into two parts, just as the ditone derives from the division of the fifth [3:2]."] But nobody should expect to find here the division of the tone into two equal parts. In the end, he verified the ratios on the strings of the lute, or another stringed instrument. Only the most perfect consonances, like the octave and the fifth, but not the fourth, succeeded in making the other strings tuned accordingly vibrate. As well as dichotomies, our French mathematician preferred, like Kepler, to close the monochord in the form of a circle (Fig. 9.6). The main numbers of the ratios became 2, 3, 5, while 4 and 6, were their derivatives.[103]

From the recent orthodox tradition for music, Descartes also took up the theory of strokes: "... sonum aures ferire multis ictibus, idque eo celerius quo sonus acutior est." ["... the sound strikes the ears with many strokes, which are more frequent, the more acute the sound is."] Like Maurolico, the philosopher of the Method justified the better consonances because in them, the strokes would coincide more often. "Tunc enim, ut sonus AB perveniat ad uniformitatem cum sono CD, debet tantùm aures ferire quinque ictibus, dum CD semel feriet." ["Sure then [for the ditone], for the sound AB to reach uniformity with the sound CD, it must strike the ears with exactly five strokes, while CD strikes it once."][104]

In 1618, the Cartesian mechanism for sound provided for: "... it is verified in practice that the tighter strings are, the more acute the sound they make is." Thus that linear proportionality was still distant from the growth of the sound pitch with the square root of the tension, verified first by Vincenzio Galilei and subsequently by Marin Mersenne. "Atque etiam potest probari ex eo quod maiori vi dividitur aer in minores partes, ex quibus exit sonus acutior." ["And it can also be proved from

[103]Descartes 1966, X pp. 89–92, 96–98, 103–105 and 116. Cf. Descartes 1979, pp. 44, 73–75, 81–82, 24, 87–89, 101, 134.

[104]Part I, Sect. 6.5. Descartes 1966, X p. 110. Also in the *Traité de l'homme [Treatise on man]*. Cf. Gouk 1991, p. 106.

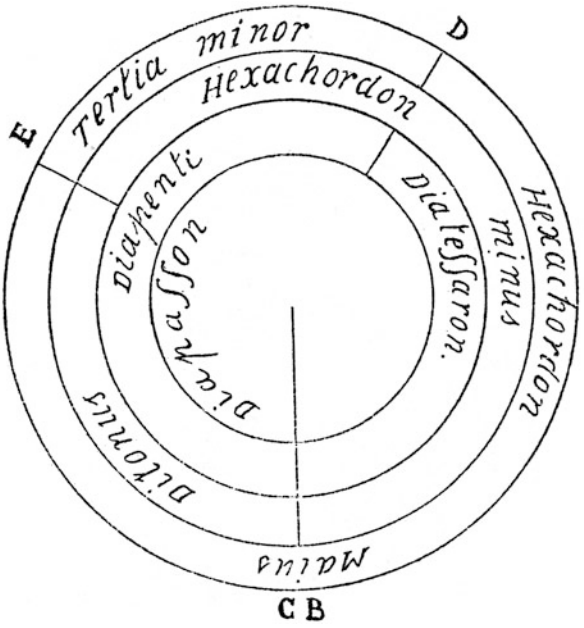

Fig. 9.6 How Descartes divided the diapason [octave] (Descartes 1964/1971, *Compendium Musicae*, vol. X 1966, p. 104)

this that the air is divided by a greater force into smaller parts, from which a more acute sound is produced."] The meaning of this passage would be that the greater force of the greater tension would divide the air into smaller, and therefore more numerous particles, which would strike the ears with a greater number of strokes, thus generating a more acute sound. Here Descartes remained at a distance from the continuous model of a wave composed of oscillating air.[105]

Seeing that, for Descartes, music should amuse us, its variety was to be enriched, by admitting together with the ditone (major third) other intervals like the minor third, although it was imperfect. The search for variety also led him to exclude parallel octaves and fifths, as well as to prefer contrary motion among the parts in polyphony. For this reason he went so far as to sense the alternation of major and minor thirds (as was to happen with the subsequent affirmation of tonalities), although he insisted on calling the former ditones, thus, choosing the word used at the time of the Pythagoreans (when they were made equal to twice the tone, that is to say, 81:64 and not 5:4).[106]

[105]Descartes 1966, X p. 115. Cf. Descartes 1979, pp. 132, 100, 134. This was the corpuscular model of Beeckman, from which he was to move away, to embrace sound waves. Descartes 1974, I pp. 104 and 109. See below in this section.

[106]Descartes 1966, X pp. 108, 132–133 and 140–141. Cf. Descartes 1979, pp. 118–119, 126–127.

He often invoked the ear as a judge, and referred to practical music. He also explained the Hand [of Guido D'Arezzo] and the relative stave with the ♯, the ♭, the ♮ and the treble clef, the bass clef and the alto clef. It is interesting that he did not understand one of the customs of musicians. "How and why that happens, I do not recall sufficiently in order to explain it; nor can I explain why, raising a single note above *la*, *b* flat [♭] is applied."[107] Like us today on the piano, musicians of his period had already merged *la*♯ with *si*♭, adopting the equable temperament. Clearly, this way of tuning and playing instruments could not fit in with the theoretical, orthodox, discrete model of music re-proposed and summarised by our young French mathematician, at the time enrolled in the army.

Descartes reasoned on the length of resounding strings because he made the larger whole numbers correspond to the deep notes. The numbers of his figures and tables were those that were found perhaps also in other treatises of the period, seeing that some have been found also in Maurolico. His scale of 21° derived from the icosichord of Guido D'Arezzo.[108] He reduced the musical intervals to the numbers 2, 3, 5. If they were small, consonances were obtained, with a few exceptions that he tried to explain; if they were big, they gave dissonances. Our French natural philosopher wanted to reduce the rules of practical and theoretical musicians, among whom he only quoted Zarlino by name, to their numerical bases. From these, he believed that it would then be possible to deduce the causes of consonances, of the composition of music pleasing to the ears, and of traditional modes. "I think that, once acquired, all the reasons can easily be deduced from what has already been said, even the most subtle ones, and once I had deduced many of them, but by now they have been lost in the course of my peregrinations." Descartes would have liked to exclude from music the excessively artificial counterpoints, because they were incapable of arousing emotions, like plays on words or on letters called acrostics and palindromes in poetry.[109]

With Beeckman, the interlocutor of this paper, Descartes was in agreement at that time. But he was to end up by quarrelling with him over the theory of strokes, which the Dutchman claimed as his own invention to explain consonances, when the Frenchman had pointed out that it was known enough. As we above already met the theory of Maurolico, we can only conclude that Descartes was in the right.[110] Music was to return at times in the thoughts and the writings of the French philosopher. He was to defend his own theory of 1618, rejecting the equable temperament. His constant interlocutor was to remain Mersenne, and we can follow this topic through the famous, abundant *Correspondance [Correspondence]*.

"Pour les raisons que disent vos musiciens, qui nient les proportions des consonances, je les trouve si absurdes, que je ne sçaurois quasi plus repondre. Car

[107]Descartes 1966, X p. 124. Cf. Descartes 1979, p. 109.

[108]Maurolico 2000, Maurolico 201?.

[109]Descartes 1966, X pp. 133 and 139. Cf. Descartes 1979, pp. 120 and 125. Evidently, he did not appreciate the intricate polyphonic games of the Flemish tradition.

[110]Tonietti 2006b, pp. 157–158. Cf. Hooykaas 1981.

de dire qu'on ne sçaurois distinguer de l'oreille la difference qui est entre une octave et trois ditons, c'est tout de mesme que qui diroit que toutes les proportions que les architectes prescrivent touchant leurs colomnes, sont inutiles, …" ["As regards the reasons that your musicians give, who deny the ratios of consonances, I find them so absurd that I hardly know what to answer. Because saying that the ear could not distinguish the difference between an octave and three ditones [six tones] would be like saying that all the ratios prescribed by architects for their columns were useless, …"]. Descartes repeated to Mersenne that musicians "… n'ont jamais rien compris en la theorie de la musique." ["… have never understood anything about the theory of music."] "Pour vos Musiciens qui nient qu'il y ait de la difference entre les demi-tons, c'est ou par desir de contredire, ou parce qu'ils ignorent le moyen d'en examiner la verité. Mais si j'estois auprès d'eus, j'oserois bien entreprendre de leur avouer, s'ils n'avoient l'oreille extremement dure." ["As regards your musicians, who deny that there is any difference between the semitones, this happens either as a result of their desire to contradict, or because they ignore the means to examine the truth. But if I were close to them, I would run the risk of trying to get them to recognise it, if their were not extremely hard of hearing."] Our natural philosopher even arrived at the point of dividing the octave with his whole numbers into 18 parts, but without any great success in convincing those hard of hearing, except one, perhaps. "In instrumentis musicis, ut perfecta sint et accurata, octava ex mea sententia dividi debet in 18 intervalla, …" ["In order that musical instruments may be perfect and accurate, in my opinion the octave has to be divided into 18 intervals, …"]. From his *Compendium of music*, we already know that he was capable of perceiving what we call today the higher harmonics of a sound (that is to say, the octave, the fifth, …), but his explanation that they derive from the imperfection of the strings is no better than the one proposed by Mersenne. They were both to learn from Mr. Théodore Deschamps (1588–after 1648) that strings were capable of vibrating in various different ways.[111]

From the numerous letters dedicated also to music, we can understand how the comparison between the equable temperament and the neo-Pythagorean theory based on rational ratios evolved. The majority of natural philosophers continued to prefer the latter, which was justified by the corpuscular theory of the strokes transmitted by sound to the ear. This happened despite that sound was not infrequently represented in writings as a wave of air.[112]

The problem of calculating proportional means between magnitudes, with which, for example, to duplicate the famous cubic altar of Delos, or to trisect the angle, had arrived from ancient Greece. It now returned in music, to divide the range of the octave into twelve semitones that were all equal. To do this, some sought the help of Mersenne, and our Minim got in contact with Descartes, to obtain an easier method

[111] Descartes 1974, I pp. 286 and 288–289; 1976, IV pp. 678–683 and 722–725. Albert Ban adopted it; see below in this section. Mersenne 1946, III p. 458; 1955, IV pp. 51, 99, 143–145, 149–151; 1959, V pp. 570–571; 1970, XI pp. 81–82. Descartes 1979, p. 31.

[112] Mersenne 1959, V p. 566; 1970, XI pp. 84 and 90.

than the one transmitted by Menaechmus (IV cent. B.C.). Eratosthenes (284–280 a.C.) had used a mechanical instrument called the mesolabe. Descartes invented another proportional compass which ended up in his *Géométrie*.[113]

In his *Harmonie Universelle*, when discussing the difficulties involved in dividing the octave for the organs into 11 proportional means Mersenne wrote "... je veux icy adiouster un moyen de les trouver Geometriquement, puis qu'il depend d'une seule Parabole, & qu'il a esté trouvé par l'un des plus excellens esprits du monde, dont la modestie est si grande, & si extraordinaire qu'il ne veut pas estre nommé. Je n'eusse icy mis que la Construction qu'il m'en a donnée, n'eust esté que Monsieur de Roberval, ..., en a fait promptement la demonstration; ..." ["... I would like to add here a way of finding them [proportional means] using geometry, as this depends on a single parabola; it has been discovered by one of the most excellent spirits of this world, whose modesty is so great, and so exceptional that he does not want to be named [Descartes]. Here, I would have presented only the construction that he donated to me, if Mr. de Roberval had not immediately provided the demonstration ..."].[114]

In spite that the Minim father was, as we know, against this method of tuning which was preferred by practical musicians, he inserted Descartes' procedure for determining proportional means into his theory of music. The philosopher of the Method, on the other hand, ignored the origin of the request, as well as his *Compendium of music*, and treated proportional means as a geometrical problem, like, for example, the trisection of the angle. For him, this was to be an important impulse leading him to the invention of his new analytical geometry with coordinates.[115] Using algebra, with the new notation of Descartes, the problem of dividing the octave into 12 equal parts was made simple and accessible to every future student.

$$\frac{1}{x} = \frac{x}{y} = \frac{y}{\dots} = \frac{\dots}{\dots} = \dots = \frac{\dots}{z} = \frac{z}{2}$$

By calculating x, y, ..., z, it was possible to divide the octave 1:2 into however many equal parts one desired, by means of algebraic equations or derived curves.

But with music, this natural philosopher did not use his new method, because he did not approve of the equable temperament. Thus, with him, all the power and the automatism of the algebraic procedure were to find expression in problems of geometry for curves. Not only was the division of the octave in accordance with proportional means *not* considered in the *Compendium*, but on the basis of what we have seen above, we may conclude that he even ignored it.

Also his constant interlocutor, Beeckman, was against it. "Idcirco illam Stevini nostri sententiam de sex tonis continue proportionalibus, olim a me diligentissime

[113] Mersenne 1945, I pp. 201, 206, 256–259, 269–273. Descartes 1983, p. 567.

[114] Mersenne 1636, [tr. 5] pp. 407–412.

[115] Shea 1994, pp. 64–67.

excultam, ante multos annos penitus rejeci. Practici autem iis divisionibus utuntur quod faciles sint et errores exigui multitudine consonantiarum occultentur." ["Therefore many years ago, I totally rejected the affirmation of our Stevin about the six tones proportional with continuity, which I once attentively practised. On the contrary, practical musicians make use of these divisions, seeing that they are convenient, and small mistakes are hidden in the quantity of consonances."] In 1624, the Flemish scholar had been able to consult Stevin's manuscript, *On the theory of the art of singing*.[116]

We may perhaps hypothesise that in their meeting at Breda in 1618, Beeckman and Descartes had also talked about this, and that the Frenchman based his judgement about Stevin on this. "Et certes je m'en estonnerois, si je n'avois vû tout de mesme de bons musiciens qui ne veulent pas encore croire que les consonances se doivent expliquer par des nombres rationaux, ce qui a esté, si je m'en souviens, l'erreur de Stevin, qui ne laissoit pas d'estre habile en autre chose." ["And I would certainly not be surprised if I had not seen good musicians who still do not want to believe that consonances are to be explained by rational numbers, which, if I remember correctly, was the error of Stevin, who continued to be competent in other things."][117]

Consistently, Beeckman derived his theory of strokes from an atomistic, material idea of sound, which was contrary to every wavelike model. "Causa vocis auditae est idem numero aer qui erat in ore loquentis; non, ut vulgo putatur, vicissitudo sphaeralis motus in aere, quem circulis ex projecto in aquam lapide factis assimulant." ["The cause of the heard voice is like the air itself that was in the mouth of the speaker; not, as is usually believed, the change of the spherical movement in the air, which is compared to the ripples made by a stone thrown into the water."][118] With the corpuscular theory of strokes, the Flemish scholar rejected the possibility of dividing the octave with irrational ratios, that is to say, with proportional means, because in this way the strokes would never be able to coincide. "... irrationales autem numeri aut ictus propter nullam terminorum coincidentiam cerebro grati esse non possunt, ..." ["... moreover, due to the total lack of coincidence in the terms of the strokes, irrational numbers cannot be pleasing to the brain, ..."].[119]

Among the musicians who, on the contrary, preferred the equable temperament, Mersenne mentioned Gabriel de la Charlonye (c. 1570–1646). He recalled that "... Aristoxenes ayt voulu diviser le ton en deux parties esgales. Vous me pourriés dire par advanture qu'encores que la voix exprime les tons esgalement, neanmoings les prenans mathematiquement, c'est à dire en proportion de musique, ilz se treuveront inegaux; mais cette raison ne me plairroit pas beaucoup." ["... Aristoxenus wanted to divide the tone into two equal parts. You may perhaps tell me that, although the voice emits equal tones, all the same, taking them mathematically, that is

[116]Mersenne 1945, II pp. 276 and 286. See above, Sect. 8.1; see below in this section.

[117]Mersenne 1945, II pp. 276 and 286. Descartes 1974, I p. 331.

[118]Mersenne 1945, II pp. 282 and 293–294. Cf. Cohen 1984, pp. 120–123.

[119]Mersenne 1945, II p. 452. Cf. Cohen 1984, pp. 127–139.

to say, with the musical ratios, we would find them unequal; but I would not appreciate this reason very much."] In these years, 1633–1634, Mersenne admitted "… l'on n'a pas encore demonstré que la raison de la quinte soit de 3 à 2, et l'on rencontre d'excellens geometres qui composent tres bien en musique, qui nient toutes les raisons des consonances et des dissonances que les Pythagoriciens, Euclide, Ptolemee, Boece, Zarlin, Salinas et les autres ont expliquees, et qui croyent que les raisons de tous les degrez et intervalles sont inexplicables, ou sourdes et irrationnelles. Car ils maintiennent que tous les tons et les demy-tons sont égaux; …" ["… it has still not been proved that the ratio of the fifth is 3 to 2, and we meet some excellent geometricians who compose music very well, but who deny all the ratios of consonances and dissonances explained by the Pythagoreans, Euclid, Ptolemy, Boethius, Zarlino, Salinas and the others, and who believe that the ratios of every degree and interval are inexplicable, or surd and irrational. Because they maintain that all the tones and semitones are equal; …"]. But in the end, he too wrote of Aristoxenus, with his equal tones and semitones, "… comme l'on pratique encore maintenant sur le luth et sur les violes, ce qui répugne neantmoins aux lois de l'harmonie et de la raison." ["… as they are practised still today, on the lute and on violas, what is, however, contrary to the laws of harmony and reason."][120]

Together with the various Pythagorean and neo-Pythagorean ways to divide the octave, the astronomer Ismael Boulliau (1605–1694) also considered the division of the octave into 12 equal parts on the basis of the proportional means. Did he perhaps prefer it to the others? One of his treatises of music has remained in manuscript form until recently. He frequented the circle of Mersenne, anyway, (after all, the former was a Catholic priest) and the Minim friar was to use his calculations to divide the octave in his *Harmonie Universelle*. Also Jean Beaugrand (1595–1640) evaluated the 11 proportional means between 1 and 2 for the equable temperament with his proportional compass, thus obtaining the right to be inserted into Mersenne's treatise he too.[121]

The medical doctor, Christophe de Villiers (1595–between 1661 and 1670) was a musical *amateur* who invented a practical method of his own for singing: "… mon passetems à la musique n'est que de chanter passablement ma partye, sans m'arrester à tant diversitez d'instrumens, …" ["… my hobby with music is simply to sing my part discreetly well, without considering so many different instruments, …"]. He must have formed the (ambiguous?) idea that Mersenne, with all his tables for the equable temperament distributed throughout the *Harmonie Universelle*, was inclined towards it with other musicians.

"Vous faites, dictes vous, un systeme de tons egaux et je crois que ce sera le plus asseuré et moins dificile quand on y sera acoustumé, quoyqu'il faille avouer que ces tons inegaux (majeur et mineur) font cette belle diversité de proportions qui se trouvent en l'octave harmonique et qui agree tant à l'oreille, laquelle toujours

[120]Mersenne 1946, III pp. 323–327, 414–415 and 418–419.

[121]Mersenne 1636, [tr. 5] pp. 384–385 and 37–38. Mersenne 1946, III pp. 447–452; 1955, IV pp. 435–437; 1945, II pp. 514 and 519–520. See above, Table 9.2.

stable au systeme (ou octave) proposé par mes precedentes, fait que je m'y tiendrois volontiers jusq'à ce que cette egalité de tons soit recognue autant consonante et agreable à l'oreille que l'antienne inegalité, à laquelle les voix estant ja acoustumee, il semble qu'elle doive toujours avoir de l'avantage, si vostre authorité n'y intercede, outre celle des instrumens qui deja sont pour vous." ["You say that you make a system of equal tones and I believe that this will be the most certain and the least difficult when we become used to it; although we risk having to admit these unequal tones (major and minor) create that fine diversity of ratios found in the harmonic octave, which give so much satisfaction to the ears; as they are still fixed in the system (or octave) proposed by my predecessors, they lead me to accept them willingly, until this equality of tones is recognized to be consonant and pleasant to the ears too as the old inequality, to which voices have already become accustomed; and it seems that it always must have a certain advantage, if your authority does not intercede as well as that of the instruments, which are already on your side."]

The provincial doctor told also to the churchman of Paris how organs were tuned in his area, reaching tones that were sensibly equal by ear. It is true that Mersenne, exploiting his numerous correspondents, distributed among his Latin or French books tables where he divided the octave into 12, 24 or even 31 equal parts.[122] However, we have seen above he repeated that he preferred to use the mathematics of rational ratios. Perhaps the Minim friar felt that the equable temperament was becoming increasingly popular, and at least in his letters to Villiers, was he moving to a position in favour of it? Another table with the relative numbers was placed by Mersenne at the end of the *Universal harmony*.[123]

On November 27th, 1637, Marin Mersenne wrote to Galileo Galilei: "I am sending you Mr. Descartes' book from Batavia [Holland]; let me know if you have received it …". Therefore the Italian had in his hands, very probably immediately before publishing his own *Discorsi e dimostrazioni matematiche [Discourses and mathematical demonstrations]* (1638), Descartes' *Discours de la méthode [Discourse on Method]* and *La Géométrie [Geometry]*, just published (anonymously) at Leiden that year. In the book, he appeared to be irritated by all the discussions and the various philosophical opinions expressed on the same subject, because he believed that there was only one truth. Reason, on the other hand, should have succeeded in distinguishing the truth from falsehood.[124] Thus he was in search of a method to find it, and as he only trusted evident truths because they should be clear and distinct, he believed that only geometricians and mathematicians were capable of constructing a reliable model of reasoning. No matter how difficult or complex a problem was, they proceeded by means of a chain of several "easy and simple" passages. If the investigation regarded a single case, it might be made simple and distinct by using the lines of geometry, whereas for the multiple case, the terms of algebra were to be used, correcting "the defects of the one by means of the other".

[122]Mersenne 1946, III p. 586; [1959], V pp. 53–54 and 69–73.

[123]Mersenne 1636, [tr. 8] pp. 21–22. Mersenne [1959], V pp. 615–617.

[124]Descartes 1925, pp. 21 and 27.

Descartes believed that the soul was distinct from the body, and as a result, our Frenchman succeeded in imagining that it was possible to dissolve the world without causing it to cease to exist: the pure thought was sufficient for him. The 'perfect being' necessarily implied its own existence, and therefore God existed with the same certainty with which the proofs of geometry could be believed. For years, by means of trips to places that were often the scene of war, he gained experience on the differences of customs, philosophies and laws, among which he often included China as the maximum distance. Was he therefore compelled to doubt everything, when, on the contrary, he was in search of sure truths? He came out with his famous *Cogito ergo sum* ["I think, therefore I am"]. Thus he would find his first truth.[125]

But all this was set in Heaven; how did our Frenchman get on with corporeal matter? Did he, too, deduct some 'slight' impediments? Descartes went beyond this, imagining a completely virtual world. He declared that he was only interested "... in what would happen in a new world, if God created now, in some part of the spaces, sufficient matter to compose it, and stirred ... the different parts of that matter, so as to make a confused chaos out of it, as poets imagine it, and then did nothing other than lend his normal support for nature, letting it act according to the laws that He has established?".

Descartes went on to illustrate the project: "... I caused to see what the laws of nature were ... how the majority of the matter of that chaos should, as a result of those laws, arrange and order itself in such a way as to make it similar to our skies; and how some of its parts should compose an Earth, and others the planets and the comets, and yet others a Sun and the fixed stars. [...] although I had expressly supposed the God had not given any weight to the matter of which it [the Earth] was composed, all its parts did not stop tending exactly towards its centre; ..."].[126] Having cancelled the sensible appearances, such as smell, heat, taste, and colour, he reduced the world to extension and movement. But also in its movement, the world of Descartes had no weight or mass, and therefore his physical model in the end became pure geometry. The movement towards the centre of the planets was not generated by forces, but they ended up by being dragged by the whirlpools of an impalpable fluid.

This French scholar was in search of "principles or first causes of everything that exists, or can exist, in the world"; he deduced from these causes the effects, such as the stars, the planets, or the elements. He arrived at the "truth in the sciences". Whoever discovered it was like a man who became rich, and was then able to "... make great purchases ..." with less effort than he had used when he was poor, to obtain much less. Also Descartes, like Galileo Galilei, must have found the comparison between science and money appropriate. Our Frenchman did not trust discussion or dispute, as he always expected not to be understood well. "... it

[125]Descartes 1925, pp. 38–39, 52–55.

[126]Descartes 1925, pp. 63–65.

is truly to join battles when we try to overcome all the difficulties and the errors that stop us from arriving at a knowledge of the truth ...".[127]

And to win the battle, Descartes sharpened his mathematical weapons. There were three large appendices that followed the *Discourse on Method*. *Dioptrics* dealt with the geometry of light, arriving, among other things, at the law of refraction, independently, it seems, of Willebrord Snell. *Meteors* considered atmospheric phenomena; among other things, he presented the rainbow as a figure of light that was formed in raindrops. For the third one, our Frenchman took up ancient mathematical studies performed between 1619 and 1621, while dreaming in his famous *poêle* near Ulm, and christened them *Geometry*. Thus Descartes, too, like Galileo Galilei and Kepler before him, seemed to use for his mathematical model the usual eternal geometry of Euclid. But even if he still considered it to be the centre of his method of investigation, now the Frenchman followed decidedly new procedures to solve these problems: those of algebra.

Here, Descartes stated at the beginning that he did not want to make himself understood by everyone, but only by scholars of geometry. "It is often not at all necessary to trace these lines on paper like this, but it is sufficient to indicate them with letters ... $a, b, a+b, a-b, ab, \frac{a}{b}, aa = a^2$". He showed that it was possible "to use figures in geometry". Our French natural philosopher translated the geometrical problem into equations, considering it already solved, thus following Pappus (c. 300) in his so-called method of analysis. By expressing the same quantity in two different ways, he then solved the algebraic equation, and reinterpreted the solutions by means of geometry, finally plotting them in a design. "Everything that comes within the field of geometry may be reduced to a single kind of problem ... trying to find the value of roots ...". At the beginning of his *Geometry*, he went beyond the well-known prohibition, and jumped over all the fences put up by the *quadrivium*. Together with the four arithmetic operations with whole numbers, he now added also "... the extraction of roots, which may be considered to be a kind of division."[128]

Descartes announced that he would calculate "... one, two or more proportional means between unity and some other line, which is equivalent to the extraction of a square root or a cubic root, etc." Drawing a circle, he showed why he considered the operation with numbers and the geometrical construction equivalent. He underlined the different behaviour of the Greeks in this connection: "... the hesitations of the ancients in using arithmetic terms in geometry, which originated only from the event they did not have a sufficiently clear understanding of the relationship between the two sciences, ...". When he made use of his above-mentioned proportional compass, he obtained new classes of "... lignes courbes [...] qu'on peut nommer Géométriques, cette a dire qui tombent sous quelque mesure precise & exacte, ont necessairement quelque rapport a tous les poins d'une ligne droite qui peut estre exprimé par quelque équation, en tous par une mesme." ["... curved lines [...],

[127]Descartes 1925, pp. 82, 84–85.

[128]Descartes 1983, pp. 525, 531, 528–529. For Descartes' three dreams of 1619, see Shea 1994, pp. 125–127.

which we can call geometrical, that is to say, which come under some precise, exact measure, [the points of which] necessarily have, with all the points of a straight line, a certain relationship, which can be expressed by a certain equation, all with the same one."][129] From now on, these curves were to be represented by an algebraic equation (a polynomial equalized to 0), and consequently called 'algebraic'. Those that the Frenchman refused, because they were "mechanical", and could not be constructed like the algebraic ones, will be, on the contrary, considered by Leibniz to be 'transcendent', e.g. the 'sine', which does not benefit from this property.

The first traces of this new conception of geometry went back at least to the context of Breda from which we started, when the philosopher of method discussed with Beeckman, partly about music, in 1618–1619, and composed his *Compendium*.[130] At the critical main moment of the invention, it would have therefore been difficult, at least for us, to forget that if proportional means had been fully accepted, they would have changed the division of the monochord with only the rational ratios, admitted by the prevailing Greek orthodoxy. It is interesting, then, that thanks to those same proportional means, constructed with compasses and now represented in curves and equations with the new symbols of algebra,[131] Descartes would be able to reduce at least a part of geometry to algebraic equations.

However, at the subsequent moment of completion of the *Method*, the initial impulse sustained by music would be forgotten, without the new invention giving its fruits, also as regards how to divide the octave. Thanks to the algebra of the Arabs, our French natural philosopher had shuffled the cards of the old *quadrivium*, which was blocked at arithmetic, geometry, music and astronomy. He could easily have made music, too, participate in that same reason which allowed him to connect arithmetic and geometry in other proportions. Instead, Descartes left it inside the same traditional *quadrivium*, which he was by now showing to be inadequate for the other mathematical sciences. As in the case of astronomy, may he have been afraid that the Church of the Counter-Reformation, so worried about the effects of religious singing on the faithful, as is confirmed by the acts of the Council of Trent, would be annoyed at a new theory for music? Was the philosopher of Method self-censuring his own *La Musique ou Traité du Son [Music, or a treatise on Sound]*, perhaps, as he did with *Le Monde ou Traité de la Lumière [The World, or a Treatise on Light]*? As no trace has remained of the former of these two titles, may this hypothesis be excessively bold, or else was the censure well too successful?

In any case, Descartes was also 'modern' in the event that he finally started to separate music from the category of mathematical sciences. He seemed to have substituted it with optics, for he concluded "Book Three" of his *Geometry* by demonstrating the construction of lenses with their relative luminous properties, depending on their concave, convex, oval, or elliptic shape, and so on. He wanted to help artisans who constructed spectacles and telescopes. In this way

[129] Descartes 1974, II p. 392. Descartes 1983, pp. 529–530, 546 and 569.

[130] Descartes 1974, X pp. 154–155.

[131] Descartes 1983, pp. 632–634, 664–665 and 682–685.

(unfortunately?) he finished his *Geometry* with the algebraic equations of "Book Three". What caused him not to deal with music any more may have been the disputes that arose with Beeckman, as we shall see below. It is curious that his greatest success in the field of optics, when he explained the rainbow, depended on the refraction index between air and water: 4/3. He also obtained the colours of the rainbow using glass with an index of 3/2. And yet, for all that I have read, he did not develop the analogy between colours and sound, even if he had discovered, behind the *sines* of the angles of incidence and refraction, the fundamental musical ratios for the fourth, 4:3, and the fifth, 3:2.[132] Also in this respect, as we shall see, Newton did not let the opportunity escape, on the contrary. And when he dreamt, unlike the Scipio of the classical dream, Descartes did not hear the music of the spheres, either.

His geometrical sense of algebraic equations remained so strong that certain solutions, where it was not present for example negative roots, were discarded. Almost all the algebraic notations that we learnt at school go back to him: x, y, z for unknowns, a, b, c, d, for known terms, the powers like $x^3, c^4 \ldots$ indicated by means of Arabic numbers higher up on the right, $+$ and $-$, instead of $\bar{p}$ or $\bar{m}$, division as a fraction $\frac{a}{b}$, the square root $\sqrt{\ }$, which he had learnt from Christoph Rudolff (XVI cent.) and so on. In "Book Three" of his *Geometry*, he classified curves, discussing not only the algebraic ones (that is to say, those that could be expressed by means of algebraic equations, like conic sections), but also some of those called 'mechanical', when they are generated by means of a regular movement, like a spiral. Today, the text could be mistaken simply for pure mathematics. But for Descartes, it made no sense even to speak of applied mathematics, because for him the study of geometry and the study of natural phenomena simply coincided, in that world of his without weight. The Ovals of Descartes, as they will be called after him, were described in relation to the form to be given to lenses or to refraction. Geometry seemed to have become the science of light. *Le Monde ou Traité de la Lumière [The world, or a treatise on light]* was the title that he was to give to his work, which however he was to avoid publishing, as he was afraid of undergoing consequences like those suffered by Galileo Galilei. Geometry ran the cosmos, in the whirling movements performed by subtle matter, but the kind expressed in the language of Euclid was no longer sufficient to indicate its laws.

In a letter to Mersenne of 1638, he claimed that he could provide "demonstrations" of refraction, even if he could not be asked to give "… demonstrations geometriques en une matiere qui depend de la Physique, c'est vouloir que je fasse des choses impossibles." ["… geometrical proof in a subject that depends on physics, that would mean expecting me to do the impossible."] In that case, logical reasoning would have to be sufficient, starting from assumptions that were "… pas exactement vrayes …" ["… not exactly true …"]. He acknowledged his debt to Kepler for his research on optics, while he ignored the elliptical planetary model.[133]

[132] Shea 1994, pp. 156, 210, 223 e 250.

[133] Mersenne 1962, VII pp. 231–232. Gingerich 1970, pp. 298 and 308.

In another subsequent letter of 1640, again to Mersenne, he claimed, more boldly, that he could demonstrate, "Pour la Physique, … l'ayant reduite aux lois des Mathematiques, …" ["For physics, … having reduced it to the laws of mathematics …"], not only how physical phenomena occurred, but also that they could not occur in any other way. One novelty, pregnant with future developments, became, as we shall soon see, using in physics, as in geometry, equations written with literary symbols. Thus they were not only equations of curves, because as they represented the movement of bodies, the algebraic formulas now also appeared to be the trajectories of movement. God was believed to maintain the cosmos by means of the same action by which he had originally created it. For Descartes, that ancient initial impulse continued to remain unchanged in its effectiveness, establishing itself in the universal law that preserved movement. This was the Frenchman's model for the (elastic) collision between bodies.[134]

Compared with the discursive dialogues of Galileo Galilei, or the Latin of Kepler, with his *Géométrie*, Descartes had effected a change of style. Here, the Frenchman expounded the truth for whoever was capable of understanding it, he did not discuss it any longer: it was the truth, and there was nothing more to say. He did not proceed, like Kepler in his *Harmonices Mundi Libri Quinque*, by means of definitions, axioms and a numbered list of propositions. Although his mathematical models remained largely qualitative, he had started to use the algebra of the Arabs in a different way. He used the new language coming from a different culture, which was considered to be hostile by many Europeans at that time, and therefore to be conquered, or at least converted. Thus, the geometry of Euclid and Pappus had been changed by Descartes into what was to be called "analytical geometry": with its famous, albeit implicit and not orthogonal, coordinates.

A new symbolism had now been invented, which was absent in Euclid, as in the Arabic culture. But it would have to prove its effectiveness and obtain a consensus of opinion in order to be adopted among the community of natural philosophers. To help us understand the process better, the *Discours de la méthode* will offer us some precious help.

As in the case of Kepler and Mersenne or Galileo Galilei, mathematics was the instrument used by Descartes to arrive at an understanding of the world, but unlike the first two, and despite music, he did not perceive as much harmony. Based on the countless number of times that the word appeared (I have numbered at least 30), the truth was simply what the philosopher of Method was searching for, with clarity. In order to find it, he would like to separate it from falsehood, and therefore proceeded to divide everything into two parts, starting with his own person, which he discovered was separated into soul and body. "… I could simulate that I did not have any body […] I was a substance whose essence and nature consist of thinking, which had no need of any place in order to exist, nor does it depend on anything material. As a result, this I, that is to say, the soul, which makes me what I am, is totally distinct from my body, …". Unfortunately, the dichotomy

[134]Mersenne 1965, IX p. 193. Mahoney 1981a.

thus became incurable and, despite his pineal gland, he never again succeeded in recomposing satisfactorily what he had cruelly divided. Thus he left a series of insoluble problems: for example, how would the soul succeed in moving the body?[135]

However, he obtained an advantage from this, i.e. that in this dualistic way his knowledge, which had now become "… the laws of nature established by Him [God] …", could become disembodied, and abstracted, far away from a contingent, imperfect Earth, to be transformed into absolute expressions, eternal and universal. Observing the phenomena, he would not see practical rules, or curious recurrences, but transcendent truths, guaranteed by God Himself. The best way to indicate them was to use the term "laws". "Mais je me contenteray de vous avertir, qu'outre les trois loix que j'ay expliquées, je n'en veux point supposer d'autres, que celles qui suivent infailliblement de ces veritez eternelles, sur qui les Mathematiciens ont accoûtumé d'appuyer leurs plus certaines & plus évidentes demonstrations: ces veritez, dis-je, suivant lesquelles Dieu mesme nous a enseigné qu'il avoit disposé toutes choses en nombre, en pois, & en mesure; & dont la connoissance est si naturelle à nos ames, que nous ne sçaurions ne les pas juger infaillibles, lors que nous les concevous distinctement; ny douter que, si Dieu avoit crée plusieurs Mondes, elles ne fussent en tous aussi veritables qu'en celuy-cy." ["But I will content myself with informing you that besides the three laws [of movement] explained by me, I do not want to assumer others, apart from those that follow infallibly from the eternal truths on which mathematicians are used to basing their most certain and evident proofs: I mean, those truths according to which God Himself taught us that He had arranged everything by number, weight and measure; the knowledge of which is so natural for our souls that as soon as we succeed in conceiving distinctly of them, we are forced to judge them infallible; nor can we doubt that if God had created many worlds, they would be in every detail equally true as in this one."][136]

Besides, he still must have got a certain acquaintance of laws while studying jurisprudence up to become a lawyer. His studies did not concern only civil laws, but also those of an ecclesiastical nature.[137] A certain legal competence also transpired from the *Discourse on Method*.[138] Now, the same God who had fixed the moral laws was also giving, with Descartes, the laws of nature. What kind of deity, then, inhabited the pages of this philosopher, often reduced to his mechanical half? He maintained the religion of his infancy "… in which God gave me the grace to be instructed …". According to his logic, the proof of God's existence derived from the event that He was "… infinite, eternal, unchanging, omniscient, omnipotent, …". "… it is at least as certain that God, who is such a perfect being, is or exists, as any geometrical proof can be." And yet, this idea of deducing a God who exists from

[135]Descartes 1925, pp. 52–53.

[136]Descartes 1967, XI p. 47.

[137]Shea 1994, p. 19.

[138]Descartes 1925, pp. 32, 36–37 and 44.

the list of His perfections, as if non-existence made Him imperfect, and therefore no longer God, was too reminiscent of St. Anselm (1033–1109), not to give a medieval flavour to one of the philosophers considered most modern. "... les verités mathematiques, lesquelles vous nommés eternelles, ont esté establies de Dieu & en dependent entieremant, aussy bien que tout le reste des creatures. [...] c'est Dieu qui a establi ces lois en la nature, ..." ["... the mathematical truths, which you [Mersenne] call eternal, were established by God, and totally depend on Him, like all the other creatures. [...] It is God who established these laws in nature, ..."].[139]

This way of comparing God to geometry was interesting, however, not taking it for granted that the "clear and distinct" truth of geometry did not leave any doubts about the existence of its figures. "..., but I did not see anything here that assured me of the existence of a triangle in the world; whereas, returning to examine the idea that I had of a perfect being, I found that existence was included in it, in the same way as the idea of a triangle includes the idea that the sum of its three angles is equal to the sum of two right angles, ...". Here, Descartes seemed to open up that discussion about the difference between theorems of existence and constructive theorems which was to become crucial at the end of the nineteenth century: from David Hilbert and Hermann Weyl to Kurt Gödel (1906–1978).[140]

From the time depths of the prohibition contained in the *Bible* regarding not mixing good with evil, now Descartes made the precepts for knowledge explicit. The first precept would be not taking anything for true which was not evident, clear and distinct. "The second [is] that of dividing each of the difficulties to check into its component parts, which is necessary in order to solve it more easily." Studying the heart, this involved a gory dissection, to make the truths of anatomy clear and distinct. In the case of disputing opposing hypotheses between philosophers, the division created the hierarchy where the truth had to be separated clearly from what was false. If Descartes admitted variety, and therefore dissonances, as a source of pleasure for music, here for the religious, mathematical and philosophical truths, that advantage of variety became a defect to get rid of: "... having considered that the same man, with his same intelligence, due to the event that he grew up from his infancy among French people or Germans, becomes different from what he would have been if he had grown up among Chinese people or cannibals; and that, ..., the same thing that we liked ten years ago ... now seems to us extravagant and ridiculous; that habit and example serve to persuade us more than any certain knowledge, and that nevertheless the consensus of many is not a proof that has any value for the truths which are somewhat difficult to discover, because it is more likely that one man will find them out by himself than a whole people; ...".

Descartes granted that there might perhaps be "... equally sensible men among the Persians or the Chinese as among us, [consequently] it seemed to me that the most useful thing was to act in line with those with whom I would have to live,

[139]Descartes 1925, pp. 43 and 55–56. Descartes 1974, I p. 145.

[140]Descartes 1925, pp. 55–56. Descartes 1975, III p. 274. Mersenne 1962, VII pp. 242–244. Tonietti 1983a; 1985a; 1985b; 1988; 1990.

…". Here, he was referring to political and social uses, or obedience to the juridical and religious laws of one's country. How would the philosopher of Method react to the *Xiantu* [Figure of the string], which had been used in the Country-at-the-centre to demonstrate the property of right-angled triangles? Undoubtedly, the Jesuits, we know, would never bring it to him so that he could examine it. But could the programmatic scepticism of the French philosopher arrive at a reconsideration of the geometrical truth offered by the theorem of Pythagoras?[141]

China had by now appeared on the horizon of these Europeans, who were gradually penetrating into its various realities. And yet their scientific aspects seemed to be of little interest for them. If they had gained a knowledge of them, how would they have reacted? Would they have drawn teachings from them? Would they have refused them because they were different from their own, and therefore 'wrong'? If Descartes had taken refuge in the "clear and distinct" ideas of geometry in order to avoid the "vain" discussions of philosophers incapable of demonstrating any "truths", how would he be able to admit the different geometry of the Chinese? It is a pity that history, or destiny, did not give us the chance to witness another exhibition of the rhetorical capacities of the mathematician-philosopher.

Separated from the thinking rational soul, the material body was reduced by the French philosopher to a pure machine, thus popularising the image of the world as a clock, which was destined to attract both many followers and noble adversaries. To measure time in their experiments on movement and music, Galileo Galilei and Marin Mersenne used the heart instead of a clock; vice versa, now for Descartes, the movements of blood and the heart would be like "… the movement of a clock, in view of the force, the situation and the form of its counterweights and its wheels." Men were able to construct automata, imitating God, who had made the animals like a more complex machine "… incomparably better arranged … […] is nature, which acts in them in accordance with the arrangement of their organs: like a clock, which is simply composed of wheels and springs, …".[142] Naturally (exactly), the philosopher of Method made enormous the dichotomic distance between man and animals, which were made incapable of speaking or thinking.

Descartes wrote *La Méthode, La Géométrie* and his other treatises in his mother tongue: French, just as Galileo Galilei expressed himself in Italian. Not only so, but here, the character born in France had become an explicit supporter of his own Gaulish language. "And if I write in French, which is the language of my country, rather than in Latin, which is that of my teachers [the Jesuits], it is because I hope that those who use only their natural reason will judge my opinions better than those who only believe ancient books; and as regards those who unite good sense to study, the only ones that I wish to have as my judges, they will not be so prejudiced towards Latin, I am sure, to refuse to understand my reasons because I explain them in the language of the people."[143]

[141] Descartes 1925, pp. 37, 68, 35, 44.

[142] Descartes 1925, pp. 70, 75, 77.

[143] Descartes 1925, p. 93.

But how was it possible for a transcendent truth of a religious nature, which had taken the form of eternal, universal laws, made independent from the contingent impulses of the body through dualism, to be expressed in a historical, local vermacular like French? As natural philosophers gradually began to use their national languages instead of Latin, these would, however, remain only a part of the scientific laws. Inasmuch as they were considered to be transcendent truths, they now needed a new artificial language, divine and mysterious for the non-initiated, as Descartes sustained. "... je me suis expressément rendu un peu obscur en quelques endroits, afin que telles gens ne se pussent vanter d'avoir sceu sans moy les mesmes choses que j'ay écrites." ["... I have deliberately expressed myself in obscure terms at certain points, so that those people cannot boast that they have understood without me the same things that I have written."][144] That symbolism of mathematical sciences was just taking shape, in which Latin letters, which had lost their historical purpose, combined in other new ways, far from the rules of grammar; or else new signs were invented for it, such as + or the square root $\sqrt{\ }$. Within a 100 years, all that symbolism was to appear and spread, which nowadays crowds the scientific manuals of the world.

Having left the tutelage of the Jesuits, Descartes stated: "... I spent the rest of my youth travelling, observing courts and armies, ... [...] I was in Germany at the time, on the occasion of the wars which have still not finished, ...". Subsequently, he was to move "... to a country [the Netherlands] where the lengthy duration of the war has created such order, that the armies that are maintained there seem to serve only to enjoy the fruits of peace in greater safety, ...". We have already seen, a few pages ago, that he conceived of truth itself as a "battle" against errors. In the mechanical model to explain the refraction of light, he narrated experiments of bullets fired through the waters of a river. In his historical context, it was not difficult for him to come across a war, and effectively, he too, as he stated, set his sights on the most important one: the Thirty Years' War. While Kepler had lived through it with great suffering, Descartes had enrolled on a voluntary basis. He also took part in the famous siege of La Rochelle (1627–1628), where Cardinal Richelieu (1585–1642) defeated the forced of the Calvinist Huguenots, leaving them only freedom of worship, without any military organisation.[145]

Another feature typical of our French natural philosopher, which makes him particularly modern, was his (Jesuitical?) shrewdness in self-censuring himself, when he expected that expounding his ideas openly would lead to troubles with the authorities. Without mentioning the name of Galileo Galilei, he explained well twice, in his *Discourse on Method*, why he had decided not to publish his *Le Monde ou Traité de la Lumière [The World or Treatise on Light]*: "... when I came to know that some people who I admire, and whose authority influences my actions no less than my own reason influences my thoughts, had expressed their disapproval of an opinion of physics published a short time before by another [Galileo Galilei]: I am

[144]Mersenne 1962, VII p. 241.

[145]Descartes 1925, pp. 28, 31 and 50. Descartes 1983, p. 216. Shea 1994, p. 20.

not saying that I was a supporter of this opinion, but that I had not noticed anything, before their censure, that I could consider harmful to religion or to the state, … […] … I have hardly ever met any censor of my opinions who did not seem to be either less rigorous or less fair than myself." "Mais comme je ne voudrois pour rien du mond qu'il sortir de moy un discours, où se trouvast le moindre mot qui fust desaprouvé de l'Eglise, aussi aymé-je mieux le supprimer, que de le faire paroistre estropié." ["But as I would like not for the world that even the least word of any of my discourses were disapproved by the Church, then I prefer to suppress it, rather than allow it to appear spoilt."] He was so attached to the image he projected that he tried to conceal his date of birth "… because I am not fond of those who cast horoscopes, either …".[146]

However, the philosopher of Method went beyond a mere opportunism dictated by survival or by a desire for peace and quiet, or at least, he transferred his attitude into his scientific ideas. In other words, he, a Copernican, transformed the relativism of the Jesuits, making the movement of the Earth relative to the reference system. "… esse translationem unius partis materiae, sive unius corporis, ex viciniâ eorum corporum, quae illud immediatè contingunt & tanquam quiescentia spectantur, in viciniam aliorum. […] Et dico esse translationem, …, ut ostendam illum semper esse in mobili, …" ["[the movement] is the transportation of a certain quantity of matter, or of a body, from the proximity of those bodies immediately contiguous to it, which are considered at rest, to the vicinity of others, … And I say that it is transportation, …, to show that it is always in the mobile, …"]. For the Earth, he used the image of a ship carried away by the stream. "… putemus quidem illa quiescere, ac nullam habere propensionem ad motum, quandoquidem nullam advertimus. Sed ne putemus hoc obstare, quominus ab isto coelo deferatur, & ejus motibus immota obsequatur: ut navis, nullis ventis nec remis impulsa, nullisque anchoris alligata, in medio mari quiescit, etsi fortè aquae ingens moles, occulto cursu delabens, ipsam secum ferat. […] Terram, propriè loquendo, non moveri, nec ullos Planetas, quamvis a coelo transferantur." ["… we believe at least that it [the Earth] is still, and has no propensity to move, because we do not perceive any movement. But we do not believe that this stops it from being carried away somehow by this sky, and abandoning itself, unmoving, to its movements: as a ship, when it is not propelled either by the winds or by oars, and it is not held back by any anchor, remains still in the middle of the sea, even if, by chance, the enormous quantity of water carries the ship away with it, drifting along in its hidden course. […] The Earth, properly speaking, does not move, nor does any Planet, although it is transported by the sky."][147]

Various historians of physics, led by their faith in progress and in (their) truths, have written at length about the countless defects of the Frenchman, compared with the Englishman, Newton. And yet in these pages, Descartes recalled on the one hand

[146]Descartes 1925, pp. 62–63, 79 and 86. Descartes 1974, I pp. 285–286; 1975, III pp. 258–259 and 270. Mersenne 1946, III p. 558. Shea 1994, p. 361.

[147]Descartes 1964, VIII-1 pp. 53–54 and 89–90. Shea 1994, pp. 328–330.

Aristotle, and on the other Albert Einstein who would authorise us in the twentieth century to place the reference system from which to measure the movement of the Earth either on the Sun, or on the Earth itself. But both the Frenchman, at odds with religious orthodoxy, and the inventor of the Theory of Relativity, who lived and worked in an equally hostile environment, still maintained their absolutes.

Those wanderings of his, which started when he was a soldier, continued for the rest of his life, in search of a physical and intellectual peace which he rarely found, perhaps only in short periods in the countryside of the Netherlands.[148] Attracted by the interest in his philosophy shown by Queen Cristina of Sweden, he had passed from the liberal Netherlands to a freezing Stockholm, where he died of bronchitis as a result of the cold weather. Unfortunately, he had forgotten the golden rule that he often quoted: "Bene vixit, bene qui latuit." ["He lived well, who hid himself well"]. The vanity of the court had killed him, the one who claimed to be able to lengthen man's life at pleasure.[149] Undoubtedly, his soul lived in a superior world heated up by light, but he had neglected his body, forcing it to live in that part of the Earth where bipeds without feathers see too little of the winter sun, with consequences which for him proved tragic. "Laws of nature" destroyed his "body", though we can still today take a glance into his "soul" by reading his books, if curiosity so leads us.

The books of history and of the philosophy of science are full of discussions about the extent to which Descartes was of genius, or had borrowed from (many?) others, how far he followed the Greek-Arabic-medieval tradition in the footsteps of François Viète, or he introduced a real paradigmatic revolution. We will recall here only the numerous disputes of Arnauld, Beeckman, Desargues, Fermat, Pierre Gassendi (1592–1655), Thomas Hobbes (1588–1679), Roberval about almost everything that the philosopher of Method did, hardly ever quoting his natural philosopher colleagues. Beaugrand called him an "impertinent man of method" and a *miles philosophus*, probably alluding to the *miles gloriosus* [conceited, boastful soldier] by Plautus. But on the other hand, in his letters, he frequently dedicated himself to criticising, rightly or wrongly, almost all the other interlocutors, including the good Mersenne. Of the Minim father he said: "… qui ungere potius non pungere valeat." ["… who it is better to anoint than to needle."] He sometimes went so far as to insult his adversaries, likening them to animals like flies or dogs; their works became "toilet paper" or "dung".[150]

Beeckman was the one who had perhaps influenced him most as regards music. In this connection, also the Flemish scholar was co-opted into Mersenne's *Harmonie Universelle*. The French churchman, following him, explained the double quantity of vibrations produced by a sonorous string half as long by means of Beeckman's diagram turned upside down, without mentioning his name (Fig. 9.7).

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[148]Mersenne 1962, VII p. 241.

[149]Descartes 1974, I p. 286. Crombie 1981. Shea 1994, pp. 345–346.

[150]Descartes 1974, I pp. 303–306, 375 and 377; 1975, II pp. 380–405, 586; 1975, III pp. 249–250 and 257–258. Lenoble 1971, pp. 414–418, 420, 426, 582, 587 and 598. Descartes 1983, pp. 638ff. Shea 1994, pp. 381, 184, 186–187, 353 and 405–406.

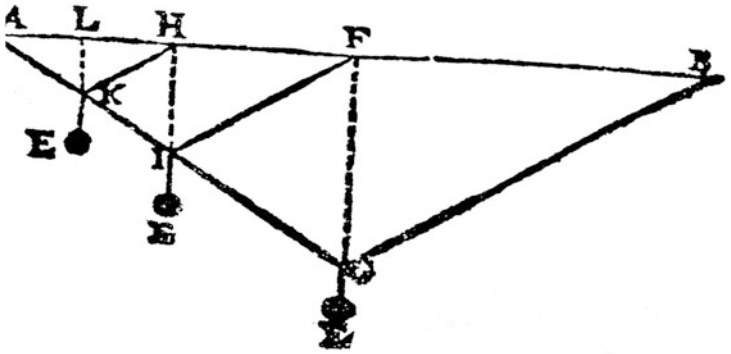

Fig. 9.7 The diagram of the vibrating string which Mersenne borrowed from Beeckmann (Mersenne/Beeckman, *Harmonie Universelle*, p. 157)

And yet among the three scholars in contact with each other, both personally and by correspondence, a controversy arose based on rivalry and misunderstandings, especially as regards the theory of strokes.[151]

It is curious, or perhaps it is not, that the worst clash arose between those who had a knowledge of music that was above all theoretical. Both Beeckman and Descartes admitted that they possessed little ear to distinguish intervals, "… de fait je ne sçaurois distinguer la quinte de l'octave, …" ["… in fact I could not distinguish the fifth from the octave, …"], and they could not sing musical scales. Indeed, "J'ay vu la nouvelle epinette de Mr. Bannius avec le systeme parfait qui le contente extremement, et quoy que je sois presque sourd elle me semble avoir quelque chose de plus que le commun." ["I have seen the new spinet of Mr. Ban [Jan Albert, 1597–1644] with the perfect system which completely satisfies him, and although I am almost deaf, it seems to me that it possesses something more than the common one."][152] There is little doubt that, as regards theory, the Flemish scholar had a greater experience, whereas we have seen that the French philosopher displayed, at most, an honest knowledge of neo-Pythagorean orthodoxy. They both belonged to the same tradition, and for this reason, Descartes was loath to recognize that Beeckman could claim too many merits.

In 1575, the Italian Jesuits had published the *Musicae Traditiones [The teaching of music]* written by Francesco Maurolico. The book was undoubtedly used in the numerous Colleges where the religious congregation taught the sciences of the period in accordance with the frame of reference of the classical *quadrivium*. We cannot say with certainty at present whether the *Musicae Traditiones* was also

[151]Mersenne 1636, [tr. 1] pp. 157–158. Mersenne 1945, II pp. 231–236. Descartes 1974, I pp. 29 and 31–32. Cohen 1984, pp. 187–201. Shea 1994, pp. 90–104.

[152]Descartes 1974, I p. 142; 1975, II p. 699; 1975, III pp. 825–834. Cohen 1984, pp. 119–120. Shea 1994, p. 85.

used at the Jesuit college of La Flèche, where both Mersenne and Descartes had studied, or whether this text had arrived in their library, but the chances seem to have been not quite low. Therefore, it just would be here that Descartes might have read, and become convinced, that "Consonantias consistere in proportionibus commensurabilibus. Nam incommensurabiles sonos impossibile est concordare: … sive consonantia fit ex ictuum correspondentia." ["Consonance arises from commensurable ratios. For it is impossible for incommensurable sounds to harmonise: … seeing that consonance derives from the correspondence of strokes."][153]

This exactly was the explanation claimed by Beeckman as his own, but challenged by the philosopher of Method. It is possible, therefore, that the same theory flashed through the mind of the Flemish scholar independently, but perhaps Descartes would have remembered some music lesson he had heard at La Flèche. In a long letter, the tone of which were more worthy of a psychologist than a historian, the Frenchman reproached Beeckman, writing that what he claimed to have taught him went back as far as Aristotle. "… jactares me saepius à te doceri consuevisse? … malitiosum artificium …" ["… do you boast that I was accustomed to being often instructed by you? … cheating deception …"]. "An tibi tribuere debui, quod omnes Ludimagistri, praeter te, ab Aristotele didicerunt? […] Scias autem saluberrimum remedium fore ad purgandam bilem, quae te vexat, si diligenter, quam apte tibi conveniat istud exemplum. […] Nempe altius ponis subsellium, quod vis calcare, ut tanto magis emineat vanitatis tuae thronus?" ["Or that I should have attributed to you what all the schoolteachers, excluding you, showed from Aristotle? […] You should instead understand that it will be a highly beneficial remedy to purge the bile that afflicts you if you observe with attention how appropriate this example is for you. […] Do you thus place the chair that you wish to occupy higher, so that the throne of your vanity may all the more dominate?"]. It appears that the purpose of this attack also was to use an elegant Latin rich in metaphors to ridicule the scholastic Latin of the poor Beeckman. To Mersenne, about it, he wrote: "… combien la sotte gloire d'un Pedan est ridicule, …" ["… how ridiculous the foolish glory of a pedant is, …"].[154]

Ever since his *Compendium on Music*, Descartes had maintained that the 'strokes' of sounds had to coincide in the ear in order to obtain consonances. Again at the end of 1629, he presented consonances to Mersenne as due to the correspondences of the "retours" ["returns, oscillations"] of strings. Here, Descartes wrote that he had already maintained this in a "… petit traité ou je l'explique, […]; il a demeuré unze ans entre les mains du Sr. Becman, & si ce tans la suffist pour la prescription, il a droit de se l'attribuer." ["… little treatise [the *Compendium*] where I explain it […]; it remained in the hands of Beeckman for eleven years, and if this length of time is sufficient for prescription, then he has the right to attribute it to himself."][155]

[153]Maurolico 1575, pp. 150–151.

[154]Descartes 1974, I pp. 24, 30, 110–111, 155–167, 172 and 177–178.

[155]Descartes 1974, I pp. 94, 100–101 and 224–227.

In reality, the theory of coincidence did not entirely convince the philosopher of Method, because, solicited again by the Minim friar who was to bear it in mind for his *Harmonie Universelle*, he ended up by taking away from numbers the decisive criterion for succeeding in distinguishing the more pleasant sounds from dissonances. The "agreeing" and the best "meeting" of sounds makes the most simple consonances, but not necessarily the most pleasant ones. "… il faut supposer la capacité de l'auditeur, laquelle change comme le goust, selon les personnes; …" ["… we must suppose the ability of the listener, which changes, like the taste, in accordance with the individuals; …"]. "… pour ce qui les rend plus agreables, cela depend des lieus ou elles sont employées; & il se trouve des lieus ou mesme les fausses quintes & autres dissonances sont plus agreables que les consonances, …" ["… as regards what makes them [the consonances] more pleasant, it depends on the places where they are employed; there are passages where even false fifths and other dissonances are more pleasant than consonances, …"].[156] In saying this, Descartes was returning to his initial *Compendium*, where he judged music as a source of pleasure for the ear. He was to make this sense prevail over theory, even over his own theory, which provided for the division of the octave into 18 unequal parts. Sure, he was to prefer the music of the Frenchman, Antoine Boësset (1585–1643) to that of his Dutch friend, Jan Albert Ban, who followed the criteria of Descartes in the field of theory.

Despite the event that our French philosopher presented himself as "… tam rudem ut nullam consonantiam auribus dijudicare …" ["… so unsuit as not to distinguish any consonance with my ears …"], he set about defending "… non immerito ego Gallus etiam Gallo …" ["… I, a Gaul, non unjustly, another Gaul …"]. "… sed patere Gallos meos, si tibi inferiores sint in Musicae scientiâ, saltem in amatoriis nugis intelligendis esse magis exercitatos, …" ["… but it is clear that, if my Gauls are inferior to you [to Ban] in the science of music, they are more expert at least in understanding the tattle and prattle of love …"]. Actually, the competition consisted of composing the most expressive music for a sextolet that sang of a disappointment in love. "At moriantur profecto isti nugatores si ipsis placet, nos vero interim rideamus. Et scias haec me ludibundum effudisse non ut tibi serio contradicam, sed ut tester istiusmodi rationes quae non tam a Musicae scientia quam ab interpretatione carminis Gallici pendebant, nec mathematicas, nec physicas, sed morales tantum mihi videri, talesque ut illorum ope non solum alteri, sed etiam mihi ipsi contradicere facile possim." ["Let these chatterboxes die out of love, if they so desire; in the meantime, sure we shall laugh about it. And take note that I would have abandoned myself to this out of amusement, and not to contradict you seriously, but to testify to the ratios of this kind; they depend not so much on the science of music, as on the interpretation of Gaulish poetry; they are not mathematical, or physical, but only moral, they seem to me, and such that I can easily criticise the work of those, not only with others but also with myself."][157]

[156]Descartes 1974, I pp. 88, 108–109, 126 and 223.

[157]Descartes 1975, III pp. 255, 261–262 and 825–834. Huygens 1889, II p. 547.

Music had to be capable of moving the sensibility of our French natural philosopher in depth. Clearly, this also let show us his great pride and vanity. Even towards Galileo Galilei he would behave in the same way as with Beeckman. "Aussy ne voy-je rien en ses livres qui me face envie, ny presque rien que je voulusse avouer pour mien. Tout le meilleur est ce qu'il a de Musique; mais ceux qui me connoissent peuvent plutost croire qu'il l'a eu de moy, que moy de luy: car j'avois escrit quasi le mesme il y a19 ans, auquel tems je n'avois encore jamais esté en Italie, & j'avois donné mon escrit au Sr. Beecman, qui, comme vous sçavez, en faisoit parade & en écrivoit çà & là, comme de chose qui estoit sienne." ["Nor do I see anything in his [Galileo Galilei's] books which arouses envy in me, or hardly anything which I would like to claim as my own. The best things are those that deal with music; but those who know me would rather believe that he took them from me, rather than vice versa: given that I had written almost the same things nineteen years ago, in a period when I had not yet visited Italy, and I had given my manuscript to Mr. Beeckman, who, as you [Mersenne] know, made a great show of it and wrote about it here and there, as if it were a thing of his own."][158]

However, all this would not authorise us to conclude, as other historians have, that Descartes had taken a large part of his theory for music from Beeckman, even denying that he had done so, overwhelmed by his "... obsession with 'praise' ... colossal vanity ...".[159] We may admit that the man was this one, with all his unsupportable defects, but all the same, the theory of coincidences for consonances was also written in Jesuit teaching manuals, and was therefore at the disposal of interested scholars. Thus our French natural philosopher might not have invented it by himself, but he might not have learnt it from Beeckman, either. May the Flemish scholar have brought it back to his mind by means of his discussions?

We are seeing how the heretics and innovators reshuffled the cards of scientific orthodoxy in the seventeenth century. The game could also be variously guided by the various potentates and by human weaknesses: someone was bound to be a victim of this. However, the story of a scholar like Beeckman does not seem to be any worse compared with others, such as the loss of a historical memory for sciences which affected the musician Vincenzio Galilei. In any case, in that circle of characters assembled by Mersenne, also the Flemish scholar had played a role that is worth recalling, without falling however into the historiographical vice of quarrelling about the priority, devoid of any value or sense. How would it be possible to compare together the theories of strokes brought by sound to the ear? One of these had been left, by Maurolico, together with the arithmetic of the *quadrivium*, another was incarnated by Beeckman in the mechanical corpuscles of air, a third one of Descartes was rendered useless by the personal tastes. During the next few decades, the 'phase' of the sound wave (i.e. when the first stroke arrived) and the equable temperament, accepted by musicians, were to make this dispute become antiquated, in a new context.

[158]Descartes 1974, II pp. 388–389.

[159]Cohen 1984, pp. 190–197. Shea 1994, pp. 82–106.

Beeckman had read Stevin's *Vande Spiegheling der Singconst [On the theory of the art of singing]* in 1624. Of it, he criticised the irrational fifth in his *Journael [Diary]*. "… waerom de proportie van de vyfde, … $\sqrt[12]{\frac{1}{128}}$, so soet in ons ooren klynckt. … soude my oock verwonderen daervan reden te sien." ["… why the proportion of the fifth …, as 1 to $\sqrt[12]{\frac{1}{128}}$, sounds so sweet to our ear. … I would indeed be surprised to see a reason thereof."] "Simon Stevin verò meo juditio nimis addictus fuit mathematicae, ac rariùs physicam ei adjunxit." ["Simon Stevin, in my opinion, was addicted too much to mathematics, and seldom combined it with physics".] On the division of the octave in accordance with geometrical proportions, he gave the following judgement. "In geometricâ verò proportione intervallorum musicorum, nihil nisi pythagoricum et nugatorium occurrit." ["In actual fact, nothing happens with the geometrical ratio of musical intervals, apart from Pythagorean things of no value."][160]

It can easily be understood why the musical theory based on the sonorous-mathematical *continuum* of all the numbers (which we would nowadays call real), which followed the ear, was judged badly by Beeckman: he had become a supporter of sonorous atoms counted by means of first whole numbers. But it would not be necessary to have read all the preceding chapters to realise that, between Stevin and Beeckman, on the contrary the latter was the better Pythagorean. Furthermore, the choice would not be between doubtful mathematical and mechanical theories, but rather between these and the ears of musicians, which, as we have read, were more refined than those of Beeckman and Descartes.

Beeckman sought explanations, imagining models composed of material corpuscles. In his *Journael*, written in Latin and Flemish, but published only in the twentieth century, he criticised those who did not provide any, like Kepler. For this reason, he has been placed by some historians among the champions of the so-called 'mechanistic approach'.[161] In actual fact, this solitary Flemish scholar, who had various jobs, and displayed various inclinations, besides being a Latin teacher, wrote in his *Diary* things about the oscillation of strings that were comparable to those of Mersenne, and details about the isochronism of the pendulum comparable to those of Galileo Galilei, including the fall of bodies.[162] Like Descartes, he too preferred dichotomies, which were sometimes pleasant, like the bisections which generate octaves in music.

Beeckman tried to extend his para-Pythagorean corpuscular model also to the sounds of pipes and to the voice. "Haec expressio in parvis tubis fit celeriter, in magnis tardè, adeò ut in parvis expressio bis fiat eo tempore, quo in duplo majoribus expressio tantùm semel fit, quia in his resultus aeris tam tardè fit ob duplicem aeris quantitatem." ["This process of pressing out goes fast in small pipes, slowly in

[160]Mersenne 1945, II pp. 276 and 286. Cohen 1984, pp. 185 and 286.

[161]Cohen 1984, pp. 186–187, 286 and passim.

[162]Mersenne 1945, II pp. 205–206. Cohen 1984, pp. 116–161. Lenoble 1971, pp. 424, 427–429 and 434.

large ones, such that in small ones the pressing out occurs twice in the time during which in those twice as large the pressing out occurs once, because in the latter the springing back of the air goes slowly, owing to the double quantity of air."] Here the Fleming tried to measure, by means of numbers, the lowering of sounds in the "larger" pipes. But, at least in the pages of the *Diary* that I know, if he had limited himself to the "double quantity of air" in order to obtain half of the "pressing out", any schoolteacher would know, by calculating the volume of the cylinder, that following the Pythagoreans, it would have much easier changed only the length, and not the diameter of the pipes too. It would not be clear, therefore, by what kind of dependence he linked the effect with their length and their breadth. Undoubtedly, if he had continued to experiment without stopping, which, unlike Mersenne, he did not do, he would have had to disrupt his theory of strokes, obtaining different ratios from those that were familiar to him.[163]

"Videbis musicam dulcedinem omnem constare in identitate et in declinando paululum ab hac identitate ac reditu ad eam." ["You will see all sweetness in music consists in identity, and in deviating a little bit from this identity, and in returning to it."][164] Beeckman actually had some experience above all of the *Psalms* sung in the rites of Calvinists, which were rich above all in consonances and avoided chromatisms. To execute them, our expert only needed small modifications to the Ptolemaic-Zarlinian scale, with an appropriately larger number of modes. We already know from his correspondence that he refused the equable temperament, and in his *Diary* he did not even know how to calculate the proportional means between the terms of tones.[165]

John Wallis (1616–1703) rebuked the French philosopher of Method because he would have taken a lot of his theory for algebraic equations from Thomas Harriot (1560–1621).[166] But we, faithful to our style, will not follow the scientific controversies between this Anglican (Puritan) pastor, who occupied the chair of geometry at Oxford, and the French people, who included Fermat and Pascal. Moreover, as usual, these were marred by their various nationalisms and various religions.[167] We prefer to continue to narrate their theories of music.

His edition for the Ptolemaeus of the *APMONIKA*, published in 1682 (used by me in Chap. 2), was followed by a substantial "Appendix. De Veterum Harmonica ad Hodiernam comparata." ["On the harmony of the ancients compared with that of the

[163]Cohen 1984, pp. 280, 148. H.F. Cohen attributed to him the idea that the pitch of sounds from pipes depended on their volumes, but without specifying the ratios. This philosopher of science considered the inverse proportion with the cube $1:2^3$, followed by the Chinese and by Vincenzio Galilei, a "mistake". On the contrary, he is the one who unfortunately perpetuated the same mistake committed by D. P. Walker, ignoring, like him, the modern manuals of acoustics, which justify the *end-effect*, or, worse still, cancelling the clear cultural differences between Chinese, Indians, Arabs and Europeans in music. Cohen 1984, pp. 82–83 and 92–93. See above, Part I, Sects. 3.2 and 6.7.

[164]Cohen 1984, pp. 279, 143.

[165]Cohen 1984, p. 156.

[166]Descartes 1983, p. 638f.

[167]Scriba 1981.

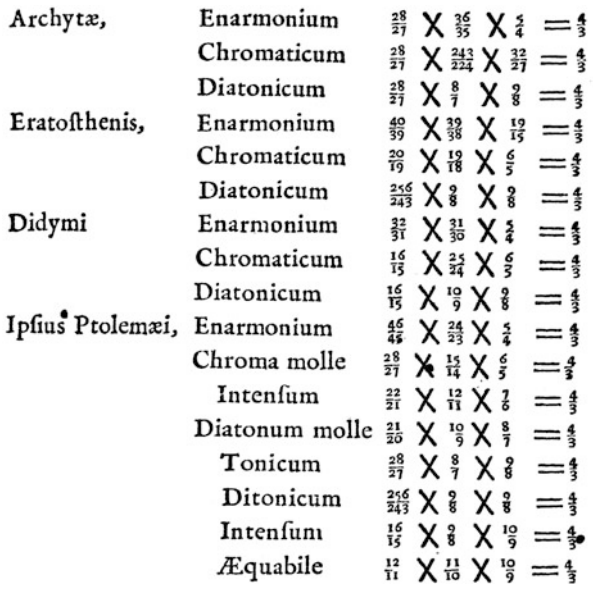

Fig. 9.8 How John Wallis divided the tetrachord [fourth] (Wallis, in Ptolemaeus 1682 Oxford, p. 302)

moderns."] Among the classical Greeks, he often considered Aristoxenus, and not only the Pythagorean variants of Euclid or Ptolemy. Consequently, he continually put also the tones, semitones and quarter-tones of the Aristoxenians, together with the numerical ratios of the Pythagorean school.

He wrote ratios as proportions, "4 to 3 ... 3 to 2", but he performed calculations with the fractions "$\frac{4}{3} \times \frac{3}{2} = 2$". He referred that the ancient Pythagoreans admitted only multiples and superparticular ratios for consonances, excluding the others. "... multo adhuc minus quae sunt ($\alpha\lambda o\gamma o\iota, \alpha\rho\rho\eta\tau o\iota$) numeri Ineffabiles." ["... even less those that are inexpressible [irrational] numbers."][168] He summed up in a table (Fig. 9.8) the breakdowns of the tetrachord 4:3 in the enharmonic, chromatic and diatonic genres, according to various Greek theoreticians.

He continued to repeat in parallel that the octave, divided into 5 tones and 2 semitones by the Aristoxenians, and called not the "Di-octo" but the "Dia-pason", should instead be defined by the Pythagoreans as the double proportion, because "$\frac{4}{3} \times \frac{3}{2} = \frac{2}{1}$". "Aristoxenus vero, ejusque sequaces, nimiam hanc esse subtilitate rati, ad Aurium judicium provocabant, primaque inde principia desumenda volebant, unde deducerentur reliqua. Frustra esse, clamitantes, eos qui Rationum subtilitates sectabantur; quas & Phaenomenis saepe contrarias esse, aurium judicio comprobatum esse asserebant: (Ideo forsan, quod ex Pythagoreis aliqui non tam

[168] Ptolemy 1682, pp. 283, 302, and 321.

caute, quam par erat, rationes nonnunquam assignaverant; quod & causatur aliquando Ptolemaeus.) Sensus itaque judicium, Principii loco habet Aristoxenus, indeque Demonstrationes deducit suas; secus quam Geometrae, qui ab Intellectu incipiunt."

["Indeed Aristoxenus and his followers, having decided that the subtlety [of the ratios] was excessive, challenged to submit them to the judgement of the ears, and wanted to deduce from that the first principles, so as to work out the remaining ones from there. Proclaiming with emphasis that they were wrong who followed the subtleties of ratios, and that these were often contrary to phenomena, they asserted this was confirmed by the judgement of the ears: (about that, for this reason perhaps, some Pythagoreans sometimes assigned the ratios without the caution that would have been appropriate; in this, Ptolemy sometimes presents excuses.) Thus Aristoxenus puts the judgement of the senses in the place of principles, and deduces his proofs from this; unlike geometricians, who start from the intellect."][169]

"An vero (...) Dia-tessaron sit praecise duorum tonorum cum semisse; an eo proxime accedat, tantillo inde distans; dubitanter aliquando loqui videtur Aristoxenus. ... An Dia-tessaron à minorum intervallorum aliquo (...) mensuretur; an vero sit omnibus incommensurabile: ... [...] Sed haec forte ab eo dicta non sunt, quasi de hoc dubitaverit ipse; sed quod hoc nondum demonstraverit (?) adeoque impraesentiarum hoc tantum postulat; nempe, vel tantundem esse, vel ab eo saltem pauxillo differre." ["Indeed if (...) the diatessaron [fourth] is not precisely two and a half tones, if it does not come close to it with approximation, thus moving away from it a little, at times Aristoxenus seems to speak with some doubt. ... If the diatessaron is measured by someone else with smaller intervals, or if indeed it is incommensurable with all of them: ... [...] But maybe these things are not said by him, as if he himself had doubts; however he did not demonstrate it yet (...) and on the contrary, he only postulates it for the moment; that is to say that it is either equally large, or it differs at least by a small quantity."][170]

Having reported the argument of Aristoxenus, as Ptolemy had also summarised it, Wallis concluded. "Atque ostendit, quam sit infirmum. Quippe, in tam exigua discrepantia, quanta est (...) Hemitonii à Limmate (...) impossibile est (...) ut possit auris tam accurate distinguere. Et quidem, si tantundem errare possit auris una vice; multo magis id fiat, in toto illius Argumenti processu; ubi ter sumendum sit (aurium judicio) Dia-tessaron, & bis Ditonon idque in contrarias partes. Ut non sit, in tam subtili negotio, aurium judicio standum." ["It also shows how weak it is. Without doubt, in such a small difference as that (...) of the semitone from the *limma* (...), it is impossible (...) for the ear to distinguish with such accuracy. And then, if the ear can be so wrong in a part, it will err much more in the whole process of that argument; where it (the judgement of the ear) is assumed three times for the diatessaron and twice for the ditone [third], and that in contrary parts. Consequently, in such a subtle question, we must not depend on the judgement of the ears."]

[169] Ptolemy 1682, p. 305.

[170] Ptolemy 1682, pp. 306–307.

Naturally the *limma*, the value of which was $\frac{256}{243}$, was less than the semitone. "Sed neque posse Tonum in duo plurave aequalia dividi (concinna, intellige) & quae sunt hujusmodi." ["But neither can the tone be divided into two or more equal parts (that is, of [rational] musical ratios) which are of this type."] "Hanc autem rationem 2 ad 1, cum in duas aequales (numeris effabiles) partire non possent, (quippe nulla numeris effabilis ratio, potest bis posita duplam praestare) in duas proxime aequales (...) dividebant." ["[The Pythagoreans] divided this ratio 2 to 1 into two parts which were almost equal (...) when they were not able to separate it into two [exactly] equal parts (using expressible [rational] numbers); clearly, no ratio of expressible numbers multiplied by two can arrive at the double ratio)."][171]

Having followed the method of the Pythagoreans, having cancelled some of its venial sins, and having sometimes accepted the judgement of the ears, Ptolemy was considered by the Englishman to be "potior [better]" than Aristoxenus. "Posteriores tamen Musici, ad nostra usque tempora, (praesertim *Practici*) cum Aristoxeno plerumque loquuntur; per Tonos & Hemitonia metientes Intervalla." ["However, down to our times, the following musicians (in particular those who are *practical*) for the most part speak together with Aristoxenus, measuring intervals with tones and semitones."] But our English pastor made very few explicit references to these musicians, remembering very little, and rarely Mersenne. There is no doubt that he wanted to leave Aristoxenus aside, who in the meantime had become cumbersome, confining him to the role that was assigned to him by the orthodox tradition inspired by Pythagoras. We obviously find no trace, therefore, of Stevin. Vincenzio Galilei was quoted only once as an adversary of Zarlino, who was much more highly considered, of course. As the physical cause of consonances, Wallis too assumed the theory that in these cases, the strokes transported by sounds to the ear coincided more often.[172]

The statutes of his chair at Oxford included the suggestion that he should also teach the theory of music. In actual fact, besides the appendix considered here, Wallis dedicated some articles and letters for the Royal Society journal to the subject. For the aim of tuning organs and other keyboard instruments, this English natural philosopher admitted the need for the equable temperament, as commonly practised, above all for modulation. He also commented on the book by Thomas Salmon (1648–1706), *Proposal to Perform Musick, in Perfect and Mathematical Proportions*. With the help of Robert Hooke (1638–1703), Wallis performed, at the

[171] Ptolemy 1682, pp. 307–308 and 321.

[172] Ptolemy 1682, pp. 308–309 and 318–320. As others had also begun to think, our mathematician realised that the strokes had to start together, otherwise the argument founded on coincidences would fall apart. Nevertheless, he maintained it undaunted, affirming, without giving any proof or justification, that if the strokes started without coinciding, after a while they would start coinciding again. Cf. Wardhaugh 2008, p. 160.

Royal Society, the same experiments previously carried out by Mersenne (without deigning to mention him), measuring and calculating the vibrations of strings.[173]

In rejecting the division of the octave into equal parts, like many others from the times of Euclid onwards, it should be noted that our English geometrician had added in brackets that it could not be done using "expressible numbers". Therefore, he could have succeeded perfectly well by using "inexpressible", that is to say, irrational numbers, which by now had been admitted to the category of numbers. Our Oxford mathematician had clearly shown that he interpreted these in a manner that was not at all in line with tradition. In his *Arithmetica infinitorum [Arithmetic of infinites]* he had long before succeeded in representing one of the most famous and indomitable "inexpressible" ratios as an infinite product of numbers

$$\frac{2}{\pi} = \frac{1}{2} \times \frac{3}{2} \times \frac{3}{4} \times \frac{5}{4} \times \frac{5}{6} \times \frac{7}{6} \times \dots$$

We should notice that the first fractions (all superparticular) of the development are those of music.

Although he would never have admitted it, our English pastor was developing considerably what Descartes had started, with different intentions, even if his way took other directions. The Frenchman generally used algebra as an instrument, always leaving geometry at the centre; the English scholar, on the contrary, tried to oust it, reducing it to the numbers of the former. The islander operated, above all, with numerical magnitudes, thus succeeding in solving classic problems for the geometry of curves.[174] In their theological and nationalistic discussions and diatribes, Wallis, Fermat, Pascal and the others by now spoke unashamedly even of infinity (and others will discover subsequently that they lacked great rigour, either): a concept which until then had been considered "inexpressible" and divine by all these natural philosophers, who were deeply involved, even as a result of their job and their vocation, in the wars of religion of their times.

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[173] Scriba 1981, p. 152. Gouk 1989a, p. 159. Other interesting details about Wallis's musical endeavours can be found in Wardhaugh 2008, pp. 156–166–179. In fact, our famous mathematician maintained that "the ear might be a legitimate source of knowledge for what was none the less definitely a part of mathematics." Unfortunately, it remains a mystery how he reconciled the judgement of the ear with his mathematics for music. Maybe, like his Ptolemy, he had faith in a superior *a priori* coherence?

Thomas Salmon engaged in a dispute with a musician who relied only on the ear, and did not believe that music consisted of ratios. While he seemed to be distant from the real English musical environments, he cannot have been very close to the mathematical culture, either, because he had to ask Wallis for explanations about the calculation of ratios for music. Salmon made numerous attempts to adapt the tuning system of Ptolemy-Zarlino (which the English insist on calling the "just" intonation scale) to the requirements of modulations, proposing various interchangeable fingerboards. However, his conclusion was the same as we shall find further on in our story, that the equable temperament "may be allow'd by such Ears as are vitiated by long custome".

[174] Scriba 1981, p. 148. Boyer 1990, pp. 436–440.

But now a war of a civil nature, in other periods known as a revolution, had broken out in England. Oliver Cromwell (1599–1658) had deposed Charles I, having him beheaded. In his youth, Wallis had been attracted by the new parliamentary powers, helping them to decipher letters written in code. In that period, he was awarded the chair at Oxford, which had been lost by his Royalist colleague. This prestigious academic appointment was confirmed, however, also after the restoration of Charles II, perhaps because he had opposed the execution of the previous king, and he would ever be useful as a code-breaker. This reverend Anglican was not the first person (nor would he be the last) to display a flair both for mathematical sciences and for mysterious texts in cipher, as they are called. Also the jurist, François Viète, who served the French king Henry IV of Navarra in the war against Spain, had been equally good at cracking the enemy's secret codes. In his free moments from serving the crown, this Frenchman calculated using decimal fractions and algebra, which, forgetting the Arabs, he traced back to the Greek method of "analysis". He had invented new literary symbols to write equations, similar to those of Descartes, albeit less successful.[175] In the war against the Germany of Hitler, centuries later, another English mathematician who decoded enemy messages with a comparable effectiveness was Alain Turin (1912–1954), who became a co-inventor of modern computers.

We have seen that in his commentary on Ptolemy, this seventeenth-century Englishman often substituted traditional Greek geometrical proportions with numerical fractions, as if they were arithmetic. In his *Treatise of Algebra, both Historical and Practical*, he not only succeeded in almost cancelling Descartes, but he, too, traced this Arabic discipline back to the Greeks, who, however, would have kept it secret. This was an antihistorical attitude (as well as anti-Arabic, anti-French, ...) which we shall soon find also in his most illustrious English successor, Newton. With Wallis, the tendency to abandon traditional geometry, with its Euclidean certainties, became explicit, beginning the still uncertain adventure of irrational numbers, infinities and infinitesimal quantities. Among other things, the symbol of infinity ∞ still in use today was introduced by him. The best-known English critics of the new course included Thomas Hobbes (1588–1679), an expert not so much on mathematics as on political pessimism, who attacked the translation of geometry into the new numbers, calling the *Arithmetica infinitorum* "a scab of symbols".

Even if irrational numbers were used when necessary, many continued to share the opinion of Michael Stifel of a 100 years earlier. Because an infinite number would not be a number, so an irrational number would not be a true number. Also Pascal and Newton continued to try to justify them by means of their, for them, more certain geometrical magnitudes.[176] Instead, our English churchman, with a certain good luck and self-assurance, calculated pieces of the area of a circle, using the first rudiments of infinitesimal analysis. He was as fond of separating from his

[175] Boyer 1990, pp. 348–358.

[176] Boyer 1990, pp. 439 and 441. Kline 1972, pp. 318, 251–252. See above, Sect. 6.5.

colleagues on the continent as he was of uniting with his fellow-countrymen of the island. Thus, thanks partly to him, the glorious Royal Society was founded.

In his *Treatise of Angular Sections*, in order to calculate the chords of a circle in the style of Ptolemy, he generalised the theorem of Pythagoras, in a style comparable to that of Thabit ibn Qurra, illustrated by us in Chap. 5, but a few centuries later.[177] It seems to be unlikely that Wallis had access to his writings in Arabic, but this cannot be totally excluded: a colleague of his at Oxford translated for him the considerations of another Arabic mathematician regarding the axiom of Euclid for parallel lines. The English pastor and mathematician published them, criticised them, and of course gave another argument of his own.[178]

An experimentalist like Beeckman took the liberty of making various observations about that Francis Bacon (1560–1626) who was destined to become particularly famous as the English philosopher of experimentalism, sometimes in competition in this field with others. Among the criticisms, the Fleming pointed out errors committed by him in claiming that water could be transformed into air, or that its volume diminished when it turned to ice.[179] However, we cannot doubt that this philosopher, too, partly dealt with music. Only he did not recognize himself in the main Pythagorean orthodoxy based on first whole numbers, and he preferred to follow his senses in the style of Aristotle. Of course, Bacon rejected the music of the heavenly spheres. And yet even he recognized the great power of music over human people, and he was forced to include it in his project of exploitation, dominion and control over the 'natural forces'. His famous Biblical House of Solomon, contemplated in the *New Atlantis*, included "houses of sound". In the *Sylva Sylvarum [Forest of Forests]*, Bacon planned to unite "the contemplative part with the practical part" of music.

He divided the octave into 6 tones and 12 equal semitones. May he have read Aristoxenus, whose works were available in those years? In 1562, Antonio Gogavino had printed *Aristoxeni Musici Antiquissimi [Of Aristoxenus, a very ancient musician]* at Venice. May he have taken his inspiration from Vincenzio Galilei? We do not know. With him, unfortunately, unlike the Tuscan musician, his rejection of the Pythagorean idea had not been transformed into a sufficient attention for the phenomena of musical sounds (in spite of all his experimental claims). He left us only vague promises, as his Stoic kind of analogy between sounds and water ripples was ambiguous. He affirmed that a kind of spirit, which recalled the ancient *pneuma*, was propagated in sound. Not only in music, but in general, this English philosopher proclaimed projects which he did not realise.

In the *Sylva Sylvarum*, Bacon maintained that the makers of musical instruments did know about music more than many theoreticians. He appeared to be a good example of this, himself: he believed that the pitch of sound in pipes depended on their diameter, and not on their length. Even the Royal Society failed to investigate

[177] Scriba 1966.

[178] Kline 1972 , p. 864.

[179] Hooykaas 1981.

the question attentively. As communicated by Wallis, the people at Oxford capable of discovering the nodes of the vibrating string were the two musicians Thomas Pigot and William Noble (together with Francis North): these were the points at which it remains still. We shall shortly see that only Robert Hooke, perhaps taking his inspiration from Bacon, was to carry out experiments on how glasses and metals vibrated.[180]

Henry Oldenburg (1618–1677), the secretary of the Royal Society in London, reviewed the *Speculazioni di musica [Speculations on music]* written by Pietro Mengoli (1625–1686). He was a pupil of Galileo's pupil Bonaventura Cavalieri (1598–1647). Having settled in Bologna, among various other arguments of mathematics, he left his own regular manuscript about music. It is not so interesting Mengoli imagined a spiritual *aura*, of Stoic inspiration, that served to propagate sound, through the necessary air. Rather, he did not believe in the theory that the strokes of sounds coincided better in consonances: "... perché trà tutti i supposti havevamo per assioma indubitato, che avviene la consonanza, per la unione frequente de' due suoni à percuotere il timpano esterno dell'orecchio nello stesso istante di tempo: il che ho poi scoperto essere falsissimo." ["... because among all the hypotheses, we held as an undoubted axiom the idea that the consonance takes place as the frequent union of the two sounds in striking the external eardrum at the same instant of time: what then I discovered to be very false."] He described an alternation of strokes in the ear: "Le une tra le altre succedono alternatamente à toccare il timpano esterno" ["The ones follow the others alternately in striking the external eardrum."] Above all, as an ordained Catholic priest, desirous of reconciling the sciences with theology, he assigned the most important role in judging consonances to the soul: "... volenterosa del diletto, nell'attentione attiva del suono." ["eager for delight, in paying active attention to the sound."]

To arrive at the soul, Mengoli introduced into the physiological mechanism of the ear an inclination of the external eardrum. "L'inclinazione di questi due timpani è di muoversi in proportione dimidiata della doppia" ["The inclination of these two eardrums is to swing in a ratio of half 2:1"], that is $\sqrt{2}$:1. Not only did a root come in here, as in the equable temperament, but the inclination strongly recalled the *clinamen* of Lucretian memory, in contrast with the insoluble Cartesian contrast between mind and body. "Sono diversi frà di loro i movimenti de i timpani, secondo la proporzione dupla; e parimente sono diverse le inclinazioni al moto, secondo la dimidiata della proporzione dupla: è manifesto che l'aura sonora haverà diversi movimenti, e diverse inclinazioni al moto; ...". ["The motions of the drums are different, according to the double ratio; and likewise, the inclinations to motion are different, according to half the double ratio. It is therefore clear that the sonorous *aura* will have different motions and different inclinations to the motion; ..."].

[180]Gouk 1989a, pp. 157–158, 160–163; 1989b, pp. 188–190; 1991, pp. 99 and 113. Cf. Wardhaugh 2008, pp. 159 and passim. Resnick & Halliday 1961, pp. 414 and 431. See Part II, Sect. 10.2. For the scales proposed by John Pell (1611–1685), with 21 or 33 notes per octave, with the relative numbers, see Wardhaugh 2008, pp. 154–155.

As a musician who prefers to add up the intervals between notes, instead of multiplying the relative ratios, our priest from Bologna introduced base ten logarithms, which made it possible for him to calculate by means of additions, rather than by performing multiplications. Thus the octave 2:1 became 30104 [$\log_{10} 2 = 0.30102999$] and $\sqrt{2}$:1 half of that, 15052. Mengoli thus would take us out of the usual orthodox ratios of the Pythagoreans, to lead us into others, as perceived by the soul and by the ear. "… la prima ragione, secondo la quale l'anima è costretta ad avvertire quel che ne' timpani dell'orecchio, e nell'aura eteria trà gli timpani per il suono si fà. Onde hà ragione per essere il primo di tutte le ragioni a scoprirsi, di essere anche la misura di tutti gli altri." ["… the first ratio, according to which the soul is brought to perceive what happens on account of the sound in the drums of the ear and in the sonorous *aura* between the drums. Thus it is rightly the first of all ratios to be discovered and also the measurement of all others."]

We do not know whether the distant pupil of Galileo Galilei intended these numbers as measurable oscillations, or whether he had read Vincenzio Galilei, too. However, Mengoli, knowing Aristoxenus, and rejecting the coincidence of strokes, tended to justify the equable temperament both by means of the physiology of the ear, and with his theology of the soul, thus introducing an element that he considered eternal into the judgement of consonances: "… l'anima ragionevole nel corpo umano, fà il temporale eviterno." ["… the rational soul in the human body makes the temporal eternal."] This is what we are discussing here, whether the soul is immortal and eternal, or also subject to time and to the different cultures. The event that this natural philosopher from Bologna was largely ignored, at that time as well as subsequently, can be explained by the resistance of neo-Pythagoric orthodoxy.[181] He conquered a place in the history of mathematics, however, for his ability in calculating infinite series.[182] Maybe his *clinamen* towards the equable temperament was parallel to the one towards these new infinite numbers?

Few other historians of science have considered all this interest in music on the part of the natural philosophers of the seventeenth century and afterwards. Rejecting the pseudo-experimentalism of Francis Bacon, Stillman Drake identified in this "… the origins of the experimental aspect of modern science …". However, if we desired to adopt his definition of experimental method, it would also fit Al-Farabi,

[181] Gozza 1991, pp. 122, 126–129. The research of Paolo Gozza is developed, with some extra details, also by Benjamin Wardhaugh. In reality, this English scholar practically always concluded by raising bitter objections to Mengoli's theories. However, even Wardhaugh stated that the majority of theoreticians of music with mathematical criteria were incapable of explaining contemporary musical practices, because, on the contrary, these were based on the ear and on the equable temperament; Wardhaugh 2008, pp. 83–96.

When also a professional musician like John Birchensha (c.1605–1681) attempted to present a theory of music to the fellows of the Royal Society, he remained distant from the practice followed by his colleagues: and even from his own practice. It seems that he used the equable temperament, whereas his theoretical scale had remained that of classical Pythagoreanism; Wardhaugh 2008, pp. 147–148 and 154–155.

[182] Boyer 1990, pp. 426–427.

as we have seen in Chap. 5. Furthermore, the question should not be simplified to the contrast between experiment and mathematics, but extended to the question of irrational numbers, in connection with the ear and equable tuning. "The experiments of Vincenzio Galilei, …, were true scientific experiments …: the manipulation of physical objects for the purpose of verifying a mathematical rule preconceived as applicable to their behaviour, … […] … The first conscious experiments to test a pre-existing mathematical theory were probably the musical experiments of Benedetti and Vincenzio Galilei. They were extended into mechanics by Galileo, …".[183]

But this offered only a part of the history: its obscure, ambiguous face. Sure enough, the musical theory of his son, Galileo, remained Pythagorean, whereas his father, Vincenzio, had rejected it, seeing that he had followed Aristoxenus. Thus the son could not have followed his father in his experimentation, which, as we know, was often overlooked, in favour of theory. We should return, then, to how other historians considered the role of music in the scientific revolution of the seventeenth century.

> … nulli facilius ad magnam pietatis famam perveniunt,
> quam superstitiosi vel hypocritae.
> [Nobody succeeds more easily than the superstitious people and the hypocrites
> in gaining a great reputation of piety.]
>
> René Descartes

9.3 Constantijn and Christiaan Huygens

Constantijn Huygens (1596–1687) had written to Descartes "… je suis jaloux de cest'honest'homme, en faveur duquel vous avez autresfois escrit le *Traicté de la Musique*, …" ["… I am jealous of this honest man [Beeckman] in favour of whom you once wrote the Treatise of music, …"]. With music, this noble functionary at the Dutch court of the Orange was clearly more practical than the other two put together. He played the organ, and had also written a book about it: *Gebruyck of ongebruyck van 't Orgel in de Kerken der Vereenighde Nederlanden [The use or abuse of the organ in the churches of the united Netherlands]*. He asked Descartes for an opinion about it. "Et je pretends le faire imprimer, pour introduire ce que je croy utile, ou faire abroger ce qui est scandaleux sans doubte." ["And I claim on having it printed, to introduce what I believe to be useful, or to remove what is undoubtedly scandalous."] Huygens senior championed the cause of sacred music, and would have like to fill his Protestant churches with organs.

The French philosopher praised his presentation. "Mais pour ce que l'orgue est l'instrument le plus propre de tous pour commencer de bons accords, permettez à mon zele de dire icy *omen accipio* sur ce que vous l'avez choisi pour suiet." ["But

[183] Drake 1970, pp. 483, 497 and 499. Drake 1981.

as the organ is the most appropriate instrument of all to start good chords, allow my zeal to say here that I interpret it as a favourable omen that you have chosen it for your subject."] Here, unfortunately, Descartes was alluding less to the tuning of organs, and more to his desire that the Protestant churches of the hospitable Holland and his Catholic churches should be reconciled. In Paris, a collection of Psalms and hymns compiled by Constantijn Huygens was published under the title *Pathodia Sacra et profana [Sacred and profane odes to passion]*.[184]

To the noble secretary of His Highness the Prince of Orange also happened to have to follow affairs of war. Mersenne took advantage of this to get him to perform experiences with cannons. But in the end, the Dutch musician commented: "Et icy je conclus nostre conference Canonique, en ayant tant d'autre icy autour de mes oreilles jour et nuict, qui reciproquement ne faict que coupper bras et jambes, que je n'ay plus loysir de traicter ces matieres en jouant. Quand vos Canonistes Royaulx seront revenus de la campagne j'attendray qu'à Paris vous leur fassiez faire un essay à leur mode, afin que puissions conferer les experiences, …" ["And I conclude our conference of can[n]ons here; having so many in my ears here day and night, which do nothing other than sever arms and legs, I no longer have any possibility of dealing with these subjects for pleasure. When your Royal cannonic men return from the campaign, I will wait for you to get them to carry out an attempt to the best of their ability in Paris, in order to compare the experiments, …"]. In the life of Huygens senior, the musician and functionary, musical harmonies and warlike massacres were mixed, and fortunately, he preferred the former.

A copy of the *Compendium Musicae [Compendium of music]* was to arrive among the papers of Huygens which are now at Leiden, in the library of the University.[185] His son Christiaan (1629–1695) possessed a copy of the *Harmonie Universelle* by Mersenne. As regards the inspiration, or the lack of it, that he had received from the Minim father, the Dutchman stated: "… fait mention de ceste invention de diviser l'octave en 31 parties esgales, mais ce n'est que pour la condamner et le Pere Mersenne la rejette de mesme, d'ou l'on pourra bien me croire que ce n'est pas de ces autheurs que je l'ay prise." ["… he mentions this invention of dividing the octave into 31 equal parts, but this is only to condemn it, and Father Mersenne likewise rejects it; as a result, it will not be difficult to believe me when I say that I did not take it from these authors."][186]

Also Huygens junior received letters from that tireless organiser of the Minim friar. "Et parceque Monsieur vostre pere se plaist grandement à la Musique j'espere que vous prendrez plaisir pour l'amour de luy de resoudre une dificulte harmonique que vous experimenterez vous mesme sur vostre Luth, ascavoire pourquoy il faut qu'une chorde de Luth tendüe à un certain ton, soit tendüe 4 fois dayantage pour

[184]Descartes 1975, III pp. 756–758. Musical works by Constantijn Huygens were to be published, or re-published, at the end of the nineteenth century. Huygens 1888, I pp. 30 and 554; 1889, II pp. 547–548, 552, 555–556, 559 and 565. Shea 1994, pp. 336 and 412.

[185]Mersenne 1960, VI p. 306. Descartes 1966, X p. 82.

[186]Mersenne 1986, XVI p. 549–551.

monter à l'octave, attendue que l'octave n'a sa raison que de 2 à 1." ["And seeing that your good father is very fond of music, I hope that for his sake you will take pleasure in solving a harmonic difficulty that you yourself may experiment on your lute, that is to say, why it is necessary for a lute string, stretched to a certain note, to be tightened four times more to rise up to the octave, seeing that the ratio of the octave is only 2 to 1."] Mersenne recognized the extraordinary capacities displayed by Huygens junior, who he called, in the letters to his father, "vostre Archimede" ["your Archimedes"], as Constantijn proudly described him, almost exalting him to take the place of Descartes. In answer, the young Dutch mathematician submitted him the solution, which calculated the shape assumed by the catenary, which was not a parabola, as Galileo Galilei believed, but a transcendent curve. By him he could be to some extent influenced. Even Descartes, generally restrained in his compliments, gratified him when he met him at his father's house, considering him a member "of his race".[187]

Christiaan Huygens learnt how to play the lute and the harpsichord, and was educated in music by his preceptors.[188] In this art, he undoubtedly had a better knowledge than the famous Frenchman. Music often appeared in his letters, especially in those he exchanged with his elder brother, who had the same name as his father, Constantijn. They often exchanged airs to be accompanied on the lute, even if they did not show the same tastes. Christiaan wrote: "Ce qu'on estime le plus dans les Airs c'est la nouveauté, . . . " ["What I appreciate most in an air is its novelty . . . "]. The mathematician complained that those that were sent to him were ". . . toutes de la legere sorte, que je scai estre plus selon vostre fantasie." [". . . all of the light kind, which I know to be more to your taste."] In return, during his journey to France in the summer of 1655, Christiaan sent him some, the text of which leaves us few doubts about the type of his songs.

["But it is true, I confess, your beauty holds my soul captive,
Alas, if I could spank your buttocks, I would be free,
And if you were ready to rest on my bed, to satisfy my desire,
I would kiss your eyes and your mouth a hundred times, and I would die of pleasure."]

This continual irrepressible entwining of music with sensual impulses returned in the friendship of Constantijn Huygens senior, at that time an ambassador in France, and Christiaan with Mademoiselle Ninon de l'Enclos, who in that period would have been 'nobly' defined as a libertine who was also an expert lute-player. "Il me monstra un tableau ou elle estoit peinte toute nuë, lors qu'elle estoit encore raisonnable, mais à cet heure elle est passée, et converse, à ce qu'on dit, avec les femmes honnestes apres que l'age luy a fait quiter son premier mestier. Mon Pere la voit assurement pour l'amour de la musique et je n'en veux point avoir d'autre pensée." ["He showed me a picture where she was depicted completely nude, when

[187]Mersenne 1983, XV pp. 24–39. Huygens 1888, I pp. 30–31, 34–35, 87–89 and 83–86; 1889, II pp. 554–559. Lenoble 1971, p. 585. Shea 1994, p. 332.

[188]Huygens 1888, I pp. 537–554.

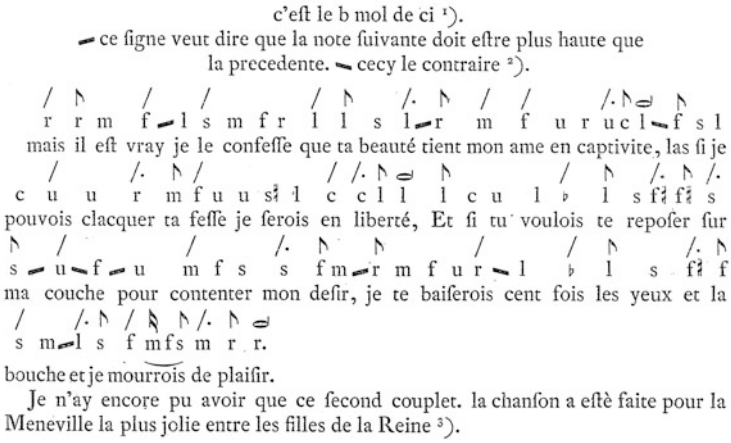
c'eſt le b mol de ci [1]).
ce ſigne veut dire que la note ſuivante doit eſtre plus haute que la precedente. cecy le contraire [2]).
mais il eſt vray je le confeſſe que ta beauté tient mon ame en captivite, las ſi je
pouvois clacquer ta feſſe je ſerois en liberté, Et ſi tu voulois te repoſer ſur
ma couche pour contenter mon deſir, je te baiſerois cent fois les yeux et la
bouche et je mourrois de plaiſir.
Je n'ay encore pu avoir que ce ſecond couplet. la chanſon a eſtè faite pour la Meneville la plus jolie entre les filles de la Reine [3]).

Fig. 9.9 How Huygens wrote the musical notes (Huygens 1888, vol. I, p. 359)

she was still reasonable, but since then she has withered, and she converses, by all accounts, with honest ladies, after being forced by her age to abandon her original profession. My father sees her for musical reasons, undoubtedly, and I do not want to think of anything else."] Without doubt, Plato would have got very angry, seeing that he had failed: because he had educated young men, by means of music, to something different from war. It should be noted that for his songs, Christiaan Huygens did not write the notes on the stave, but employed a special notation of his own as his tablature (Fig. 9.9).[189]

At Paris, the natural philosopher consorted with the most famous composers, harpsichord-and lute-players of the time. Among them, he met Jacques Champion de Charbonnières (1602–1672), who had founded the private concert association, Honnêtes curieux, where the young Huygens also went. Charbonnières had transferred the instrumental technique of the lute to the keyboard of the harpsichord, composing several pieces for it, and thus acquiring an international reputation. In the French capital, the Dutchman met the composer Henri Dumont (1610–1684), who was likewise a master in keyboards for the organ and the harpsichord, as well as in the composition of sacred music. Among the members of the Gaultiers family, he knew the lute-player Jacques (c.1600–c.1670), who was an acquaintance of his father, Constantijn. In his correspondence with him, Huygens junior also mentioned the name of Michel Lambert (1610–1696), the court musician whose daughter was to marry Giovanni Battista Lulli (1632–1687). This renowned Florentine composer was to enjoy success at the court of Louis XIV, and as a collaborator of Molière (Jean-Baptiste Poquelin 1622–1673). Christiaan copied a book by Charbonnières for his father, probably made up of pieces for the harpsichord, and had learnt to play a jig; he was to remain in contact with him by correspondence. As regards

[189]Huygens 1888, I pp. 111, 135, 347 and 359. 1891, IV pp. 183–184.

the tablature for the lute indicated above, he asked a professional like Dumont for his opinion, and sent him a dance, a *courante*, by his father, "... qui luy semble beaucoup plus facile que l'ordinaire de l'epinette non seulement à escrire, mais aussi à jouer. Pour moy je le scai par experience, et serois bien aise que vous fussiez de ce mesme advis; ..." ["... which [tablature] seems to him much easier than the ordinary one for the spinet, not only to write, but also to play. For my part, I know this by experience, and would be glad if you were of my same opinion; ..."].[190]

During another stay in Paris, the 30-year-old Huygens went to see the opera *Xerse* by Pietro Francesco Cavalli (1602–1676), the Venetian composer called to Paris by Cardinal Giulio Mazzarino (1602–1661) to write another opera, specifically for the marriage of Louis XIV. However, these were eclipsed by the ballets of Lulli which accompanied them, and which satisfied the tastes of the court better. Unfortunately, in the fragment of the rough copy that is extant, Huygens commented on the architecture of the theatre, and the nieces of the Cardinal, without leaving us any judgement about the music. He probably preferred "... les belles Opera de Baptiste, ..." ["... the fine operas by Battista [Lulli], ..."].[191]

Cavalli senior, a pupil of Claudio Monteverdi, mainly composed music for dramatic texts, suitable to be sung, seeing that he could sing well, and in spite of the event that he also played the organ at St. Mark's in Venice. His lyric works abounded in virtuoso arias and recitative passages devised for theatres and courts. Among his 40 works, also *L'Ercole amante* was staged in Paris. But here it did not enjoy much success, and the younger Lulli, who had become Lully, the *protégé* of the Sun King, was preferred to him. Particularly skilful as a courtier, the Florentine emigrant established himself with his instrumental music suitable for ballets, which were particularly appreciated at Versailles. It was only when he met the poet and librettist Philippe Quinault (1635–1688) that Lully dedicated himself (with royal privilege) to that lyric kind of music destined to become the model of the *Grand opera* in French, rich in dramatic emphasis and in *obbligato* dancing scenes, which we shall find with Rameau in Chap. 11. Here, the spectacle, the scenic effects, and now also the female singers, contributed to the success. Huygens junior was an enthusiastic patron of this environment, and it is no surprise that he appreciated it. Moreover, our Dutch natural philosopher had moved away from polyphony, and preferred to cultivate the harpsichord, the lute or the flute, which were instruments to accompany monodic singing, as we shall shortly see below.

It is particularly interesting that, like Galileo Galilei, also this descendant of the Dutch culture was the son of a good amateur, almost professional, musician. He began by reading and commenting on the classic literature about musical theories, above all through the writings of M. Meibomius (1630–1711), who, among other things, had edited the most important Greek texts. Thus the Dutch natural philosopher ended up by introducing a new way of calculating the musical ratios that

[190] Huygens 1888, I pp. 342, 349–350, 352, 356, 371–372, 508–509; 1890, III p. 228.

[191] Huygens 1890, III pp. 199–200 and 252; Huygens 1899, VIII p. 241. Rasch 1992.

have accompanied us since the beginning. The procedure had never been followed so far, except by his forerunner, Mengoli.

Musicians reason above all about the intervals between notes, which are added (the fifth added to the fourth forms the octave) or subtracted (the fourth taken away from the fifth leaves the tone). Whereas, translated into proportions, they become ratios (3:2, 4:3, 2:1), and as we know, they must then, rather, be multiplied or divided. A short time before, John Napier (1550–1617), Jost Bürgi (1552–1632) and Henry Briggs (1561–1630) had been calculating the ratios of geometric successions, by first translating them into arithmetic successions, and then, instead of tiring themselves out in complex multiplications, they performed simple additions. In other words, they had invented logarithms [numbers of ratios]. The relationship with the calculation of musical ratios was so clear and so close that the French historian of mathematics Paul Samson Tannery (1843–1904) for them hypothesised an origin within music that was hidden and disguised.[192] Anyway, also Nicolaus Mercator (Niklaus Kauffman 1619–1687), not to be confused with the Gerhard Mercator (Kremer, 1512–1594) of the naval maps projected on to a plane, wrote explicitly in his book of logarithms that what was true "in Musicis" ["in the things of music"] was equally valid "in hac nostra logarithmo-technia" ["in our art of logarithms"].[193]

Now, also Christiaan Huygens started to use these new logarithms successfully, just in the field of music. For example, the ratio of unison is 1:1, whereas the distance of a sound from itself is obviously equal to 0: it is therefore correctly represented by $log1$, which is equal to 0. Sure, whatever the base a is, $log_a 1 = 0$, because $a^0 = 1$. A ratio does not change when it is multiplied by 1, which is also true for a number added to 0. In this way, calculation with logarithms brought the ratios within music again, translating them into the current reasoning of musicians, based on intuition. "Je me suis occupé pendant quelques jours à estudier la musique, et la division du monochorde à la quelle j'ay appliqué heureusement l'algebre. J'ay aussi trouvé que les logarithmes y sont de grand usage, et de la je me suis mis à considerer ces merveilleux nombres ..." ["For some days, I have been engaged in studying music and the division of the monochord, to which I have successfully applied algebra. I have also discovered that logarithms are extremely useful, and from that point, I started considering these wonderful numbers ..."].[194]

The Dutch natural philosopher was convinced that "... les hommes par toute la terre chantent par les mesmes intervalles ... et la musique devant donner du plaisir e non du chagrin elle ne pouvoit se chanter par d'autres intervalles que ceux là." ["... men sing the same intervals all over the world ... and as music must

[192]Tannery 1915 (1902) p. 71.

[193]Huygens 1940, XX p. 11. On logarithms, see Wardhaugh 2008, pp. 40–42ff. On the musical manuscripts of Nicolaus Mercator, where among other things he divided the octave into 53 parts using logarithms, see Wardhaugh 2008, pp. 47–53 and 152–154. Wardhaugh's speculation about the hypothetical use made by Descartes of logarithms for his circular figures of the musical scales, does not sound very convincing.

[194]Huygens 1890, III pp. 307–308; 1940, XX p. 12.

procure pleasure, and not sadness, it can only be sung by means of these intervals [Pythagorean consonances].”] In order to discuss what these should be, he wrote the notes using the capital initials of U[t], R[e], M[i], F[a], S[ol], L[a], C[i]. He was referring explicitly to Stevin, whose manuscript *Vande Spiegheling der Singconst*, kept in the library of his father Constantijn, he clearly knew. He used the well-known theory that the strokes of sounds coincided in the ear better when they were consonances. But he was forced to modify it, admitting also the higher octaves generated by the harmonics. Otherwise he would not have succeeded in explaining why his ears judged the major third (5:4) better than the fourth (4:3).

Thus he seemed to reject Stevin, and fall back on the current Pythagorean orthodoxy, like Galileo Galilei, Beeckman or Zarlino. “Il est constant par l’experience, et ceux qui ont tant soit peu d’oreille pour la musique ne peuvent nier, que les consonances suivant les proportions sudites ne soient tres parfaites et meilleures que quand on s’ecarte de ces veritables proportions numeriques. Et ceux qui ont osé soustenir le contraire et que la 5 ne consistast pas dans la raison de 3 à 2 ou n’avoient pas l’oreille capable d’en juger ou croioient avoir une raison pour cela, mais ils concluoi[en]t mal.” [“It is certain on the basis of experience, and those who have any ear for music however mediocre cannot deny that the consonances in accordance with the above-said ratios are not the most perfect, and better than when we move away from these true numerical ratios. Those who have dared to maintain the contrary, that the fifth does not consist of the ratio 3 to 2 either do not possess an ear capable of judging, or would believe that they give valid arguments, but conclude wrongly.”] Here, then, in the margin he put a reference to “Stevin”.

And yet he must have maintained some doubts about the classic Pythagorean numbers. “… tous les gouts ne s’accordent pas tout à fait en ce jugement. […] Car peut estre nous pourrions faire la mesme faute que les anciens.” [“… not all tastes agree with this judgement. […] Because we may be making the same mistake as the ancients [who excluded thirds and sixths from the consonances].”] He also tried to overcome the Ptolemaic-Zarlinian ‘perfection’ of the number six. “Cependant à bien examiner la chose et sans prejugé l’on trouvera que le nombre 7, comparé à d’autres, n’est pas incapable de produire une consonance, …” [“However, if we examine the question carefully and without prejudice, we will find that the number 7, compared with various others, is not incapable of producing a consonance, …”].[195]

The Hemony brothers of Antwerp had constructed famous *carillons* with bells, which Huygens junior had heard play. Another musician who influenced him was a no-better-specified organist, Mr. de Montalent, whose manuscript on the division of the monochord was known to him. Thus our Dutch mathematician now made his own regular calculations, to choose into which notes to divide the octave. Starting from Pythagorean presuppositions, the notes were generated by successive fifths, transferred back to the initial octave. But, as a practical musician of his period, he desired that every fifth on any note should sound the same, and that furthermore, succeeding one another, they should end up by arriving at exactly any octave higher.

[195] Huygens 1940, XX pp. 30–37.

Therefore he left the exact value of his starting fifth undetermined (it could no longer be equal to 3:2) in order to satisfy these conditions that were indispensable for a music that was successfully becoming "tonal". He took a string a long; the octave was thus $\frac{a}{2}$, while he placed the undetermined fifth in the ratio $\frac{x}{a}$. It is clear that to solve his little musical problem, this Dutch natural philosopher was using the new mathematical symbolism introduced by Descartes.

Following this procedure, he eventually obtained for the length of the string x the value of $\sqrt[4]{\frac{1}{5}}$, with the constraint that the major third should be equal to 5 to 4. On a length taken as 100,000, this is equal to approximately 6,687, which is thus greater than the 6,666 that derives from $\frac{2}{3}$. Consequently, the fifths should diminish "pauxillo" ["a tiny bit"] in their pitch. Using the powerful means of algebra, Huygens had re-obtained the compromise between Pythagorean theory and practice followed in that period by musicians or theoreticians like Zarlino, called the "mean tone".[196]

This way of his of dividing the octave was not appreciated by Mr. Cousin, who among other things did not make use of logarithms, and was probably the Louis Cousin (1627–1702) who at that time edited the *Journal des Sçavants*. The interlocutor maintained: "... la meilleure division doit estre celle qui est la plus conforme au temperamment de l'orgue, soit que les consonances se trouvent plus ou moins eloignées de leurs proportions arithmetiques ordinaires, toutes, ou quelques unes d'elles, ... [...] ... la raison doit recevoir la loy de l'oreille sur un instrument à clavier bien accordé, pour partager la ligne harmonique, ..." ["... the best division must be the one that conforms most closely to the temperament of the organ, whether all the consonances prove to be more or less distant from their ordinary arithmetic ratios, or only a few of them ... [...] ... reason must accept the law of the ear on a well-tuned keyboard instrument to divide the harmonic line ..."]. But, in spite of all his references to the ear and to experience, Cousin remained within a variant of rational ratios, tuning the major semitone of the organ to 16:15. In the margin of the letter, the Dutch natural philosopher commented: "Ce partage n'estoit pas bon, 1673." ["This division was not good, 1673."][197]

Christiaan Huygens believed that the ancients sang more quickly than his contemporaries, but he did not consider the modern notation of the time dedicated to notes sufficiently precise: breve, semibreve, minim [quarter], crotchet [eighth], quavers, etc. Thus he considered it necessary to determine it by means of fixed measurements "... comme sont les pendules ..." ["... as pendulums do ..."]. The organist of the Hague, Quirinus van Blankenburg (1654–1739) met him personally, and gave written confirmation of the event that effectively, Huygens junior regulated the singing with a pendulum, as he ended up by doing himself, as well. The organist later recalled also what the music teacher Etienne Loulié (16??–1702) wrote in his *Éléments ou principes de musique [Elements or principles of music]*, published in

[196]Huygens 1940, XX pp. 43–60.

[197]Huygens 1897, VII pp. 368–369.

1696, shortly after the death of Huygens, about the "clock" for music: which was nothing other than a pendulum. If this Frenchman was considered to be the first inventor of the metronome, shouldn't we compare him with Huygens? But we will let the question drop, because disputes over priorities are of no interest for us.[198] Anyway, after Galileo Galilei had measured the oscillations of the pendulum with music, now also Christiaan Huygens measured music with the pendulum, in the wake of Mersenne.

Particularly taken by music, our natural philosopher was among those who were starting to think that the major tonality (based on the major third $do - mi$) and the minor tonality (based on the minor third $la - do$) were sufficient, instead of the traditional 12 ecclesiastic or Greek modalities. In this respect, he was different from Mersenne, who, though discussing the possibility of this solution, had preferred to maintain the ancient theory.[199] Consequently, Huygens believed that the other tonalities were simply the transpositions on other notes of the major and the minor ones. This was to become the decisive reason for the definitive affirmation of the equable temperament, with the modulations between the tonalities.

And yet, also in the division of the octave previously calculated by him, every now and then the "correct" ratios were not found; while the addition of other notes would not have been sufficient for him.[200] With its complications, the Dutchman's manuscript well represented the reason why musicians preferred the equable temperament, which would completely dissolve the problem. The voice could adjust things by choosing from time to time the suitable different notes in the *continuum* of the vocal cords, but organs and harpsichords with a fixed tuning were not able to do the same. Anyway, also for our natural philosopher, transpositions on the different tonalities seemed to be necessary. "Les tons transposez servent encore aux compositeurs pour faire plus de variété, …" ["The transposed tonalities serve so that composers can obtain an even greater variety, …"]. Regarding the whole question, Huygens did not fail to read and comment on the other theoreticians present in the age, such as Ptolemy, Giovanni Maria Artusi (circa 1545–1613) or the usual Zarlino.

In contrast with the opinion of Francisco Salinas (1513–1590) and accepting that of Wallis, Huygens believed that the ancient Greeks did not sing polyphonically, because neither the division of the octave into 12 semitones, nor that of Ptolemy, in his opinion, would be suitable for this purpose. As regards the famous, well-known ability of Flemish musicians in the field of polyphony, this Dutchman even knew the opinion of Lodovico Guicciardini (1523–1589), who had written in a book of his dedicated to the Netherlands: "Questi sono i veri maestri della Musica & quelli che l'hanno restaurata & ridotta a perfettione …" ["… They are truly masters of music, those who have restored it and reduced it to perfection …"].[201]

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[198] Huygens 1940, XX pp. 68–69; 1901, IX p. 567.

[199] Mersenne 1636, [tr. 4] p. 187. Huygens 1940, XX pp. 69–76.

[200] In the edition, this escaped the notice of the editors of the page, by their own admission.

[201] Huygens 1940, XX pp. 78–81.

He read, annotated and commented on the ancient theoreticians, whose printed editions circulated at that time: Marcus Meibomius, *Antiquae Musicae Auctores Septem*, published at Amsterdam in 1652 apud Ludovicum Elzevirium; or the version edited by Wallis for the *APMONIKA* of Ptolemy. The major third 10:8 (5:4) was divided into a major tone 9:8 and a minor tone 10:9. "At secundum divisionem Temperamenti intervallum tertiae majoris, quod est 10 ad 8 seu 5 ad 4, dividitur in duos tonos equales qui sunt rationis 5 ad $\sqrt{20}$. Quae ratio quanquam non sit numero explicabilis hoc nihil refert, quia de intervallo consono non agitur." ["But on the contrary, according to the division of the temperament, the interval of the major third, which is equal to 10 to 8, or 5 to 4, is divided into two equal tones, which stand in the ratio 5 to $\sqrt{20}$ [5:$\sqrt{20} = \sqrt{20}$:4]. Although this ratio cannot be expressed by means of a [rational] number, it does not matter, because it is not a consonant interval."]

Huygens sought a 'good' note to use as the physical unit of the others. "Definitio toni certa ac constans ex longitudine fistule seu cylindri cavi, cujus sonus semper idem qualicunque sit crassitudine, saltem si non major ea fuerit quam pars decima vel circiter longitudinis." ["The establishment of the note in a certain, constant manner, with the length of a pipe, or a hollow cylinder, whose sound is always identical with any breadth, at least if this is not greater than approximately one tenth of the length."][202]

Did the Dutch natural philosopher ever reflect, therefore, on how to tune pipes? Maybe. He left us a figure (Fig. 9.10) with the "Tons de ma flute" ["Notes of my flute"]. But he does not seem to have given any mathematical theory about how to tune it, nor did he show any signs of knowing about the proposal of Vincenzio Galilei. In the page, the numbers referred to the fingers used to close up the finger-holes of the instrument. Yet he must have tried to carry out some experiments personally. "Si le ton d'une flute change lorsque l'air est plus ou moins comprimé. En sorte que le ton est plus bas dans l'air moins comprimé. Non. [...] Si en sufflant a l'embouchure d'un creux fait de deux cylindres d'inegale grosseur, le ton est le mesme qu'il seroit dans un cylindre qui seroit par tout aussi menu que celuy ou l'on souffle et d'egale capacité avec ces deux inegaux (je ne suppose d'ouverture qu'a l'embouchure). Ouy." ["Whether the note of a flute changes when the air is more or less compressed. With the result that the note is lower with the air less compressed. No. [...] Whether, blowing into the mouthpiece of a cavity formed by two cylinders of different sizes, the note is the same that there would be inside a cylinder that was wholly of the thinness of the one where one blows, with a capacity equal to those of the two different cylinders (I suppose only one opening at the mouthpiece). Yes."][203] As the main factor that determines the pitch of the sound, here, the Dutchman seems to indicate the volume: cylinders "of equal capacity". The Tuscan had done the same

[202] Huygens 1940, XX p. 91. As this was clearly not a case of equal tones like those of Aristoxenus, the footnote on the Greek musician placed at this point by the editor of the edition appears to be inappropriate.

[203] Huygens 1940, XX p. 104; 1937, XIX p. 377.

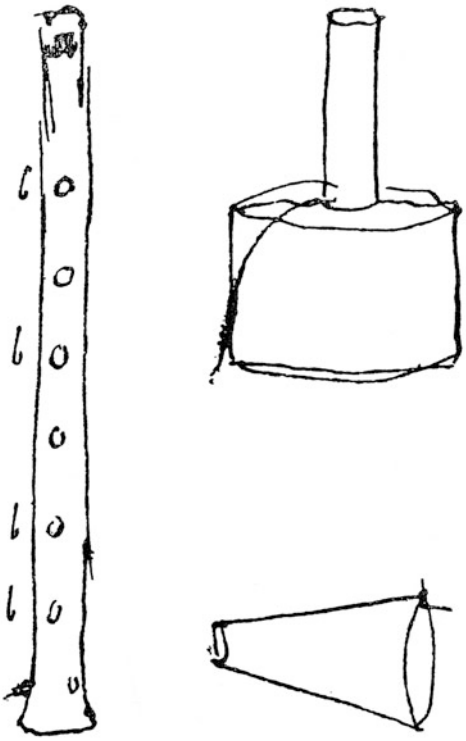


Fig. 9.10 Huygens's flute (Huygens 1937, vol. XIX, p. 377)

with his cubic proportions, which now proved to be confirmed by the observations of Huygens.

Aristoxenus was criticised by our Dutchman, because "Ipse nihil definit" ["He himself does not define anything"], but he was read with attention. "Aristox.[enus] lib. 2 pag. 46 multos deceptos fuisse ait quod putarent ipsum dicere tonum in 4 aequalia divisum cani." ["Aristoxenus book 2 pag. 46 [ed. Meibomius] says that many were deceived because they believed he himself had maintained that the tone divided into 4 equal parts were sung."] In actual fact, the Greek musician had written: "In the first place, this essential detail must not be ignored, which has already caused many people to err, because they believed we were saying that it is possible to perform the interval of a tone, dividing it into three equal parts. They have fallen into error, because they did not observe that it is one thing to use the third part of a tone, and another thing to perform the interval of a tone, dividing it into three parts." The natural philosopher of the seventeenth century took various items of information from the ancient musician, maltreated by the mathematical theoreticians, e.g. about the 40-stringed instrument invented by Epigonius, or about the diatonic genres.[204]

[204]Huygens 1940, XX pp. 94–97. Aristoxenus 1954, p. 67.

Of course, what he apprehended from Euclid or Ptolemy was quite different. Surely he realised that putting together the intervals considered "just" by Ptolemy or Zarlino, he exceeded the octave 2:1 by the *comma* of Didymus 81:80. "Nempe vel ipso temperamento, quod adhiberi solet, vox utitur vel paulum diverso, sed quod idem efficiat tamen. Itaque non canuntur intervalla consonantia perfecta." ["In other words, the voice is used either in the same temperament which is usually emploied, or else in one that is slightly different, but which, however, produces the same result. And thus the intervals are not sung in perfect consonance."] Lastly, the Dutchman took the liberty of contradicting Wallis, when the Englishman maintained that the intervals 9:8 and 10:9 were more pleasant than those composed of larger numbers, like the classic Pythagorean 256:243. "Nam nec istae proportiones quidquam auribus gratum consonant." ["Actually, these proportions do not sound pleasing to the ears either."][205]

The judgements formulated by Christiaan Huygens on the theoreticians of music, who were almost his contemporaries, or not too much older, were always careful. Among these, the most interesting criticisms were those reserved for Salinas, who had commented on the new Italian harpsichord, as a result of its "prava constitutione" ["bad construction"]. Invented by Nicola Vicentino (1511–1572), this instrument had 6 keyboards and 125 keys, dividing the octave into 31 intervals in order to distinguish the sharps from the flats, and thus to play also the enharmonic genres, with their thirds and quarters of a tone. The Protestant Dutchman rebuked the Catholic Spaniard for calculating a theory for it which did not divide the tone into exactly five equal parts, as he desired. He had not even carried out good experiments to verify it; in spite of the event that Salinas wrote he had found so many imperfections in the consonances of the Italian instrument that his ears could not stand them. "Hanc tamen divisionem, atque hoc temperamentum optimum esse nec sensibiliter ab illo? quod in usu est, differre, ostendimus inventa per Logarithmos vera divisione octavae in 31 partes aequales; … vult in violis semitonia omnia esse aequalia, ut olim puto Aristoxenus. Sed nihil veta quin efficiantur inaequalia ut in cymbalis nostris et organis, … ut vera semitonia efficiant." ["However, having found by means of logarithms the true division of the octave into 31 equal parts, we show that this division and this temperament are excellent, and do not differ substantially from those in use; … [Salinas] wants all the semitones on violas to be equal, as I believe Aristoxenus once [did]. But nothing forbids them from being generated unequal, as in our harpsichords and organs, …, so that they produce true semitones."]

And Zarlino was not treated much better either, as regards the problem that had become central of the temperament. "… demonstrare contendit aliud temperamentum tolerabile non dari, et improbat illud quod tono majori adimit $\frac{1}{2}$ comma ac tantundem tono minori addit. Quod tamen verum atque optimum est temperamentum." ["… he strains himself to demonstrate that the other temperament

[205]Huygens 1940, XX pp, 98–102. If we take away the major third 5:4 from the fourth 4:3, instead of subtracting the Pythagorean ditone 81:64, what is left is a diesis of 16:15.

were not tolerable, and rejects the one which takes away half a *comma* from the major tone, while it adds the same amount to the minor one. However this is the true, and excellent temperament."] Huygens concluded. "Mon opinion est que le Temperament veritable a esté longtemps auparavant pratiqué par les organistes &c. seulement à l'ouie en diminuant un peu les quintes, sans examiner aucunement la proportion de cette diminution qui n'appartenoit qu'aux geometres." ["My opinion is that the true temperament has already been practised for a long time by organists, etc., only by ear, diminishing the fifths a little, without examining the ratio of this decrease at all, which is a thing that belongs to geometricians."] Yet Huygens, too, was to recognize that in the *Dimostrazioni Harmoniche [Harmonic Demonstrations]*, the Venetian theoretician had taught "… le bon et veritable temperament …" ["… the true, good temperament …"]. For him, then, this is what is described above as the 'mean tone'.[206]

The division of the tone into three equal parts, performed by the organist Titelouze, whom we have already seen with Mersenne, was of no value for him. Through the Minim friar, he recorded: "Aristoxene divisoit l'octave en 12 demitons egaux, ce que Vincent Galilée maintient estre la meilleure division. Mersenne s'en sert pour le luth." ["Aristoxenus would divide the octave into 12 equal semitones, which Vincenzio Galilei considers the best division. Mersenne uses this for the lute."] Huygens followed Mersenne in [badly] tuning organ pipes of the same length, varying only their diameter. But when the French natural philosopher was to sustain the equable temperament for organs, the judgement of the Dutchman was *trenchant*. "Que pour cela il faut faire les 12 demitons egaux comme au luth. Mal!" ["Is this the reason why it is necessary to make the 12 semitones equal, as on the lute? Too bad!"][207]

While noting down various considerations on the *Musurgia Universalis* by Athanasius Kircher, our natural philosopher at last abandoned himself to judgements about music proper. "Vellem etiam ut non quae artificiosissima sunt et inventu difficilia sectarentur melopoei nostri sed quae aures maximè afficerent. … Artifices istis se delectari dicunt, non tam dulcedine concentus affectos quam consideratione artificiosae compositionis; qui non recte hoc pacto melopoeam aestimant, cujus finis est delectare sono quem auribus percipimus, non contemplatione artis. … Ars autem nisi naturam moveat ac delectet nihil sane egisse videatur, ut Cicero inquit. … Quamobrem nec praeceptis ita considere debent ac si geometriae axiomata essent, sed multas exceptiones dari existiment semperque fixum illud teneant, propositum delectationem adferre auribus prius quam artis examen instituatur." ["I would like our melodists [sic! Ironic?] to choose to have contact gladly, not with those [compositions] which are extremely artificial and difficult to find, but those which were to impress the ears most. … Artists say that they take delight in these [fugues], not so much captivated by the sweetness of the singing, as in consideration of an artificial composition; they do not have any good esteem of the melody, the

[206]Huygens 1940, XX pp. 113–117.

[207]Huygens 1940, XX pp. 120–122.

purpose of which is to delight with the sound perceived by the ears, and not with the contemplation of the artifice. … Instead, if art does not move the nature [of people], or give delight, it is considered not to have achieved anything, as Cicero says. … For this reason, they [musicians] should not stop at the precepts yet, as if they were axioms of geometry, but let them grant many exceptions, and let them hold fast to this: the intention to bring delight to the ears, before the art examination begins."]

On another page of the manuscript, Huygens wrote down the name of Ludovico [Grossi from] Viadana (1564–1645), who was famous for having based his new *concerto* style on the continuous bass. Perhaps he appreciated that action in the orchestra: in which some instruments assume a melodic significance and importance compared with others which remain in the background, while the repeated chords of the harpsichord can be heard in the bass. Subsequently, the Dutchman again made a comment. "Qu'il est assez disputable si les musiciens modernes avec leur contrepoint figuré ou l'on prononce des paroles differentes en mesme temps, sont dans le bon chemin." ["It is highly debatable whether modern musicians, with their figurative counterpoint, where different words are pronounced at the same time, are on the right road."][208]

Thus, our eminent natural philosopher, so keen on music, was no longer fond of the artistic, but also artificial, polyphonic constructions of his Flemish ancestors. He was probably realising, beyond his musical tastes and better than Kepler, that musicians had started to move in other directions. One possible direction had been offered by Vincenzio Galilei, with his "recitar cantando" ["reciting in songs"] of his Florentine friends, who had largely abandoned polyphony. And yet, Christiaan Huygens did not seem to be ready to follow the new theoretical approach, decidedly favourable to the equable temperament. Was he perhaps seeking a compromise with the classical ratios of Pythagoreanism?

In any case, he did not let escape the work that appeared under his nose: that written by the scholar who, rightly or wrongly among the historians of music, was to be considered to begin the revolution of the equable temperament: the German organist, Andreas Werckmeister (1645–1706). The Dutchman commented on his *Musicalische Temperatur* of 1691, which claimed to "… wol temperirt stimmen können …" ["… be able to tune well-tempered …"]. But he considered him an "… author ineruditus ac parvi pretij; … improbat Temperamentum optimum quo quintae diminuuntur $\frac{1}{4}$ commatis." ["… ignorant author of little value; … he rejects the best temperament because the fifths are diminished by $\frac{1}{4}$ of a *comma*."] Recalculating with the help of his logarithms, Huygens was subsequently also to comment "… plus quam $\frac{3}{5}$ commatis haec diapente deficit." ["… this fifth lacks more than $\frac{3}{5}$ of a *comma*."]

However, Werckmeister performed his calculations instead without extracting roots, and always with whole numbers, proposing, after the abolition of the previous enharmonic differences between sharps and flats, the succession given also by the natural philosopher:

[208] Huygens 1940, XX pp. 125–126, 129 and 133.

$$196\ 186\ 176\ 165\ 156\ 147\ 139\ 131\ 124\ 117\ 110\ 104\ 98\ 93\ 88\ 82\tfrac{1}{2}\ 78\ 73\tfrac{1}{2}\ 69\tfrac{1}{2}\ 65\tfrac{1}{2}$$

Do Do♯ Re Re♯ Mi Fa Fa♯ Sol Sol♯ La Si♭ Si do do♯ re re♯ mi fa fa♯ sol

This is clearly only an approximation of the succession generated by $\sqrt[12]{2}$, but not too distant, all told after calculating.[209] Within a short time, musicians were to abolish all those *commas* which were impossible to reconcile with the ear, but which mathematicians still appreciated, and which justified their interest, as we see.

In the end, stimulated by all these theories which were widely criticised, but also in the desire to maintain the pleasure of music, Christiaan Huygens proposed his "*Cyclus Harmonicus - περικυκλωσις*" ["Harmonic cycle - cyclicity/revolution"]. This was published as a letter in 1691 by H. Basnage de Beauval in the journal *Histoire des Ouvrages des Sçavans*.[210] However, the proposal outlined by the Dutchman, for a temperament that divided the octave into 31 equal intervals, had been preceded, over the years, by calculations, various considerations, and notes, through which we can follow the passages. He explained the title himself: "... il constitue un parfait Cycle Harmonique, en ce qu'en y montant ou descendant tout de suite par l'intervalle de quinte ou quelqu'autre que ce soit, on revient apres certaine revolution à la chorde d'ou l'on a commencé." ["... it constitutes a perfect harmonic cycle, seeing that going up or going down successively by the interval of a fifth, or any other, after a certain revolution, we return to the chord from which we started."] Whereas, on the contrary, in all those divisions of the octave that were variants of the ancient Pythagoreanism, the others were always left with the well-known *commas*, which made it impossible to return exactly to the octave.

Our natural philosopher was to link to this temperament of his the appropriate mathematical procedure based on logarithms, because these (thanks to the patient work of those who had compiled the tables) simplified the calculations and made them precise without extracting any roots. Therefore, the note on the "Divisio octavae in 31 intervalla aequalia (per logarithmos)" ["Division of the octave into 31 equal intervals with logarithms"] must probably have gone back to the above-mentioned letter of 1661, where he praised logarithms to the members of the English Royal Society. From other sheets, we can learn the names of those from whom he took his inspiration for the problem of the temperament. "Zarlin et Salinas s'en disputent l'invention. Trouvé par experience, puis la raison. ... Mersenne et Salinas le condamnent. ... Mais cette connaisance de la *περικυκλωσις* donne moyen de faire un clavecin ou orgue avec le clavier mobile sur les batons d'egale largeure et 31 dans l'octave, lequel clavier sert à transposer avec facilité ... Contre Stevin. L'octave in 31 parties que donne Mersenne prop. 10 des Genres de musique n'est pas la nostre, et ne sont ses parties aucunement egales." ["Zarlino and Salinas dispute

[209] Huygens 1940, XX pp. 133–135. The difference from the equable temperament is explained in Cohen 1984, p. 216.

[210] Huygens 1905, X pp. 169–174.

its invention. Having found [the temperament] by means of experience, and then by means of [mathematical] reasoning … Mersenne and Salinas condemn it. … But this knowledge of cyclicity/revolution makes it possible to construct a harpsichord or an organ with a mobile keyboard on keys of equal breadth, 31 per octave, which keyboard serves to transpose easily … In contrast with Stevin. The octave in 31 parts that Mersenne gives, prop. 10 of the musical genres, is not ours, and its parts are not equal at all …"].[211]

Thus the Dutch natural philosopher wanted to construct the theory of what musicians already did in practice. He was not satisfied with the theories so far proposed by Zarlino, Salinas, Stevin and Mersenne. As regards the harpsichord with 31 keys per octave, Salinas had not revealed to him the name of Nicola Vicentino, according to whom also the enharmonic genres could be played more easily. Zarlino had written, it is true, about 12 equal semitones, but he had not provided any numerical values for them. Stevin, instead, had calculated them; and yet this kind of temperament was refused by his compatriot, who evidently felt that the equable temperament in 12 semitones must have sounded too revolutionary, seeing that it cancelled all the privileges reserved for the rational ratios. Huygens, on the contrary, wanted to remain as close as possible to them with the temperament of the mean tone, maintaining with his different, but not too different, temperament in 31 parts, a compromise with the current practices of musicians. Actually, he offered it as a less radical temperament, because it would only be a better approximation to those ratios that he considered to be 'true', fixed by tradition with rational numbers. In reality, though forced to abolish them by his own ears, and by those of musicians, Huygens still reasoned enough with *commas* in the classic Pythagorean style. In these pages, it may be noted that Vincenzio Galilei stood out for his absence.

Our music-lover and natural philosopher was particularly interested in maintaining the balance between different requirements, which he tried to reconcile. He felt that he was the heir of the long Pythagorean-Greek tradition, which he preserved without refusing it like Vincenzio Galilei and Simon Stevin, precisely because he modified it just enough to harmonise it with the musicians of his period. He himself, using logarithms, compared his own temperament with the "ordinaire" ["ordinary"] one of the mean tone, and concluded that it offered some advantages. Besides the one already mentioned, that of closing the series of fifths on the octave, it permitted transpositions of the major and minor tonalities on the keys of the scale.

And yet Huygens realised that there were not enough musical instruments to use it freely. For this reason, he even engaged himself in practical questions, inventing a "clavecin" ["harpsichord"] and a "clavier mobile" ["mobile keyboard"]. "… mon invention du clavecin a tresbien reussi, et que je ne m'en passerois pas pour beaucoup de chose." ["… my invention of the harpsichord has been very successful, and may I not pass it off as something important."][212] "J'ay fait autrefois ajuster de

[211] Huygens 1890, III pp. 307–308; 1940, XX pp. 160, 168–169. Mersenne 1636, [tr. 1] p. [vi]; [tr. 4] p. 132. See above, Sect. 9.1, Table 9.1.

[212] Huygens 1895, VI pp. 473, 484 and 515.

tels clavier mobiles à des clavecins estant à Paris, et mesme à ceux qui avoient leur clavier ordinaire où il faloit que celuy que je mettois par dessus egalast en mesme temps les hauteurs des touches et des feintes à fin que les touches pussent glisser sans empeschement. Et cette invention fut approuvée et imitée par des grands maitres qui y trouvoient de la commodité et du plaisir." ['I have had such mobile keyboards adapted differently to some harpsichords existing in Paris, and also to those that had their ordinary keyboard, where it was necessary that what I put on top of it should be equal at the same time to the heights of the keys and the fissures, so that the keys could slide without any impediment. And this invention was approved and imitated by some great masters, who find convenience and pleasure in it."]

Huygens even believed that he could go so far as to consider as a consonance the ill-famed dissonant *diabolus in musica [devil in music]*, that is to say, the tritone. It is curious that he continued to refer to the theory of the coincidence of strokes in the ear in order to justify consonances, as if even his own tempered system were not totally incompatible with it, strictly speaking, and did not demonstrate that it was untenable. "… parce que plus les consonaces sont parfaites, c'est à dire plus leurs tremblements s'unissent souvent, et moins l'oreille leur souffre d'alteration. […] ce sont les battements ou tremblements qu'elles causent dans l'air, qui vienent à s'unir souvent et reglement, et que d'autant plus frequentes que sont ces unions, d'autant plus la consonance est censée parfaite, ou du moins eminente en dignité." ["… because, the more perfect the consonances are, that is to say, the more often their vibrations combine, the less the ear suffers as a result of their alteration. […] They are the strokes or vibrations caused in the air which come to unite often, and with regularity, with the result that the more frequent these unions are, the more perfect the consonance is judged to be, or at least eminent in dignity."] Anyway, he admitted as judge the ear, which would not be able to distinguish his new tuning with all its advantages from the current practices.[213]

Among the musicians inclined to accept the theory, the requirements of the new music had led to an increase in the number of those who followed the equable temperament or the mean tone. With Huygens, also the number of natural philosophers interested in it was growing. For the Dutchman, the impulse had not only been Mersenne, but also Aristoxenus, Vincenzio Galilei, the harpsichord of Nicola Vicentino, Zarlino and Salinas. Readers interested in competitions can choose for themselves which champion to make the winner, distinguishing, when necessary: the categories of those who tempered by ear or by means of numbers, extracting roots or calculating logarithms, who approximated to the third figure or arrived at the seventh decimal, who got his calculations wrong or performed them correctly. To arrive at merciless, or presumedly decisive matches, tables could be compiled, descending from Mersenne to Frenchmen like Beaugrand, Boulliau and Gallé, but (of course?) omitting the Chinese Ming prince, Zhu Zaiyu.[214]

[213]Huygens 1940, XX pp. 161, 159 and 162.

[214]Huygens 1940, XX pp. 171–173. See Sect. 8.1.

Huygens father had made Christiaan learn many ancient and modern languages, including a little Italian, but among them, the favourite language of the Huygens family became French. In this gallant tongue, our Dutch natural philosopher listed 25 rules to accompany songs (on the harpsichord?):
"5. Il ne faut pas escarter beaucoup la main droite de la gauche, en accompagnant.
6. Quand le chante monte haut, il est bon d'accompagner vers le haut du clavier."
["5. Do not separate the right hand too far from the left hand, when accompanying.
6. When the song rises up, it is good to accompany it towards the top end of the keyboard."] Based on the testimony of his father, we can be relatively certain that he himself sang, accompanying himself on the harpsichord (Fig. 9.11).[215]

Although he preferred to reason about the most practical lengths of strings, even if he rebuked Mersenne for not wellcoming the temperament in 31 equal parts, and for not succeeding in calculating it correctly, either, Huygens accepted monk's idea that sound was generated by vibrations. But were these vibrations of waves in the air? Curiously, the Dutchman attributed to Galileo Galilei the earliest experiments to establish the numbers of vibrations, instead of to Beeckman and Mersenne. These vibrations, in his opinion, were transformed into orderly, commensurable successions of strokes. By attaching weights to it, our natural philosopher seems to have succeeded in perceiving the vibrations of a string. For the speed of sound, Huygens gave the value of 351 m/s, which was lower than the measurement of Mersenne, and therefore better. Even more precisely, he quoted that of a book that had been published in Florence.[216]

By making wheels with different circumferences rotate (Fig. 9.12), he succeeded in measuring the vibrations of an "axiculus" ["spline"] exactly. With a pendulum, he controlled that the large wheel revolved once during a single oscillation. In the same period of time, the "spline" would accomplish "609" or "547". His ear judged that the note emitted was the same "... clavecymbali mei ..." ["... as that of my harpsichord ..."]: a *Re*. Thus he had measured its frequency. Then he divided the speed of sound, taken this time as 1,080 ft/s [about 339 m/s], by 547, obtaining approximately 2 ft. What may this number have represented in the head of our Dutchman?[217] A modern student of physics, taking the formula $\lambda = \frac{v}{\nu}$ from his manual, would answer that figure of 2 ft would correspond to the wavelength of the sound. But the conclusion would be too hasty and anachronistic, unless we found in Huygens a clear, well defended image of sound as a wave. And yet his use of the theory of strokes for consonances makes this notion difficult with him, and brings him closer, rather, to the sonorous atomism of Beeckman. Actually, the former thought the means that transported sounds, like the various ethers imagined for light, magnetism, or gravity, was composed of particles. After God had created it, the world had become for him a game of collisions between very hard marbles.[218]

[215]Huygens 1940, XX pp. 559–561; 1937, XIX p. 356.
[216]Huygens 1937, XIX pp. 364–373.
[217]Huygens 1937, XIX pp. 375–376.
[218]Huygens 1937, XIX p. 351. Resnick & Halliday 1961, p. 403. Huygens 1950, XXII p. 16.

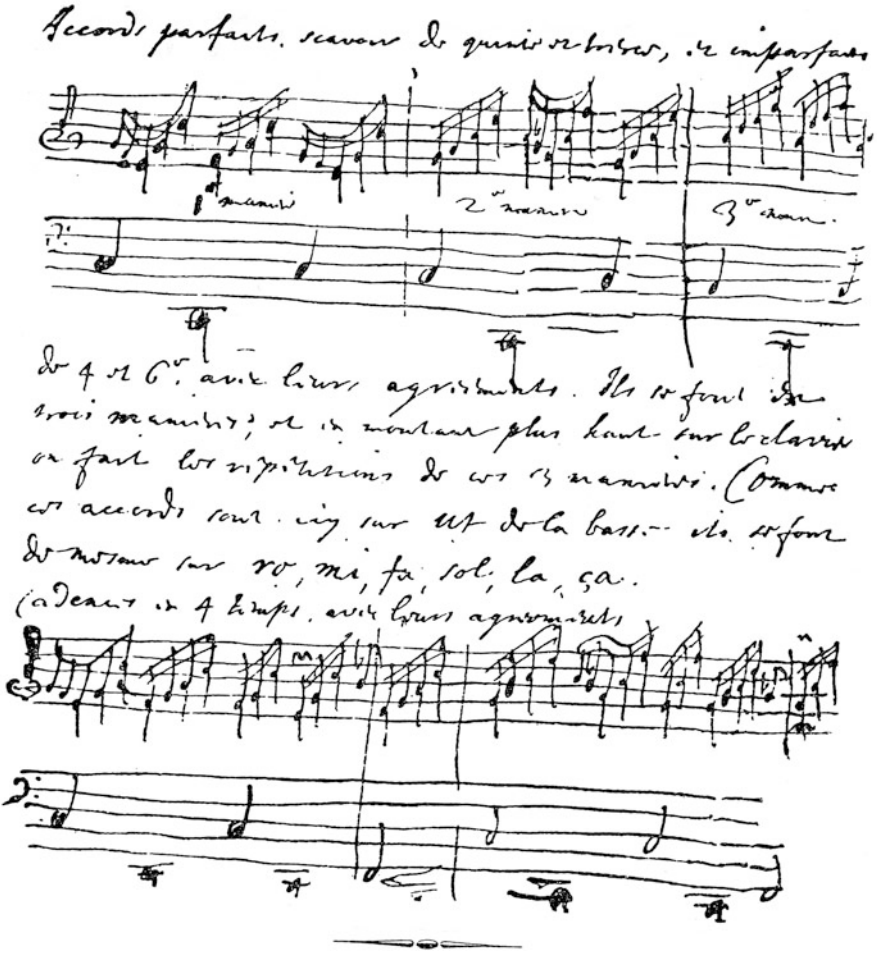


Fig. 9.11 How Huygens accompanied his singing on the harpsichord (Huygens 1940, ibid., vol. XX, p. 561)

Sounds returned again also in one of the most famous pages written by this Dutch natural philosopher, the one about the isochronism of oscillations along a cycloid: the subject was developed, comparing them with those of a vibrating string, stretched by a weight K. For this, the figure was the same as we have already seen in Mersenne and Beeckman, (Fig. 9.7). Thus he arrived at the composition of formulas for the period of the oscillations using $\sqrt{2a}$, in which $2a$ was the length. From here, we shall see, the considerations of Leibniz and Newton about the periodic movements of bodies nowadays called 'harmonic motions' were to originate.[219]

[219]Huygens 1934, XVIII pp. 489–492.

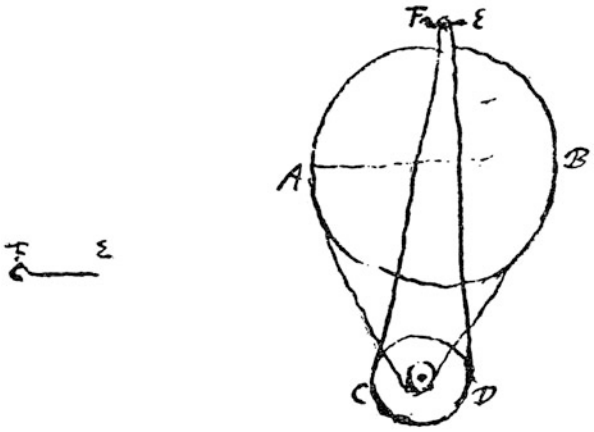


Fig. 9.12 How Huygens measured sound vibrations, comparing them with musical notes (Huygens, ibid., vol. XIX, p. 375)

Even in the *Cosmotheoros [The observer of the cosmos]*, the last work, offered to the elder brother, Constantijn, his companion of so many musical expeditions, Christiaan re-examined the art of sounds. Like geometry, it was to demonstrate that "… est veritas in Saturno, ac Jove, atque apud nos, & ex iisdem simplicissimis principiis pendens, …" ["… on Saturn and Jupiter, as among us, the truth is dependent on the same extremely simple principles, …"]. The same ratios for intervals between notes would be valid up there, as well, though the ways of singing may vary as they do between the French, the Italians and the Persians. Better than Descartes, the inhabitants of those planets would probably also have discovered the "… veriorem causam … ac si repente toni statum immutemus; …" ["… truest cause … as if we suddenly changed the tonality; …"] of perceiving as unpleasant the putting one fifth after another. Saturnians and Jupiterians could have confirmed the necessity of dividing the octave into 31 equal parts, so that the series of intervals would close the circle on the same note, thus avoiding the *comma*. "Itaque cogimur, occulto quodam temperamento uti, intervallaque ista canere imperfecta; …" ["And thus we are forced to use a certain hidden temperament, and to sing imperfect intervals; …"].[220]

The musical gifts of Christiaan were also invoked by Descartes, in the dispute between Ban and Boësset seen above.[221] Admitting, together with Constantijn senior, that he could not sing, nor could he judge the singing of others, the philosopher of Method conjectured "… touchant un suiet qui ne depend que du iugement de l'oreille. Et si M. vostre second fils se veut exercer en cete matiere, il en peut avoir occasion en nous reprenant Bannius et moy et mostrant que nous n'y

[220]Huygens 1944, XXI pp. 751–755.

[221]Remember Sect. 9.2. On Ban, see above all Rasch 1992, pp. 192–196.

avons rien entendu ny l'un ny l'autre. Car nos raisons n'estant ny mathematiques ny physiques mais seulement morales, comme i'ay dit là, il est aysé d'en trouver d'autres qui leur soient contraires. S'il escrit quelque chose sur ce suiet, ie seray bien ayse de le voir . . . " ["... regarding a subject which depends only on the judgement of the ear. And if your second-born son [Christiaan] wished to exercise himself in this matter, he might have an occasion to do so, reprimanding Ban and myself, showing that we have not understood a thing, either of us. Because as our arguments are not about mathematics or physics, but only of a moral nature, as I said there, it is easy to find others that are contrary. If he writes something on this subject, I would be glad to see it . . . "].[222]

Like the Kepler of the *Dream*, with the *Cosmotheoros* the force of imagination displayed by Christiaan Huygens had gone beyond all possible limits. He shifted those of his period by grinding lenses for telescopes with the help of his brother. Actually, he succeeded in seeing the rings of Saturn.[223] The doubts that Descartes wanted to impose on others, in order that his own clear and distinct truths might be better made free, were substituted by Huygens with greater or lesser probabilities. He did not consider to be null the chances that the planets and the stars were inhabited, like the Earth.[224]

Unlike Galileo Galilei and Descartes, who had aspired to construct very general systems of natural philosophy, the Dutchman concentrated his talents on circumscribed, limited environments with a practical significance for his historical context. Thus for them he obtained results which to a certain extent were relatively more precise. He perfected telescopes and microscopes with a greater attention than the other two. He also constructed cycloidal pendulum clocks with greater success. His letters exchanged with the Englishman R. Moray of the Royal Society revealed the interest of our skilful, cultured Dutch natural philosopher in solving the problem of calculating the longitude when out at sea, by projecting precision clocks. The question was discussed in his correspondence both with the English and with the French, and even with the famous minister of Louis XIV, Jean Baptiste Colbèrt (1619–1683). His contraptions based on a pendulum moved by weights, which were offered as better than those with a spring mechanism, were effectively used at sea.

How would it be possible for us not to notice that music returned here, too? "... je dis de ce pendule dont les vibrations sont egales à celles de la chorde ... Je ne doubte pas qu'apres ces experiences des chordes, vous ne veniez aussi aux vibrations des ressorts, et aux tons des corps durs qui sonnent. En ceux qui procedent de la division du monochorde, il y a beaucoup de belles remarques à faire, dont on en trouve quelques unes dans les livres du Pere Mersenne, mais je ne scay si vous en estes encore à la musique. Le Temperament qu'on a trouvé aux quintes est une tres belle invention à mon avis, et touchant la quelle je pourrois vous communiquer quelque chose que j'en ay escrit." ["... I mean this pendulum, the vibrations of

[222]Huygens 1950, XXII p. 53. On the death of Ban, see Huygens 1889, II p. 547.

[223]Huygens 1889, II p. 510; 1944, XXI p. 788.

[224]Huygens 1944, XXI pp. 539–559.

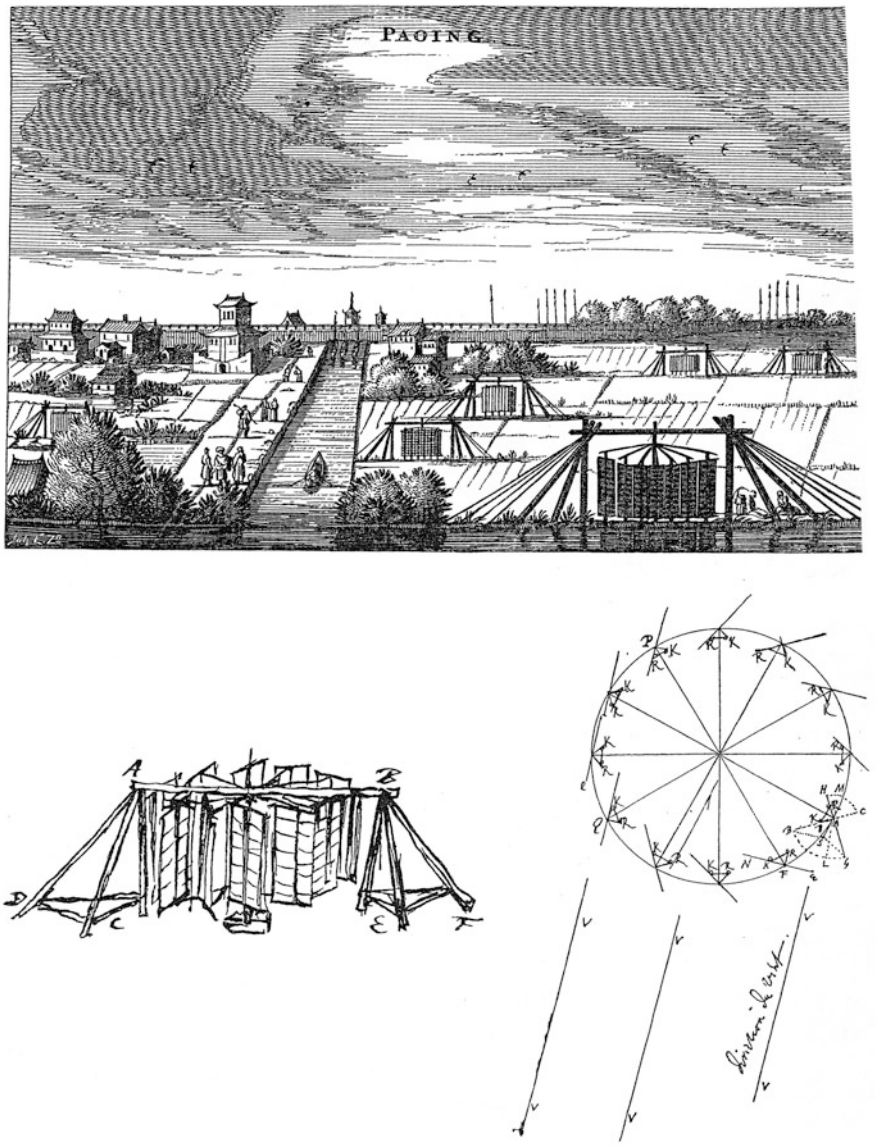
which are equal to those of the string ... I do not doubt that, after these experiments with strings, you will also arrive at the vibrations of springs [elastic bodies] and at the notes of solid bodies capable of resounding. As regards those that derive from the division of the monochord, there would be many fine observations to be made, some of which can be found in the books of Father Mersenne; but in this connection, I do not know whether you have arrived at music yet. The temperament found for fifths, in my opinion, is very good, and I could let you have what I have written about it."] At a meeting of the Royal Society, Huygens's proposal was actually discussed.[225]

Undoubtedly stimulated by the successes achieved, and by the obstacles encountered by Dutch merchants and ships in the Far East, Huygens also left us notes about China. The Huygens family was acquainted with the *Histoire Universelle du grand Royaume de la Chine [Universal history of the great kingdom of China]* written by the Jesuit Alvarez Semedo (1585–1658) and recommended by Mersenne. The Huygens brothers showed interest in certain "Figures Chinoises" ["Chinese Figures"]. With other interested parties, they exchanged information "Ex itinerario Chinensi Hollandorum Anno 1656/57" ["From the Chinese voyage of the Dutch in the years 1656/57"]. Among various things, historical details which are inexact and unreliable, storehouses full of goods and a large "bad-sounding" bell, we again come across our acquaintance, Adam Schall. Believing his report, this Jesuit would seem to enjoy great freedom in China, thanks to astronomy. The Dutch did not have it so much. Remembering Sect. 8.2, the reader knows that in reality, things were a bit different.

Christiaan Huygens also took an interest in a Chinese windmill which was different from the traditional Dutch kind. He designed and made notes for the project of a "Moulin de la Chine ... avec des ailes perpendiculaires" ["Chinese windmill ... with perpendicular vanes"], as had been narrated to him by a Frenchman (Fig. 9.13). The natural philosopher commented: "... en effect la machine est tres jolie, et sera utile selon toute apparence, puis qu'elle l'est dans la Chine." ["... effectively, this device is very pretty, and by all appearances will be very useful, because it is such in China."]

Among all the characters that we have met so far, this Dutch natural philosopher was without doubt the one who had shown most interest in China. He also lived in the European country which had good access to China, thanks to the Dutch East India Company. And yet, also in this case, it appears that little or no information of a scientific nature passed. There were described the cities of Nanjing [Nanking], Beijing [Peking] or the Great Wall; Dutch business was discussed, The Tartars and the Jesuits were mentioned. Unlike the other foreigners, the Jesuits enjoyed a greater freedom: so much so "... dat sij inde Sterrekunde ende den loop des hemels wel ervaren sijn, daer in dese Keyser en al sijne grooten een sonderlingh vermaeck scheppen, en haer selven oeffenen." ["... that they are very well versed in astronomy

[225]Huygens 1893, V pp. 63, 98–105 and 346; 1895, VI pp. 485, 500 and 518; 1897, VII pp. 313–314; 1899, VIII pp. 239 and 241; 1901, IX pp. 20, 230 and 568; 1905, X p. 598. Boyer 1990, pp. 431–436. Cf. Bos 1981, where music is unfortunately ignored.



MOULINS DE LA CHINE.

Fig. 9.13 Huygens took interest in a Chinese windmill with perpendicular vanes (Huygens 1897, vol. VII , pp. 108/109)

and the courses of heavenly bodies, in which the Emperor and all his important men take great pleasure, and exercise themselves."] But, at least here, as regards what Chinese astronomy consisted of, Huygens remained in the dark.[226]

The name of Christiaan Huygens further lengthened the list of the famous natural philosophers who, like it or not, had also had to undertake studies on law and jurisprudence.[227] For all them, it cannot have been difficult to use the word *lex* or *loi* [law] to indicate those properties and regularities which to them seemed to be true, eternal and universal. We shall soon see that a judicial similarity was to slip into the discussions with the next philosopher-jurist who comes immediately afterwards.

From the correspondence with Wilhelm Gottfried Leibniz (1646–1716), it clearly transpires that the mathematical problems present on their tables could be solved in different ways. Christiaan was particularly good at drawing and painting, as we can see from the sketches and watercolours that he left. He even made copies of works by Rembrandt and Hans Holbein. He could thus easily draw curved lines, tangents and curvatures that he continued to trace, albeit by means of Cartesian equations.[228]

The philosopher from Hannover, on the contrary, calculated by means of infinitesimal differences, which were expressed by certain new symbols entirely invented by him. They both admitted that they arrived at the solutions for isochronous curves. However, they differed in their procedures to obtain them. While Leibniz considered the solution of Huygens "enigmatique" ["enigmatic"], the Dutchman, in response, found "... d'oscurité ..." ["... some obscurity ..."] in the "... nouveau calcul Algebraique ... Vostre Analyse et Algorithme ..." ["... new algebraic calculation ... your analysis and algorithm ..."] proposed by the German in his journal *Acta Eruditorum* starting from 1684. However, compared with his own geometrical method, Huygens considered it "... methode equivalente, tant pour trouver les Tangentes des Lignes courbes où les regles ordinaires ne servent pas, ou fort difficilement, que pour plusieurs autres recherches." ["... an equivalent method, both to find the tangents to curved lines, where the ordinary rules are of no use, or very little, and for many other kinds of research."] He would have liked Leibniz to explain it "... clairement et dans un traité expres." ["... clearly and in a proper treatise."]

After studying it better, the Dutchman concluded that with his own method, he could find "... cette mesme construction et toutes les autres dans les lignes qui se forment de mesme, sans que les quantitez irrationelles m'embarassent, et à tout cela je ne me sers d'aucun calcul extraordinaire ni de nouveaux signes." ["... this same construction and all the others with the curved lines that are formed in the same way, without any embarrassment from irrational quantities; and in all this, I do not use any calculation out of the ordinary, or any new symbols."] When he was 15 years old, Huygens had demonstrated to Father Mersenne that the catenary (the curve

[226]Huygens 1888, I p. 88; 1891, IV pp. 183–190, 334, 341 and 356; 1897, VII pp. 108–110 and 122–123.

[227]Huygens 1888, I p. 17; 1950, XXII p. 411.

[228]Huygens 1888, I p. 17; 1893, V pp. 63–64; 1899, VIII p. 322; 1950, XXII pp. 405–406.

formed by a heavy chain suspended at its extremities) was not a parabola, as Galileo Galilei instead believed. Seeing that now (in 1690) Jacob Bernoulli (1654–1705) reproposed it as a problem, the Dutchman "... sans beaucoup de peine ..." ["... without too much difficulty ..."] gave the solution on the basis of his method, and impatiently waited for that of the German. But he offered it in an encoded language, so as not to deprive his adversary of the pleasure of finding it by himself, asking for Leibniz's result to be encoded, too. The three natural philosophers were to be joined also by Johann Bernoulli (1667–1748). The acrostic of Huygens, "d. h. c. q. c. p. q. i. p. e. t. i. i. p. e. r. c. i. i. i. ae.", was solved in the manuscript entitled *Adversaria* as "data harum curvarum quadratura catenae puncta quotlibet invenire possum et tangentes in iis punctis, et rectas curvis inter illa interjectis aequales." ["Given the quadrature of these curves, I can find any point of the chain, also the tangents at these points and the straight lines equal to the curves formed between them."]

More recently, the twentieth-century editors translated, without too much difficulty, the Latin and the geometry of Huygens into the formulas of modern post-Leibniz calculus. Actually, Leibniz was to insist on the advantages of his own symbolism. "Je conçois fort aisément, Monsieur, que vous avés une Methode equivalente à celle de mon calcul des differences. Car ce que j'appelle dx ou dy, vous le pouvés designer par quelque autre lettre, ainsi rien ne vous empeche d'exprimer les choses à vostre maniere. Cependant je m'imagine qu'il y a certaines vues qui ne viennent pas aussi aisement que par mon expression, et c'est à peu pres comme si au lieu des racines et puissances on vouloit toujours substituer des lettres, ... Jugés Mons. combien cela embarasseroit. Il est de meme de dx ou de ddx, et les differences ne sont pas moins des affections des grandeurs jndeterminées dans leur lieux, que les puissances sont des affections d'une grandeur prise à part. [...] Je voy que M. Newton se sert des minuscules pour les differences, mais quand on vient aux differences des differences, et au delà comme il peut arriver, il faudra ancor changer, de sorte qu'il me semble qu'on fait mieux de se servir d'une expression qui s'etende à tout. ... Aussi sans quelque chose d'approchante de mon expression, je ne scay si on s'aviseroit d'exprimer les courbes trascendentes comme la Cycloide ou la quadratrice, par des equations entre x et y abscisse et ordonnée, ou il n'entre aucun inconnue que ces grandeurs ou leur affections." ["I understand very well, sir, that you have a method equivalent to that of my calculation with differences. Because what I indicate by dx or dy, you can indicate it by means of any other letter; thus nothing will stop you from expressing things in your own way. However, I imagine that certain aims exist which can only be achieved easily by means of my expression, and it is almost as if one wished to substitute letters in the place of roots and powers, ... Judge, sir, how embarrassing that would be. The same thing happens with dx or ddx, and the differences have no less effect on the indeterminate magnitudes in their environment than the effect that powers have on a magnitude taken apart. [...] I see that Mr. Newton uses small-case letters for differences,[229] but when we arrive at the differences of differences, or beyond as may happen, it

[229]Newton 1687 (1972), Sectio II liber secundus.

will be necessary to change again, with the result that it seems to me better to make use of a notation suitable for everything. ... Furthermore, without something similar to my expression, I do not know if a way would be found of writing transcendent curves, like the cycloid or the quadratrix by means of equations between x and y, the abscissa and the ordinate, where no other unknowns appear than these magnitudes or their modifications."][230]

The German natural philosopher preferred to express the transcendent curves in symbols, with equations where the unknown appeared in the exponent, because then he could reformulate them into his "equations differentielles" ["differential equation"]. Huygens admitted the efficacy of the differential procedure, together with the correction of Leibniz on a sign, detected by the philosopher of optimism at all costs. And yet, he had "... de la peine à croire que la regle universelle se puisse trouver, sur tout quand les termes algebraiques de la construction donnée pour la Tangente sont beaucoup deguisez par la substitution des valeurs." ["... difficulty in believing that the universal rule could be found, above all when the algebraic terms of the construction given for the tangent are heavily masked, substituting their values."] Using his own geometrical-Archimedean method, if the philosopher-cum-mathematician from Holland believed that he had found a curve different from that of Leibniz, then it emerged that he had made a limitative hypothesis (would this be a mistake?). On closer examination, the relative equation would contain both of them.

Huygens admitted that he could not follow the new differential procedure perfectly. "... et ainsi vous instruisez le proces (pour demeurer dans les termes de vostre similitude) devant un juge qui n'entend pas bien vostre langue." ["... and thus you prepare a case for trial (to remain within the terms of your comparison) in front of a judge who does not understand your language perfectly."] Between the Flemish and the German, the lingua franca used was the French of diplomacy. But for their natural philosophy, they no longer had the common symbolism of the classical Euclidean and Archimedean tradition. The confrontation continued over the new "calculus differentialis". "Vos expressions pourroient estre plus courtes, mais pour l'invention je crois qu'il faudroit passer à peu pres par les mesmes chemins." ["Your expressions might be shorter, but as regards the invention, I believe the route will have to be more or less the same."]

They discussed mathematics and physics, yet Huygens did not forget music, even with Leibniz. "Si vous lisez l'Histoire des ouvrages des Sçavants qu'on publie icy de 3 en 3 mois, vous y trouverez quelque chose de moy en matiere de Musique et qui regarde un nouveau systeme de Tons. Si Mrs. de Leipsich avoient envie de le mettre dans leurs Acta, j'y pourray joindre quelques autres nouvelles considerations." ["If you read the *Histoire des ouvrages des Sçavants*, which is published every three months here, you will find something of mine dealing with music, which concerns a new system of notes. If the gentlemen from Leipzig had a desire to include it in their *Acta [Eruditorum]*, I could add some new considerations."] But the 'tempered'

[230]Huygens 1901, IX pp. 258, 366, 471–475, 497–520; 1950, XXII p. 153.

"New harmonic cycle" was not to be published in Germany, because the Dutchman no longer felt like translating it.[231]

"Harmonic", an adjective undoubtedly of musical origins, returned in the discussions between the two scholars, who succeeded in distinguishing themselves both from Descartes and from Isaac Newton (1642/43–1727), who by now was on the European stage. Huygens rejected the idea, which had come to the Englishman, that the force of gravity acted at a distance on bodies, like the Sun on the Earth. To explain the movement of the planets, instead, the Dutchman maintained the Cartesian vortices of ether. Was it not true that they could be seen to form around Saturn those fascinating rings discovered by him?[232] In one of his first letters written to Huygens, Leibniz seemed to want to move out of the wake left by the philosopher of Method. "Mon dessein avoit esté de tailler un peu de besoigne à ces bons Cartesiens …" ["My design had been to create a few obstacles for these good Cartesians …"]. But then it emerged that the German had been a follower of Cartesian physics, in this matter. Actually, in the *Acta Eruditorum*, Leibniz tried to explain the movement of planets by means of "harmonic" vortices of an impalpable fluid.

Huygens then asked him: "Je vois que vous vous este encore rencontré avec lui en ce qui regarde la cause naturelle des chemins Elliptiques des Planetes, mais comme en traitant cette matiere vous n'aviez ancore vû qu'un extrait de son livre et non pas le livre mesme, je voudrois bien sçavoir si du depuis vous n'avez rien changé à vostre Theorie, parce que vous y faites entrer les Tourbillons de Mr. des Cartes, qui à mon avis sont superflus, si on admet le Systeme de Mr. Newton où le mouvement des Planetes s'explique par la pesanteur vers le Soleil et la *vis centrifuga*, qui se contrebalancent. Outre que ces Tourbillons Cartesiens faisoient naitre plusieurs difficultés, …" ["I see that you have again clashed with him [Newton] regarding the natural cause of the elliptic orbits for planets, but as you have only seen an extract of his book [*Philosophiae Naturalis Principia Mathematica*, 1687] and not the book itself, in your study of this subject, I would like to know whether you, subsequently, have changed anything in your theory. Seeing that you include the vortices of Mr. Descartes, which, in my opinion, are superfluous if we admit the system of Mr. Newton, or if we explain the movement of the planets by the weight towards the Sun, offset by the centrifugal force. Furthermore these Cartesian vortices made many difficulties arise, …"]. The delay of the German philosopher in answering the question, which the Dutchman repeated once again insistently leads us to suspect a certain embarrassment on the part of the former. Above all because the relative letter remained in the form of a rough copy, and was never sent. In the following chapter, we shall see whether, like Huygens, Leibniz was to retain or abandon his "harmonic" vortices in the ether.[233]

[231] Huygens 1901, IX pp. 569–576; 1905, X pp. 224–225, 298–299.

[232] Bos 1981, pp. 608–609.

[233] Huygens 1901, IX pp. 258, 367–368, 472, 522–524.

The philosopher from Hannover knew and commented on the most important books written by Huygens regarding the oscillations of the pendulum, the cause of weight, and light. "L'usage que vous faites des ondes pour expliquer les effects de la lumiere m'a surpris, et rien n'est plus heureux que cette facilité, avec la quelle cette ligne qui touche toutes les ondes particulieres et compose l'onde generale satisfait aux loix de reflexion et de refraction connues par l'experience." ["The use that you make of waves to explain the effects of light surprised me; and nothing is happier than this facility with which that line, which touches all the particular waves, and composes the general wave, satisfies the laws of reflection and refraction which are known by experience."] The Dutchman had even succeeded in explaining, in this way, the double refraction visible in the Iceland Crystal. Leibniz knew "... exemplar novi, planeque excellentis operis de *Lumine*, quo mihi videtur *Cartesii* lumina prorsus extinxisse in hoc argumento." ["... the copy of the new, clearly excellent work on light [*Traité de la lumière*, 1690], with which [Huygens] seems to me to have extinguished the illumination of Descartes on the subject."][234]

The undulatory model for light proposed by the Dutch natural philosopher was to enjoy such great success that the following physicists in general forgot the supporting ether was conceived of by him as composed of particles agitated by mechanical impulses. And yet light is not formed by particles in movement, but rather by the enveloping together of all these micro-collisions, which in the end formed macroscopic spherical waves. The model will be baptised the "Principle of Huygens". Even if he considered some opinions of Descartes to be false, Huygens recognized that the Frenchman had opened up the way for his inventions. He gave different laws from those of Descartes, for the collisions between bodies, since his laws were based on relative speeds, and he succeeded in maintaining an expression in them, which included the square of the speeds. Anyway, his models for light and weight appeared to be very different from those of Newton, because he always tried to connect natural phenomena with mechanical causes. "True philosophy [is the kind] in which the causes of all the natural effects are explained by the reasons of mechanics." The assumption that only direct contact between bodies could explain phenomena sufficiently thus brought him closer to Descartes. However, the world of the Dutchman remained composed of solid, concrete particles, not at all disposed to get lost by disappearing into the geometry of space, nor could space sublime into the absolute certainties of a reasoning mind.

Huygens called "vis centrifuga" ["centrifugal force"] the tendency of a body rotating in a circle to move away from the centre as a result of inertia. He posited that it was proportional to the square of of the speed, and inversely proportional to the radius. If this *vis* [force] had been balanced, as he believed, with the attraction of the planets towards the Sun, by substituting the speeds obtained by Kepler's Third Law, we shall see in the following section that one of the famous results attributed to Newton would be obtained. About it, effectively, the Dutchman also wrote in his *Discours de la cause de la Pesanteur [Discourse on the cause of weight]* (1691). "Je

[234]Huygens 1901, IX p. 522.

n'avois pas pensé non plus à cette diminution reglée de la pesanteur, sçavoir qu'elle estoit en raison reciproque de quarrez des distances du centre: qui est une nouvelle et fort remarquable propriété de la pesanteur, dont il vaut bien la peine de chercher la raison." ["I had not thought any more about this regular decrease in weight i.e. that it was reciprocal to the square of the distance from the centre: which is a new, highly significant property of weight, for which it is worth while searching for the reason."] Among the ideas of the Dutchman the Englishman abandoned, there was that of movements described only in relation to other bodies, which could not be considered with respect to an absolute space.[235]

Accommodated comfortably with an influential family who enjoyed high-ranking protection, Christiaan Huygens did not have to undergo either religious or political censure. After running some risks when he was in France, on the death of Colbèrt, he prudently moved back to Holland. He was accepted as a member of two new institutions, which were to exert even more influence on the subsequent evolution of the sciences: the Académie Royale des Sciences at Paris, sustained by Colbèrt, and the Royal Society of London. These also published scientific journals, like the *Journal des Sçavants* and the *Philosophical Transactions of the Royal Society*. From the former institution, the Dutch natural philosopher who lived abroad received a salary, and accommodation in the capital, where he remained, even during a war which broke out between France and Holland.[236] Leaving consistent religious beliefs in the background, at least for a moment, some more far-sighted State Minister started to see that maintaining a 'good' scientific policy would above all bring advantages in many senses.

This descendant of the prevailing Dutch society and culture of the seventeenth century had displayed diverse skills, both in practical and in theoretical and philosophical activities, both in mathematics and physics, and in the arts. And yet it would not be difficult to identify the supporting hinge around which much of this research revolved. It culminated in the *Horologium Oscillatorium [Oscillating clock, oscillating proportion of time]*, or in other words, the study of mechanical movement as a periodic oscillation in time. In spite of the event that this was explicitly called "harmonic", it is sad to note that in general, historians ignored the clear significance of this for music.[237] For Huygens, instead, all those clocks, pendulums, springs, and vibrating strings that he studied followed the rhythm of music, observing the tempo with regularity.

[235]Huygens 1901, IX p. 525. Dijksterhuis 1971, pp. 492–509 and 612–620. Bos 1981, pp. 609–610, 602–603.

[236]Bos 1981.

[237]Bos 1981, pp. 607–608.

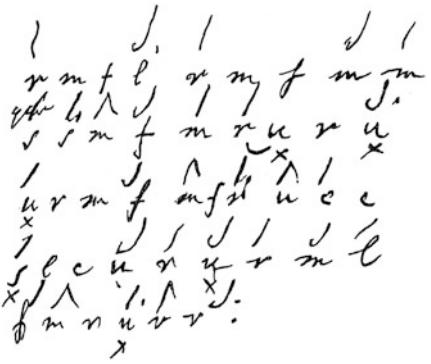


Fig. 9.14 Accompaniment for a song composed by Huygens (Huygens 1905, ibid., vol X, p. 651)

Vice versa, those who should read his theory of music, comparing it with the other scholars of his age, would have ignored his coherence with the other scientific activities.[238]

From the other books of history, we could easily get the idea that Dutch scholars were influenced by the clear inclinations towards practical activities of this culture, which had made itself independent by means of wars, commerce and sea voyages. Christiaan Huygens sealed, in beauty and in evidence, the list of those who had taken their inspiration for their research in natural philosophy, partly also from music: Stevin, Mersenne, Descartes, Beeckman, Wallis, Huygens. Some manuscripts are extant in which some of them were even capable of composing songs and pieces for the harpsichord. We know a couple of these by Huygens.

That for a tender, faithful heart
Absence is a cruel torment
It accepts everything of the best
But it is cruel above all
For a happy lover. (Fig. 9.14)[239]

The following is an extant original piece for the harpsichord by the Dutchman (Fig. 9.15).[240]

Now let us try to give a better credit to the sub-title of our book, which reads: Musical, multilingual and multicultural history of the mathematical sciences. The event that Stevin, Galileo Galilei, Kepler, Mersenne, Descartes, Beeckman, Wallis, Francis Bacon and Huygens all devoted themselves to music has been made evident

[238]Cohen 1984, pp. 206–230. What would be appreciable here is the desire to measure the theory of music against the specific tastes of Huygens; if it were not for the event that he had written, aroused by his historiographic hypothesis: "... of a Lully not a single word", p. 225. With reference to the Franco-Florentine composer, an expression of praise above has been found about him: "belle opere [fine works]". Huygens 1899, VIII p. 24. See above all Rasch 1992, pp. 200–201. Rasch 1992, pp. 185–210.

[239]Huygens 1905, X p. 651.

[240]Rasch 1992, p. 203.

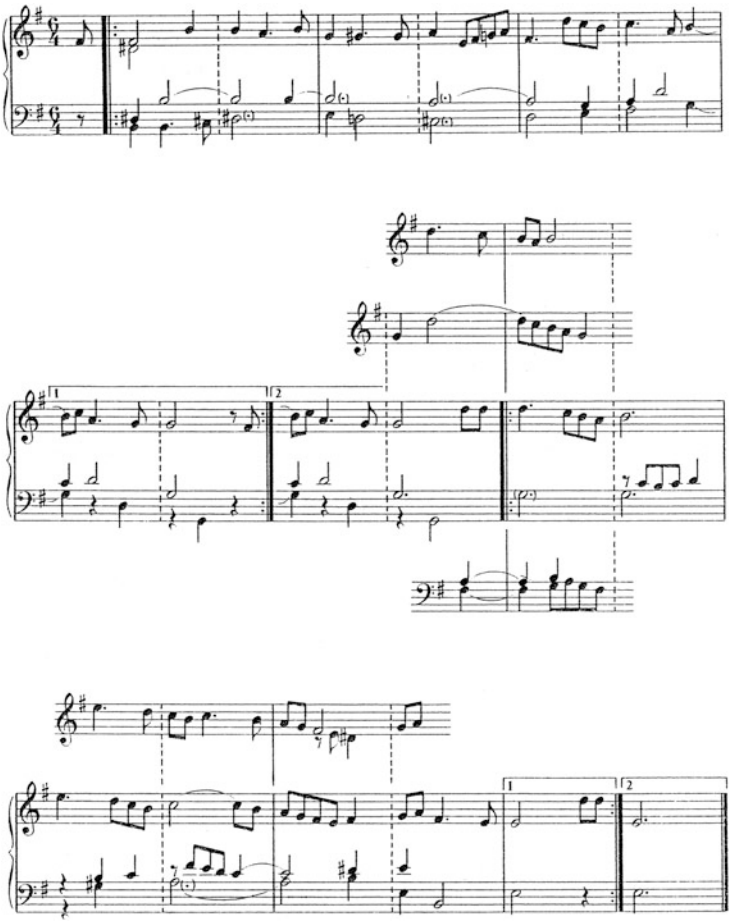



Fig. 9.15 Original *Courante* composed by Huygens for the harpsichord (Huygens, "Courante" from Rasch 1992, in Coelho, Dordrech Kluwer, p. 203)

and not negligible. All of them studied its theory, and wrote about it. With Huygens, we have recounted that he even played the lute and the harpsichord, and at times he wrote the accompaniment for songs, or composed short pieces. There is a trace of the event that he consorted with the composers of the period in Paris.

But we should go further in order to re-assign to music the place that it deserves in the history of the sciences. In reality, it has made it possible for us to understand better how the prohibition of irrational numbers was eclipsed during the seventeenth century, in connection with the affirmation of the equable temperament: initially among musicians, and then we gradually see it develop also among natural philosophers. Bearing in due consideration the subjective moment of invention in relation to the context where it took place, we have the testimony that

Galileo Galilei felt the movement of the pendulum with the help of music. Kepler restored the harmony of the solar system by means of the polyphonic singing of the planets, identifying the sesquialter ratio 3:2 for his Third Law, the same ratio as the (Pythagorean) fifth *do – sol*. And is it not true that Mersenne, in his theological and philosophical universe, was guided by the foundational role assigned to music, when he undertook difficult experiments aiming to measure the frequencies of vibrating strings and sound?

Although the efforts of Descartes clearly did not excel for their originality amid this noble company, we must not ignore what had already been noted by others. For him, the calculation of proportional means was to be a part of the route towards his new geometry. But where did he find the stimulus to take an interest in them? From Mersenne, who wanted to know a procedure to divide the musical octave into 12 equal semitones using numbers: the equable temperament! We have found that even the invention of logarithms had a close relationship with music. Addition, instead of multiplication, is just the manner of musicians to reason with the intervals between notes, unlike the traditional calculation with proportions. Actually, Mengoli and Huygens calculated musical ratios with logarithms.

This last Dutchman, so skilful in applying himself to various subjects, would deserve a position of greater importance in the sciences of the seventeenth century, compared with the one that is generally assigned to him. But not only because he (dearly?) loved our music, and practised it with great skill. Above all because he acted as a pivot around which Kepler, Galileo Galilei, Mersenne, Descartes, Leibniz and Newton revolved. He took, exploited and distributed ideas which were to be developed by others, and which he did not want to develop, as he could have done, on the contrary, with little effort. Huygens was to remain inside the traditional classical world of geometry, and the mechanics of bodies in contact. The same behaviour is precisely what he followed with music. Here, in order to flourish exuberantly and varied, the accompanied monodic singing, which he preferred to polyphony, required new harmonic spaces in relation to the new tuning systems, freeing itself from the Pythagorean prohibitions. And yet, with music too, our Dutch natural philosopher stopped half-way along the route with respect to tradition, although he had all the motivations, the reasons and the instruments to be able to pass over into another world of musical theory, more simple and free.

It has been sufficiently highlighted that the characters under examination expressed themselves in a plurality of languages. In Europe, the progressive abandonment of Latin as a lingua franca for scholars of the mathematical sciences was to raise a new problem. The event led to the invention of that particular symbolism which was destined to take root, and to represent, for better or worse, one of the characteristics of scientific discourse in the West.

And what about the East? The Near East of the Arabs was definitely present, albeit hidden and disguised in other forms. Everyone by now used the symbols from India and the Arab countries, as we know, in order to write numbers. Their algebra had become a mathematical discipline that outclassed traditional arithmetic. By means of this, though rewriting it in his new symbols, Descartes renewed geometry. Leibniz extended calculation with the finite terms, of Arabic algebra, to those

transcendent infinite procedures which were to represent the greatest mathematical novelty of the age.

Only the Chinese culture appeared to be missing. Thus, we must dwell on this event a little longer here. When the Ming dynasty came to an end, it undoubtedly had its mathematicians, who had to start to confront themselves with the Western novelties, often seeking compromises with their more ancient tradition.[241] In the passage from the XVII to the eighteenth century, the most eminent Chinese mathematician appeared to be Mei Wending (1633–1721). His many abilities deserve more than respect. A scholar who has studied him with attention and competence, comparing him in certain cases with what European mathematicians of the age were doing, or what Carl Gauss (1777–1855) subsequently did, has sometimes arrived at the following conclusion. "The superiority of Chinese technique compared with the method of European origin cannot be doubted …". He was referring to complex problems which nowadays we would define as linear algebra. "Linearity tends to a part of numerical calculus, and it seems clear that in this domain, we can speak of a Chinese originality, rich in potential."[242]

The grandson of the preceding Chinaman, Mei Juecheng (1681–1763), calculated with infinite series, as he had learnt from the Jesuit, Pierre Jartoux (1669–1720). But this Frenchman used them without offering any proof of their correctness. On the contrary, the Mongol astronomer, Ming Antu (1???–1769) sought for a long time to give a demonstration of them, but only an unfinished manuscript of his remains. By using infinite series, the Chinese obtained the length of the circumference, or of its arcs, given the diameter or the chords. As they limited themselves to geometrical procedures, it is not appropriate to make comparisons with the infinitesimal and transcendent methods invented by Leibniz and Newton. On the other hand, a comparison with the 'equivalent' geometrical calculations of Christiaan Huygens, which were generally equally effective, would throw new light on the Chinese mathematical culture of the age between the Ming and Qing dynasties. Its purpose was to calculate astronomic events, with the relative trigonometry, also spherical, when necessary. Despite the efforts of Chinese scholars, however, their sophisticated, and possibly effective, calculation procedures were becoming invisible, compared with the new Western methods. They lacked those characteristics which were successful in the West: automatism and symbolism. With these, the European scientific community reaped successes, albeit amid controversies, as we shall see, and made proselytes.

However, we should not think that autochthonous Chinese mathematics had now become extinct, having been trounced and made useless by the Western variety.[243] Rather, they had followed other routes, as we have already narrated in Chap. 3. Where were they to arrive? We might ask a similar question as regards Huygens. Was the 'geometrical' route offered by the Dutchman to have any followers in

[241] Mikami 1913. Li & Du 1987. Jami 1988b.

[242] Martzloff 1981a; 1981b; 1988; 1993. 1981b, pp. 223, 327.

[243] Needham 1959, p. 168.

Europe? If not, why not? As this is not a one-dimension history of the sciences, but it is open to many other possibilities, may we at least be allowed to raise the question.

De las cosas mas seguras [Of the surest things]
la mas segura es dudar [the most certain one is to doubt]
Fr. de la Mothe le Vayer (1588–1672).

Chapter 10
Between Latin, French, English and German: The Language of Transcendence

Un bel air de Musique qui sera tousjour chanté, est comme un beau theoreme de Geometrie rencontré.
[A beautiful musical aria, which will always be sung, is like a fine geometrical theorem found again.]

. . . when the Chinese will have learnt all they want to know from us, then they will close their doors to us.
Letter to Peter the Great

Aimer est être porté a prendre du plaisir dans la perfection de l'objet aimé.
[To love means being led to take pleasure in the perfection of the object loved.]
Gottfried Wilhelm Leibniz

10.1 Gottfried Wilhelm Leibniz

Although the Galileian school still included scholars of the first rank like Evangelista Torricelli (1608–1647) and Vincenzo Viviani (1622–1703), the barycentre of European scientific evolution had now shifted to the North. The most important characters for our history had already appeared in the correspondence of Huygens. Through the one with whom our Dutchman had the closest relationship, Leibniz (1646–1716), we can take up the thread of music which we prefer to follow here. Actually for his model of the solar system, the renowned German philosopher and diplomat had used a term which too few modern astronomers would expect: "circulation harmonique" ["harmonic circulation"].

In contrast with the explanation offered by Newton, and to answer the question of Huygens seen in the previous chapter, Leibniz continued to imagine vortices of ether that drove the planets along in accordance with the laws of Kepler. "Cependant je ne comprends pas comment il conçoit la pesanteur, ou l'attraction. Il semble que selon luy ce n'est qu'une certain vertue incorporelle et inexplicable, au lieu

T.M. Tonietti, *And Yet It Is Heard*, Science Networks. Historical Studies 47,
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que vous l'expliqués tres plausiblement par les loix de la mecanique. Quand je faisois mes raisonnemens sur la Circulation harmonique, c'est à dire, reciproque aux distances, qui me faisoit rencontrer la regle de Kepler (du tems proportionel aux aires), je voyois ce privilege excellent de cette espece de circulation: qu'elle est seule capable de se conserver dans un milieu qui circule de même, . . . " ["However, I do not understand how he [Newton] conceives of weight or attraction. It seems that according to him, it is simply a certain incorporeal, inexplicable virtue, whereas you [Huygens] explain it very plausibly by the laws of mechanics. When I carried out my reasonings on harmonic circulation, that is to say, reciprocal of the distances, which led me to find Kepler's rule again (times proportional to the areas), I saw the excellent advantage of this kind of circulation: the only one capable of being maintained in a medium which likewise circulates, . . . "].

In the "Tentamen de Motuum Coelestium causis" ["Essay on the causes of heavenly movements"] (1689), he wrote that "Kepler had in a certain sense suggested gravity with the movement of water pushed into a vortex". Therefore the "circulation" of ether would be "harmonic" when the velocities were "inversely proportional to the distance from the centre, in other words, if the velocities around the centre decrease in the same proportion in which the distances from the centre of the circulation increase, or in short, if the velocities increase in proportion to their closeness. Thus if the radii or the distances increase to the same extent, that is to say, in an arithmetic proportion, the velocities decrease in a harmonic progression: $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}\ \ldots$". The same proportions were used by Greek tradition to construct its orthodox theory of music, where the pitch of the notes increases with the reciprocal of the lengths of vibrating strings.

From this law, he believed he could deduce, by means of suitable hypotheses, that the planets were attracted by the sun in a proportion equal to twice their closeness to it, that is to say, the reciprocal of the square of the distance. "Video hanc propositionem jam tum innotuisse etiam Viro celeberrimo Isaaco Newton o, ut ex relatione Actorum apparet, licet inde non possim judicare, quomodo ad eam pervenerit. Ita habemus consensum pulcherrimum eorum quae supra a priori conclusimus ex lege radiationis attractoriae, et eorum quae nunc ex phaenomenis, id est natura motus Elliptici planetarum secundum areas ex Sole tanquam foco abscissas aequabilis a Kepler o ex Tycho nis et suis observationibus stabilita et a posteris comprobata sponte nascuntur." ["I see, then, that this proposition made already become known also to the most excellent Isaac Newton, as appears from the summary in the *Acta [eruditorum]*, and thus it is not possible for me to judge how he arrived at that. And therefore we have a very nice agreement between what we concluded above *a priori* from the law of attracting radiation and what derives spontaneously now from the phenomena, that is to say: the nature of the elliptic movement of the planets, in accordance with areas equally detached [in equal times] from the sun as the focus, established by Kepler, starting from the observations of Tycho and his own, as well as those proved by his successors".][1]

[1] Leibniz 1689.

Thus a planet would move "…dans l'ether comme s'il y nageoit tranquillement …" ["…in the ether, as if calmly swimming …"]. From his physical and mechanical model, Leibniz obtained the advantage of being able to explain why all the planets rotated in the same direction, without having to resort to the "…author de la nature …à un hazard heureux …" ["…creator of nature [or] to a fortuitous coincidence …"]. For him, this would not have been "…assés philosophique …" ["…very philosophical …"]. Our German diplomat considered two possible causes of weight: the first could be a force of attraction like light, which would be distributed over spherical surfaces, diminishing with the reciprocal of the square of the radius. For the other one, he made reference to Huygens "…à vôtre explication de la pesanteur par la force centrifuge de la circulation de l'ether, qui m'a tousjours parue fort plausible." ["…to your explanation of weight by means of the centrifugal force due to the circulation of ether, which has always seemed highly plausible to me."]

Leibniz succeeded in finding Kepler 's third law again by reflecting that "…les quarreés des temps periodiques sont comme les cubes des distances." ["…the squares of the periodic times stand as the cubes of the distances."] He maintained that "…les tendences centrifugues sont en raison doublée reciproque des distances." ["…the centrifugal tendencies stand in a reciprocal relationship to the square of the distances."] Thus, "…j'ay quelque penchant à retenir les tourbillons et peutestre ne sont-ils pas si coupable, que M. Newton les fait." ["…I have a certain inclination to maintain the vortices, and they may not be so guilty as Mr. Newton makes them."] However, our German philosopher realised that "…l'hypothèse de quarrés des vistesses reciproques aux distances ne s'accorde pas avec la circulation harmonique." ["…the hypothesis of the square of the velocities reciprocal to the distances is not in agreement with the harmonic circulation."] He then pre-announced another article, which was never to be published, and perhaps never written, where he intended to protect and perfect his "harmonic circulation" with additional hypotheses. He intended to do this by means of a second "…circulation harmonique en puissance (où les quarrés des velocités sont reciproques aux distances) …je distingue l'ether qui fait la pesanteur (…) de celuy qui defere les planetes, qui est bien plus grossier." ["…potential harmonic circulation (where the squares of the distances stand in a reciprocal relationship to the distances)…I distinguish the ether which generates weight (…) from that which carries the planets along, which is much more unrefined."][2]

The physics of Leibniz was a physics of ether. "Gravity stems from the circulation of the ether around the earth …". "Sonus non consistit in motus aëris …" ["Sound does not consist in the movement of the air …"] but absolutely "It thus consists of the movement of the ether".[3] How can we doubt that this deeply-rooted conviction, sustained at all costs, had not been inherited from the old music

[2]Huygens 1901, IX, pp. 367 and 523–527.

[3]Leibniz 1960/65, IV, p. 196. Leibniz 1962, IV, p. 243. Bailhache 1992, p. 43. Cf. Serres 1968, pp. 462–464.

of the spheres which had passed through Kepler? Whichever mechanical model is used to try to justify it, in all cases it is a question of harmony between the planets, and Leibniz could presume that he had found a higher, more general principle, to explain gravitation. This is something that Newton had not succeeded in doing, in his opinion.

The ether has always had to put up with alternating fortunes in the course of time. In the nineteenth century, it was believed to have carried light waves first, and subsequently, electromagnetic waves; for this reason, it was called 'luminiferous'. Some experiments and the special theory of relativity would have claimed to be able to do without it, as it was a fluid impossible to measure and useless as a hypothesis. However, with general relativity, some, like Albert Einstein himself (1879–1955) and Hermann Weyl (1885–1955) were to start talking about ether again, intending it as a sort of geometrical *continuum* described by the metrical tensor.[4] More recently still, others have used the same word to indicate the background 3 K radiation which pervades all the universe, perhaps referring to a beginning which was believed to have been pyrotechnical. We should not be too hostile, then, against the ether of Huygens and Leibniz, who, in writing about "attracting radiation", was preparing the ground for the notion of a gravitational field, quite different from Newton's model.

Any student of philosophy knows that Leibniz, with his particular optimism, sustained that the whole world was held together by a "prästabilierte Harmonie" ["pre-established harmony"], guaranteed by his God.

He repeatedly insisted on this, in spite of the evident, innumerable contrasts, which could be solved only with great difficulty, even if he enjoyed all those disasters that his hostile adversaries unleashed in front of him. Is it, perhaps, because this was the mortar with which he consolidated all his philosophical system that our famous German mathematician hardly left us anything well organised and extensive dealing with music? Who knows what all his papers contain, pending the publication of a complete critical edition of his works? Anyway, for me, all his work sounds like a glorious countermelody to divine, heavenly music.

Let us start from a relatively complete article, where Leibniz enunciated the most synthetic and famous definition of sound art to be found in the history of European sciences. "Musica est exercitium arithmeticae occultum nescentis se numerare animi." ["Music is a covert exercise of arithmetic, performed by the soul that is unaware it is counting".] In other words, therefore, singing is counting. He wrote this on April 17th, 1712, to Christian Goldbach (1690–1764).

"III. It is not impossible that animals may exist somewhere which have more musical sensibility than us, and take pleasure in musical ratios which create less impression on us. But I would imagine that a greater sensibility of our senses might be more harmful to us than advantageous: we would have many unpleasant things to feel with our sight, our hearing and our touch; and those who possess too acute a sensibility in music are hurt by certain deviations which are not easy to avoid in the

[4]Tonietti 1988.

practical construction of organs,[5] by which, however, the listeners are not usually hurt. We, in music, do not count beyond five ["Nos in Musica non numeramus ultra quinque"],[6] like those peoples who do not arrive even beyond three in arithmetic, for whom the expression that the Germans use for a simpleton has a meaning: *er kan nicht über drey zehlen* [he cannot count beyond three]. All our usual intervals, sure enough, are based on proportions composed of ratios between couples of the prime numbers 1, 2, 3, 5. If we were allowed a little more subtlety, we might arrive as far as the prime number 7. And I believe that in reality, such people exist. Thus the ancients did not actually openly avoid the number 7. But there was hardly anyone who went on as far as the following prime numbers, 11 and 13.

IV. I believe, on the contrary, that the reason for consonances is to be sought in the congruence of strokes. ["Rationem autem consonantiae petendam puto ex congruentia ictuum."] Music is a covert exercise of arithmetic, performed by the soul that is unaware it is counting. It actually does many things, perceived confusedly or insensibly, which it does not succeed in distinguishing with clear apperception. Therefore they are mistaken who think that nothing can exist in the soul, of which the soul is not conscious. Thus, though the soul does not realise it is counting, nevertheless it feels the effect of this unconscious counting, or in other words, the pleasure in consonances and the irritation at dissonances deriving from it. The pleasure derives from many insensible correspondences. ["Ex multis enim congruentiis insensibilibus oritur voluptas."] Usually, indeed, those who attribute to the soul only the operations of which it is conscious make a bad judgement in this matter. Hence arise many errors, not only with philosophers of the past, but also with the Cartesians themselves, and more recently with others like Locke [John, 1632–1704] and Bayle [Pierre, 1647–1706]. But to come back to the point, in the octave one stroke every two of a series of strokes coincides with any stroke of the other series. In the fifth, every third stroke of one series coincides with the second stroke of the other one. The polygons of the circle, regular bodies and other bodies of this kind allowed by the extremely subtle Kepler are employed $\alpha\grave{}\pi\rho o\sigma\delta\iota\acute{o}\nu\upsilon\sigma\alpha$ [inopportunely], because they do not extend to the arithmetic of rational numbers.

V. I do not think that irrational ratios are in themselves pleasing to the soul, except when they are distant only a little from rational ones which are pleasing. ["Rationes surdas non puto per se placere animae, nisi quando parum distant a rationalibus, quae placent."] In some cases, however, dissonances may sometimes be pleasing, and they can be usefully used, and are interposed in sweet intervals, like shadows between order and light, so that afterwards, order is found to be more pleasing. Your friend, whose profile you communicated to me, who intends to divide the

[5] . . . *organorum* . . . , an ambiguous term which indicates all the instruments, but above all were the organs and harpsichords with a fixed tuning which had to be tempered.

[6] The wisecrack that Pythagorean theoreticians could not count beyond five was a joke that I thought I had invented: until I found it in Leibniz. We are all modest commentators on the work of someone else. At the beginning, what would come out is always either the *Bible*, or the *Koran*, or the *Veda*, or the *Wujing*.

monochord into the extreme and middle ratio [the golden section], would really find intervals that thus proceed towards the sense so as to coincide almost with the major and minor sixths: that is to say, if we have $AB : BC = BC : CA$ will be approximately

$$AB - - - - - - - BC - - - - - - - - CA$$
$$8 - - - - - - - - 5 - - - - - - - - 3$$
$$minorsixth - majorsixth.$$

They are sufficiently close to 809 . 500 . 309, positing that $\sqrt{5}$ is 2.236, or rather 2.236 to the thousandth, and exactly like $\sqrt{5}+1$, 2, $\sqrt{5}-1$. Instead, $\sqrt{5}$ is irrational anyway, slightly higher than 2.236, but the error is less than $\frac{1}{1000}$. Thus, if that division possessed something pleasing, it would be taken from these close intervals."[7]

The anonymous friend of Goldbach had suggested dividing by the number 5 the interval of the eleventh, 8:3, which is the combination of a major sixth, 5:3, and a minor sixth 8:5. But $3 \times 8 = 24$ and $5 \times 5 = 25$, and therefore these would have been two parts only approximately equal. Leibniz suggested using the whole numbers $809 \times 309 = 249,981$, which come better closer to $500 \times 500 = 250,000$ to the thousandth. He wrote $(\sqrt{5}+1)(\sqrt{5}-1) = 5-1 = 4 = 2 \times 2$, which was another approximate way of dividing the double octave.

Should this calculation, perhaps, remind us of that of Kepler in his *Harmonices Mundi Libri Quinque*, where the astronomer approximated the golden section by means of Fibonacci's succession?[8] Actually, as the golden section is equal to $\frac{-1+\sqrt{5}}{2}$, by going up one octave, we obtain $\sqrt{5}-1 = 1.236\ldots$. This irrational number may come more or less close to the division into two equal parts of the eleventh, 8:3. Bringing back actually $\sqrt{24} = 4.899\ldots$ into the octave, we obtain $1.224\ldots$ Even if out of modesty he wrote it in Greek, with the preceding adverb "inopportunely", our mathematician was moving away from his fellow-countryman, the astronomer. He rejected Kepler's geometrical explanation, to put himself in the hands of numbers, but rational numbers were no longer sufficient for him. Stimulated by music, Leibniz was completing the course towards transcendence. The approximations sought by means of his new mathematical procedures, which finally succeeded in justifying the prohibited irrational numbers, not only solved the requirements of musicians to temper intervals. They now regarded many of the other phenomena which natural philosophers were faced with: above all, the movement of bodies.

Whether good or otherwise, the numbers rounded off to a few decimals only tempered, for Leibniz, the 'just' consonances expressed by the traditional whole

[7] Leibniz 1734, pp. 240–242.

[8] See above, Sect. 8.3, notes 82–90 and Fig. 8.5.

numbers. Sure enough, also the philosopher of pre-established harmony believed, like Huygens and other natural philosophers, in the coincidence of strokes in the ear as the physical reason for consonances. Yet his answer to Goldbach confirmed, if there was still any need for it, that the tuning of organs and harpsichords based on irrational ratios had by now become unstoppable. It would not be possible, therefore, simply to reject them, neither to allow rival natural philosophies, like that of Locke, to free the senses from the control of whole numbers. A musical sensibility was admitted even for animals, with the aim of belittling it. As regards the (theorationalistic?) metaphysics of Leibniz, we may think what we like, but its force must not be underestimated. The philosopher from Hannover actually offered a solution to the main dilemma: the one which, strictly speaking, was insoluble. Did the ear draw *voluptas* also from irrational ratios? Those ratios which were inexorably condemned by the theory of coincidences in the strokes: why were they made impossible by the irrational numbers, thus destroying consonances? The truth, for Leibniz, was that the ears listened to the sounds only seemingly, and outwardly (on the Earth?), while deep down, covertly, and without realising it, the soul would be counting the strokes. When it had counted, in the unconscious of the listener, whole numbers sufficiently close to the irrational numbers heard, then these would be judged consonant. And so much the worse for the clear, distinct mind of Descartes.

It arrives to us as an ironic quirk of history, that these *rationes surdae*, as irrational incommensurable ratios in numbers were called for a 1,000 years, had now become perfectly audible in the music of the eighteenth century. Whereas, on the contrary, the ratios capable of having a *logos*, that is to say in their words, the rationals had disappeared, because they were hidden and made unconscious by the metaphysics of a mathematician-cum-philosopher.

Probably inspired partly by a Catholic like Mersenne, in whose works we have already read a similar thought,[9] after consonances, our German Protestant even justified the presence of dissonances in music. The latter served to interrupt pleasure and to create those shadows, that relief, so as to find much more pleasure in finding "the order" of the former again. With his theory of music, Leibniz, as we shall immediately see, could thus sustain what his complex philosophical system required, which went well beyond the simple appearances observed in natural phenomena.

Again writing to Goldbach a few months later, Leibniz advised him to read, as regards the temperament, the letter published by Huygens.[10] But he judged what the Dutchman offered to be too subtle, with his division into 31 parts, concluding with a certain wisdom, "…most are satisfied with this division of the octave into twelve equal parts, attributed to Aristoxenus."[11] Other suggestions, to arrive at recognizing the role of music in the work of Leibniz, can be found in his exchange

[9] See above, Sect. 9.1.

[10] See Sect. 9.3.

[11] Bailhache 1992, p. 153.

of correspondence with the Court Counsellor, Conrad Henfling (1648–1716). Being stemmed from his real royal working environment, made up of princesses, imperial electors, *Hofräte* [our counsellors] and diplomats like himself, the authoritative interlocutor placed before him, in Latin, without hiding his ambitions, yet another umpteenth variant of Pythagoreanism. Offered as "... the true foundations of music, not yet established ...", however, this had to be 'tempered' now, in order to fit in with the music of his epoch.

Henfling used certain expressions invented by himself, writing letters of the alphabet, as if he were Viète or Descartes, to ennoble, or perhaps to hide the usual numerical ratios that were multiples of 2, 3 and 5 (without 7), to which we were all accustomed. The new symbolism made what had wanted to argue prolix and obscure, but in the end, he succeeded in telling what he had to say to us. "Thus the octave contains 81 different parts (including those derived from the 38 *commas*) which cannot be placed in the ordinary musical instruments. Thus it is necessary to examine some *moderamina* [guides] so as to temper the sounds so that one may be capable of performing the functions of many, without hurting the ear."

Unexpectedly remembering the ancient Aristoxenus, he consequently now divided the octave into 12, 19, 31, and 50 equal intervals distributing, in his own way, the *commas* and other micro-intervals to which he always gave new names. For this reason, he even proposed a new kind of keyboard, as Huygens had done. For those who liked games with whole numbers, 12, 19, 31, 50 and 81 could be more enjoyable than others, because by adding two successive numbers together, the following number is obtained. Thus they formed a kind of Fibonacci's succession, though the classic one was different, because it began 1, 2, 3, 5, 8, 13, ... Starting from Euclid, Henfling took the liberty of criticising his predecessors. And yet he was avowedly inspired by Descartes, whose dichotomies he used to divide the string. Therefore he remained the heir of that orthodox Greek tradition, even if he moved away from it into a curious formalism. As regards notes, he would have preferred to exclude the B, stopping at Guido D'Arezzo. Leibniz undoubtedly appreciated that he had invented new notations.[12]

The philosopher commented on his results, which were not always original, but he was to publish them at Berlin, seeing that something or someone prevented them from being published at Leipzig in the *Acta eruditorum*. I leave it to wordly-wise readers to imagine to what extent delicate relationships of power and prestige, between the various princely or regal courts, might have played a role in this. In a science "... pas encore ... assez établie et cultivée, et même la science de la pratique, et principalement par rapport à l'art d'émouvoir les passions par la Musique ..." ["... not yet made stable or cultivated, both in theory and in practice, and above all in the art of arousing passions with music ..."], among all the innumerable possible divisions, Leibniz sustained: "... nous ne devons faire aucun fonds sur ces instruments de Mathématique dont il a été parlé, *in vino et rapida scribere oportet aqua*. ... Mais l'importance est de choisir le plus commode point pour la pratique."

[12]Bailhache 1992, pp. 73–121.

["... we must not rely at all on these mathematical instruments of which we have spoken, 'we must write on wine ['the wind', in reality a quotation from Catullus] and in running water.' ... But it is important to choose the most convenient point for practice."]

More competent, undoubtedly, than many others in mathematics, it should be noticed that our German scholar admitted that his favourite discipline would not solve the question with its means, because they would be destined to change, and thus to prove useless. He was probably alluding to Henfling's initial neo-Pythagoreanism. But thanks to that covert counting of his soul, he too, in subscribing to the mechanical theory of the strokes coinciding in the ear, did not deviate too far from it.

In the papers prepared for the publication of the article by his court counsellor, the philosopher of pre-established harmony already mentioned what he was to write to Goldbach a few years later. Leibniz was thus led to specify his ideas about the crucial point of how far to divide up the octave. "Ayant considéré un jour et examiné par les Logarithmes l'ancienne division de l'octave en 12 parties égales qu'Aristoxène suivait déja; et ayant remarqué combien ces intervalles également pris approchent des plus utiles de ceux de l'échelle ordinaire, j'ai crû que pour l'ordinaire on pourrait s'y tenir dans la pratique; et quoique les Musiciens et les oreilles délicates y trouveront quelque défaut sensible, presque tous les auditeurs n'en trouveront point, et en seront charmés. Cependant cela n'empêche point que les Musiciens ne retiennent toujours et conservent les vrais intervalles; je souhaiterais qu'on pensât un peut plus qu'on fait ordinairement, aux raisons de la pratique et de ce qui plaît le plus dans les compositions, ..." ["Having once considered and examined with logarithms the ancient division of the octave into 12 equal parts, which Aristoxenus once followed; and having noted how far these intervals, taken as equal, are close more usefully to those of the ordinary scale, I believed that one might normally follow it in practice. And although musicians and delicate ears may find some sensible defect, almost all listeners will not notice any at all, and will be fascinated. However, this does not prevent at all that musicians should not always maintain and conserve the real intervals. I hope that a little more attention than usual may be given to the reasons of practice, and to what is found to be most pleasing in compositions, ..."].

Among 100 arias, our German scholar wrote that he could scarcely find a couple that moved him. "La simplicité y fait souvent plus d'effect, que les ornements empruntés. Qu'y a-t-il de plus simple que le chant de ce Texte *Ecce quomodo moritur justus*?" ["Simplicity often creates more effect than artificial ornaments. What is more simple than singing this text: 'Behold how a just man dies'?"] It is difficult to avoid wondering why he made reference to a funeral song suitable for Lent of April 1709, rather than to other more famous, attractive arias. At the same court of Hannover, one of the best-known musicians, Georg Friedrich Händel (1685–1759) was about to arrive. Would he be still too young? Perhaps the innumerable travels of the composer of operas between Italy and England would not often give him the opportunity to meet our diplomat, who was equally busy, all over Europe.

It is likely that Leibniz did not appreciate very much the music that the German musician was composing in that period. Actually, Händel was writing lyric operas at the time, with arias and recitatives, above all in the style that he had learnt during his stay in Italy in previous years, and these were highly fashionable in the courts. Immediately afterwards, the musician started travelling around England, where he was to settle down, among alternating fortunes, until he gradually became the most famous 'English' musician. When the *Water music* and the edifying oratorios on religious themes were played here, our German philosopher was already dead.

However, Agostino Steffani (1654–1728) with whom Händel had already set up a close friendship in Italy, also worked at the court of Hannover, as choirmaster. And thus the Italian musician was to leave his place at the court to him. Steffani was another man of the church who successfully carried out diplomatic functions, and enjoyed high-ranking protection in the German courts. How can we imagine that Leibniz, who did the same job, did not know him, and had never met him? The Italian had studied music, as well as theology. Another probable acquaintance of the German mathematician was Attilio Ariosti (1666–1740). This friar and musician, frequently the object of gossip as a result of certain aspects of his behaviour, had passed through the Germany of Leibniz on his peregrinations around Europe. He, too, therefore, like Huygens, must have had a good knowledge of the musical life that took place in the courts he frequented on his diplomatic activity. He could undoubtedly not be unaware of the tendencies followed by musicians, and of their preference for the equable temperament most suitable for them.[13] However, the German probably had different musical tastes from the Dutchman.

We must not forget, therefore, as in the other cases encountered in our history, that recognizing the power of music to arouse passions was often followed by an inevitable moral judgement. Neither Leibniz did abstain from expressing his own. "Mais la Théorie doit rendre raison de la fabrique et de l'effect de ces éléments sensibles, et donner l'art d'en former autrement que par instinct; il est à qui on le doit peut-être le plus souvent, lorsque la passion de quelque amant, la rêverie douce de quelque mélancolique ou la joie de quelque agréable débauché a été accompagnée d'un génie naturel pour la Musique." ["But the theory must account for the construction and the effect of these sensible elements and gave us the art of forming them in a manner different from instinct. It is to this, most often, perhaps that we owe it when the passion of some lover, the sweet dream of some melancholy person, or the joy of some merry rake have been accompanied by a natural genius for music."]

Because he was an optimist at all costs, there is little doubt that our philosopher succeeded in reconciling the conflict between the new musical tendencies of his period and the Greek orthodox tradition of the 'true' rational ratios. "Comme les incommensurabilités ne permettent point qu'on puisse garder une entière exactitude, il faut des équivalences commodes, et il y a du génie à les trouver. Notre esprit cherche le commensurable même le plus simple et il le trouve dans la Musique,

[13]Mila 1963, pp. 150–153. Luppi 1989, pp. 21–59 and 150–155.

sans que ceux qui l'ignorent s'en aperçoivent." ["As incommensurabilities do not make it possible at all we can maintain a perfect exactness, are needed convenient equivalences; it takes genius to find them. Our spirit seeks the most simple commensurable, and finds it in music, without the realisation of those who ignore it."] And yet we must recognize to him, in all cases, that remembering Aristoxenus, and also previously Vincenzio Galilei (thanks probably to Henfling), he sustained the solution of the equable temperament in 12 semitones as the best one possible.[14]

Instead of multiplying and extracting roots, we see, with him and Henfling, that they preferred to calculate ratios and *commas* with logarithms, as Huygens had already started to do. To justify, in this connection, a seemingly exaggerated numerical approximation, Henfling maintained that in tuning instruments it was necessary to involve not only the lengths of the strings, or the weights for their tension, but also "... solids, as in the construction of pipes for so-called organs: in this case, as a result of the cubic proportions between the quantities ...", the number of figures would be reduced to one third. After Vincenzio Galilei, who was not remembered (but was he perhaps known?), here we meet, for the second time in Europe, a way of tuning organs with the cube of the ratios, which was foreign to the orthodox Pythagorean tradition, and generally ignored by theoreticians.

The mention of the problem reveals that he possessed more practical knowledge than was suspected. This interlocutor of Leibniz for music probably played the organ. He spoke of this in the following terms: "... in my minor organs, or in my so-called positive [organ] (where I have taken care to include fifteen tones or registers) ...". Thus, in the end, despite all his complex construction made by means of calculations hidden behind his excessively original symbolism, it emerged that his instruments were tuned in the equable temperament of 12 semitones.[15]

A competitor like Joseph Sauveur (1653–1716), who also used logarithms, appeared on the scene, offering another different division of the octave into 43 parts, which he considered a better one. A dispute developed between them. Following pleasure and the ear, the evolution of music will do without all these odd, useless mathematical divisions, because all of them were possible, and all were arbitrary. This French scholar was to be remembered for his experimental studies on multiple audible sounds, called harmonics, already seen with Mersenne. Also his use of the

[14]Bailhache 1992, pp. 125–148. Cf. Baas 1976, pp. 69–72; on the contrary, though attractive and suggestive of some similarities, connections between Leibniz and Johann Sebastian Bach (1685–1750) are historically unlikely. See below, Sect. 11.3.

[15]Bailhache 1992, pp. 89–90 and 98–100. It is curious that the editor and French translator, from the original Latin for the Prussian Academy of Sciences, considered the cubic ratios for the pipes another error among the many that he observed in the work of the German scholar. Instead, as we have already underlined in Sect. 6.7 in note 123, a modern book of acoustics, which takes into consideration the *end effect* in the pipes would confirm, at least in this case, the correct interpretation of Henfling. Fletcher & Rossing 1991, pp. 474–477.

term “acoustique” [acoustics] for the relative art was destined to remain as a branch of physics, unfortunately cancelling music from it.[16]

Beyond their attempts to reconcile things that were incompatible by means of dogmas and *dei ex machina*, Henfling and Leibniz brought into discussion again what the Greek tradition of the *quadrivium* had maintained for 2,000 years. Now whole numbers were no longer sufficient even for music. Did Henfling want to hide them behind his letters because he knew that he would have to abandon them? He multiplied and divided those ratios, but he preferred to call his calculations sums and differences. Not only would he thus remain close to operations with the intervals of music, which are added and subtracted, but he would also maintain a famous classic procedure of ancient Greece. This was Euclid’s constructive procedure among whole numbers, called $\alpha'\nu\theta\upsilon\phi\alpha\iota'\rho\epsilon\sigma\iota\varsigma$ [subtraction in its turn], which, after being repeated several times, terminated at zero: producing the common measure or common divisor. But the Greeks had also found incommensurable cases which never stopped at zero.[17] It was precisely these cases that the interlocutor of the renowned mathematician was trying to reach in order to obtain the temperament with his curious symbolism by means of logarithms, abandoning a sure geometry. What Huygens, on the contrary, had not wanted to do, must have been appreciated by Leibniz, who dealt with similar problems more generally with his new infinitesimal calculus.

Even harmony seemed to suggest to both of them that it was possible to subdue incommensurables, starting again from whole numbers, and continuing to calculate with them. To dominate ancient Greek music, numbers with the *logos* may seemed to be sufficient to the Pythagoreans and Plato. Now, in the Europe of the eighteenth century, tempered tonal music required another solution, with new numbers: the irrationals. By means of them, to the equable temperament Leibniz made to perform a true miracle in the theory of music: to be the heir of the Pythagorean tradition, justified by the theory of strokes, but at the same time to abandon it, to follow the requirements of the new ears, educated in his times. Would a soul performing secret arithmetic be up to the task? Now, however, they could also recover Aristoxenus, without condemning him, but unfortunately, they were to forget Vincenzio Galilei and Simon Stevin. As regards what Leibniz had taken from the tradition started by the Pythagorean sects, and what he rejected of them, every discussion must understand and take into clear consideration his well-documented affirmations. For music, he tried to reconcile as far as possible, whole numbers, in the Pythagorean style, with the irrational numbers that they forbade. With these latter numbers, which he considered to be as valid as the former, he could calculate the otherwise impossible ratios of the equable temperament, which the philosopher from Hannover presented as the best one for the practice of musicians. His decision, to reserve for the unconscious of the soul the famous faculty of counting in whole

[16]Bailhache 1992, pp. 28–36. The word “Acoustique”, “Acoustics” had already been used by Francis Bacon and Gaspar Schott (1608–1666). Cf. Gouk 1999, pp. 108 and 157.

[17]See Part I, Chap. 2.

numbers, depended on the desire to harmonise the Pythagorean tradition with its superseding. Proceeding continuously towards a world by now full of irrationals, infinites and infinitesimals, he was putting aside, through them, the classic Euclidean geometry. We might say that, always tormented by dualism, he had a Pythagorean soul in an Aristoxenic body.

When Leibniz attributed a great power to music, in arousing or stifling passions, does he not recall to us the legends of Pythagoras and Plato's *Republic*? "Wenn schon die Lieder das höchste Vergnügen hervorbringen können, wenn die Soldaten durch Kriegstrompeten zur Verachtung des Todes beseelt werden, ... Die Sybariten ertheilten Belohnungen demjenigen, welcher eine neue Art von Vergnügungen erfunden hätte: ich aber glaube, daß die Christenheit demjenigen sehr viel schuldig sein werde, welcher bewirkte, daß die meisten Vergnügen mit der Frömmigkeit verbunden sei." ["When songs can already procure the highest pleasures for us, when soldiers are incited by the trumpets of war to despise death, ... The sybarites attributed rewards to all those who discovered a new form of lust: but it is my belief that Christendom will be more indebted to those who have acted in order to unite the greatest pleasures with religious devotion."][18]

Music, of course, continued to emerge here and there from the more than copious mathematical and philosophical production of Leibniz. Let us recall a few other cases. In 1666, at Leipzig, the *Dissertatio De Arte Combinatoria ... Demonstratio Existentiae Dei, ad Mathematicam certitudinem exacta. [Dissertation On the Combinatorial Art ... Precise Demonstration of the existence of God, reduced to a mathematical certainty]* was published. Like a demiurge, he seemed to invent his own world, starting from the letters and words. Then, it was only left for him to classify all the possibilities, patiently, calculating the combinations, which he called "variationes" ["variations"]. He combined the registers of organs. Under the Hebrew letter א, he included also here the harmonic succession formed by the reciprocals of whole numbers arranged in a triangle, baptised the "harmonic triangle", taking

[18]Cf. Bailhache 1992, pp. 36–50. Cf. Luppi 1989, pp. 144–154. Cf. Bailhache 1995b. In his article, the Frenchman judges the musical theories of Huygens, Henfling and Leibniz anachronistically by the standard of future justifications based on the harmonics of sound or on the physiology of the ear, like Hermann Helmholtz (1821–1894); see below, Sect. 12.2. For the division of the octave, he took for correct, true, objective and 'scientifically' proven, the so-called "just" ratios of Zarlino. But the belief that these were "just" was an arbitrary dogma, a heritage of the ancient Pythagoreanism, while musicians were successfully taking a completely different direction. In the eighteenth century, Leibniz had understood that the mathematical theory of music would have to adapt to the requirements of composing and playing. In the twentieth century, Bailhache relegated the problems of composers and players to the last position, preferring to leave his own philosophy of mathematics in the first place. For music, even if almost only in this, my essay follows Leibniz. What our French philosopher and mathematician calls, referring to Leibniz, "... sa 'retraite' sur le tempérament égal ..." ["... his 'retreat' over the equable temperament ..."] shows, on the contrary, how well the famous German philosopher and mathematician knew the music of his epoch, and how much he desired to harmonise it with his own infinite and infinitesimal numbers. Bailhache 1989; 1995a; 1996. Instead, Haase exaggerated the Pythagoreanism of Leibniz, which is called "... 'säcularisierter' Pythagoreismus ..." ["... 'secularised' Pythagoreanism ..."], almost imprisoning him inside it; Haase 1960 and 1985.

his inspiration from Pascal's "arithmetic" one. In a table indicated by another Hebrew letter, he listed the number of things, with their combinations (without any repetition). 3 ones could be combined in 6 different ways (3.2), 4 in 24 (4.3.2), 5 in 120, ... 12 in 479,001,600, 24 in 62... followed by 22 figures (24.23 ... 3.2.1 = 24! nowadays called factorials).[19] Through the art of variation/combination, he studied and reconstructed the creation of the world.

"...omnia ex intima variationum doctrina erui ...quae sola prope per omne infinitum obsequentem sibi ducit animum, et harmoniam mundi et intimas constructiones rerum seriemque formarum una complectitur, cujus incredibilis utilitas perfecta demum philosophia aut prope perfecta recte aestimabitur. Nam VIImus est in complicandis figuris geometricis usus, qua in re glaciem fregit Joh. Keplerus lib. 2, *Harmonicωv*. Istis complicationibus non solum infinitis novis theorematibus locupletari geometria potest, nova enim complicatio novam figuram compositam efficit, cujus jam contemplando proprietates, nova theoremata, novas demonstrationes fabricamus, sed et (si quidem verum est, grandia ex parvis, sive haec atomos sive moleculas voces, componi) unica ista via est in arcana naturae penetrandi, quando eo quisque perfectius rem cognoscere dicitur, quo magis rei partes et partium partes, earumque figuras positusque percepit." ["...I deduced all these things from the deepest doctrine of variations, which alone now leads the mind to itself, through all the consequent infinity. Alone, it embraces the harmony of the world, the most isolated constructions of things, the series of forms, the incredible utility of which is estimated by philosophy exactly to be perfect, or better, perfect with order. Actually, it is extremely useful to put geometrical figures together, an activity in which Kepler broke the ice in his second book of the *Harmonicωv*. Geometry may benefit from this composing together, not only with infinite new theorems. Surely, a new making up together produces a new composite figure considering which we now create new theorems, new properties, and new demonstrations. But also (even if it is undoubtedly true that I composed the large things starting from the small ones, both these atoms, and the molecules and the sounds), this is the only way to penetrate into the secret things of nature. Because any person is said to know a subject more perfectly, the more he has understood the parts of the subject and the parts of the parts, and their figures and their position."][20]

Thus our German mathematician calculated the combinations with the repetition of six notes. He ignored *B* (*si*), as Henfling was to do. Here, the combinations between the things explicitly became those that are nowadays called 'variations' in a succession of musical notes. "Esto propositum: dato textu omnes melodias possibiles invenire." ["Let the aim be: in a given text, find all the possible melodies."] When he calculated them, he wrote: "I. ut, re, mi, fa, sol, la Variatio ordinis est 720. II. ut, ut, re, mi, fa, sol Variatio ordinis est 720 – 120 f. 600. Non

[19]Leibniz 1666, pp. 17, 23 and 61. Serres 1968, pp. 186–188 and 468–470. 479,001,600 will be the number of combinations with 12 notes used as a series by Arnold Schönberg; see Sect. 12.4. Tonietti 2004.

[20]Leibniz 1666, p. 34.

solum autem *ut*, sed et quaelibet 6 vocum potest repeti 2 *mahl*, ergo 6×600 f. 3600; ...” [“do, re, mi, fa, sol, la, the variations in the order are 720 [6!]; do, do, re, mi, fa, sol, the variations in the order are 720 - 120 = 600. Not only C but also any other of the six notes can be repeated twice, thus $6 \times 600 = 3{,}600$; ...”]. Playing with the repetitions, in a first calculation he obtained 187,920. Then he wondered, if he had also considered the seventh notes, B, the pauses and the differences in time: “... quantum erit mare melodiarum ...?” [“... how big will the sea of melodies be ...?”][21]

“... la perception confuse de l’agrément ou des [dés]agréments qui se trouve dans les consonances ou dissonances consiste dans une Arithmétique occulte. L’âme compte les battemens du corps sonnant qui est en vibration et quand ces battemens se rencontrent régulièrement à des intervalles courts, elle y trouve du plaisir. Ainsi elle fait ces comptes sans le savoir.” [“... the confused perception of appreciation or non-appreciation, which is found in consonances or dissonances, consists of a covert arithmetic. The soul counts the strokes of the vibrating resounding body, and when these strokes regularly coincide, at brief intervals, it takes pleasure in this. Thus it performs its counting without realising it.”][22] “La musique nous charme, quoyque sa beauté ne consiste que dans les convenances des nombres, et dans le compte dont nous ne nous appercevons pas, et que l’âme ne laisse pas de faire, des battemens ou vibrations des corps sonnans, qui se rencontrent par certains intervalles.” [“Music fascinates us, although its beauty only consists of the ratio of numbers and in counting, which we do not realise, and which the soul does not cease to perform, in counting the strokes or the vibrations of sounding bodies which meet together at certain intervals.”][23]

“Und gleichwie fast nichts den Menschlichen Sinnen angenehmer als die Einstimmung der Musick, so ist nichts dem verstand angenehmer als die wunderbare einstimmung der Natur, davon die Musick nur ein vorschmack und eine kleine Probe.” [“And as hardly anything proves to be more pleasurable to the human senses than harmony in music, so nothing is appreciated more by our understanding than the wonderful agreement in nature, of which music represents only a foretaste and a small proof.”][24] “La Musique est subalterne à l’Arithmetique, et quand on sçait quelques experiences fondamentales des consonances et dissonances, tout le reste des preceptes generaux depend des nombres, et je me souviens d’avoir un jour fait une ligne harmonique divisée en telle sorte, qu’on y pouvoit determiner avec le compas les compositions, differences et proprietés de tous les intervalles de Musique. Et on peut monstrer à un homme qui ne sçait point de Musique le moyen de composer sans fautes.” [“Music is subordinate to arithmetic; when we become acquainted with certain basic experiences regarding consonances and dissonances, all the rest of the general precepts depend on numbers. I remember tracing a

[21] Leibniz 1666, pp. 68–69.

[22] Leibniz 1960/65, IV, pp. 550–551. Bailhache 1992, p. 41.

[23] Leibniz 1960/65, VI, p. 605.

[24] Leibniz 1960/65, VII, p. 122.

harmonic line one day, divided in such a way that it was possible to determine with a compass the compositions, differences and properties of all the musical intervals. It is possible to teach a man who has no knowledge of music the way to compose without any errors."][25] Of course – it goes without saying! – it took genius, practice and the imitation of true masters to compose good music. However, Leibniz was continually involved in magnifying the rules and universal reason, because, in the event of new circumstances, theory, even without practice would always surpass all blind practice without theory.

Leibniz had written to Pierre Bayle (1647–1706): "…the supreme wisdom, which is the source of all things, acts like a perfect geometer, in accordance with a harmony to which nothing can be added".[26] Music thus became one of the arguments, small or large, in favour of the harmony that supported the cosmos, according to our renowned philosopher. It was expressed in that transcendent and symbolic language created for the mathematical sciences. One, perhaps the most important, of the recurring motives in the theo-rational philosophy of the German diplomat appeared to be a faith that mathematics expressed the harmony of the cosmos, and that God revealed Himself in harmony.

Leibniz retained that the physical and mathematical sciences were governed by a principle of continuity. Because the ellipse could be transformed continuously into a parabola, gradually moving one of the foci further away to infinity, then the theorems of the ellipse were valid also for the parabola. Because rest became an infinitely small velocity, the rule of rest was to become a particular case of that of movement. "But the will of God is regulated by wisdom, and the geometers would be equally surprised by observing this kind of irregularity in nature, as if they had found a parabola to which the properties of an ellipse with an infinitely distant focus could not be applied. Thus I do not think that these irregularities can be found in nature; the more we know nature, the more geometrical we find it".[27]

In a letter to Pierre Varignon (1654–1722), Leibniz seemed to prefigure a future physicist-cum-mathematician. "…however much I see that is regular and orderly in nature, it is necessary for physics to harmonise with geometry, and the opposite would happen if, while geometry requires continuity, physics sustained a sudden interruption. According to me, everything is connected in the universe, as a result of metaphysical reasons, whereby the present is pregnant with the future, so that no state can be explained naturally except by means of the state which preceded it immediately. If this is denied, the world would be discontinuous, which would oblige us, in the explanation of phenomena, to recur to miracles or to chance". Like Jan Hudde (1628–1704), he claimed to succeed in reproducing the features of any face by means of algebraic curves. "…It is possible to express by means of a formula …any essential property of the universe, in such a way that it would be possible to read what the subsequent states of all its parts at any future moment

[25]Leibniz 1960/65, VII, p. 170. Liebniz 1967, II, p. 747.

[26]Leibniz 1967, II, p. 759.

[27]Leibniz 1967, II, p. 761.

you may choose will be."[28] Here, our German metaphysicist already seemed to be capable of foreseeing the destiny of the cosmos, as Pierre Simon de la Place will claim later (1749–1827).

In 'suppressing' the obstacles of matter, Leibniz had gone a long way beyond Galileo. He for himself had constructed an ideal world, perfectly continuous, without flaws, without fractures, without catastrophes. But it was still, unfortunately, perfectly unstable, and thus the objections raised by the Galileian Simplicius would also apply to him. However, now Leibniz had succeeded in inventing a new mathematics, suitable for creating the future of the cosmos. From moment to moment, with ever shorter increases, that is to say, from infinitesimal to infinitesimal, he climbed up the continuous scale of time. With short but sure steps, the same principle of continuity guided him also through the mazes of space, interpreted therefore as a succession of infinite evanescent linear lengths. Having reduced movement to the ratio of such infinitesimal differences, from the cause that always produced it, it would gradually be possible to obtain the effect. There was therefore a need for a new algebra, dealing with tiny differences of time, symbolised by *dt*, and as regards space, *dx*, giving the rules to multiply them, divide them, add them, subtract them, together with the procedures to calculate the differentials of the habitual algebraic expressions. In the end, foreseeing the future of a phenomenon meant carrying out the addition of all the infinitesimal increases undergone at every infinitesimal interval of time. Leibniz called this operation the integral, using, as its symbol, a large S to indicate the sum: $\int$. The algebraic equation with differentials, in short the differential equation, entered into physics and mathematics; solving it, finding its integral, meant finding the solution to the problem. In this way, would it be possible to foresee the future? Leibniz hoped so, but the universe was to prove to be less harmonic, more complex and "catastrophic" than he desired.

However symbolic it was, to him the algebra of Viète and Descartes did not appear to be sufficient to deal with geometrical problems such as the calculation of the centre of gravity, areas, or the length or curves. What was needed was another "transcendent" kind of algebra, and also a "characteristic", that is to say, a more general writing of algebra and arithmetic to express the relationships between thoughts. This "characteristic", that is to say, a kind of logic, would cancel the chimeras, and would be impossible for the ignorant to use. "Because this writing is much more instructive than that of the Chinese, where it is necessary to be wise to know how to write". Leibniz would not stop at the "…*lingua Franca* faite du Latin que je trouvay assez intelligible, quoyqu'il n'y eut ny cas ny temps ny autres flexions, …" ["…*lingua franca* made of Latin, which I found to be fairly comprehensible, although there were no cases, or tenses or other declinations, …"]. But he would even have wanted to base his new language on a single noun, *Ens* [being] or *Res* [thing] and the single verb "to be" with adjectives. "Petrus scribit, id est: est scribens." ["Peter writes, that is to say: he is a writing one."]

[28]Leibniz 1967, II, p. 764.

From the logical transcendences of the *Mathesis universalis* to the analytical transcendences of differential calculus, Leibniz would have liked to unleash all the power of his mathematics on the "... problemata concreta ... simplicissima sunt quae exhibet Optica. Hanc sequitur Mechanica, inde Musica, post Astronomia et Geographia ac Navigandi ars, et Architectonica, et Scientia militaris. Et omnino post Opticam et Mechanicam et Musicam et Astronomiam caetera tractanda eo ordine, quem postulat usus eorum in vita, ut appareat origo inventionis; ea enim cognita mirifice et acuitur ingenium et memoria sublevatur." ["... concrete problems, among which the most simple ones are those offered by optics. This is followed by mechanics, and then music, astronomy, geography, the art of navigation, architecture and military science. In all, after optics, mechanics, music and astronomy, the rest are dealt with in that order which their use requires in everyday life, as the origin of the invention makes clear; actually, if we know this, we sharpen our wits and we support our memory."][29]

The new mathematics, which we have already seen in the correspondence with Christiaan Huygens, came out into the open during 1684 in the *Acta Eruditorum* of Leipzig, and was called by Leibniz "Differential calculus" or then, 2 years later "Analysis of indivisibles and infinites". Enunciated here, the fundamental theorem of analysis was communicated in 1690 also to the Dutch scholar in the following terms. "Et comme les racines sont reciproques aux puissances, de mème les sommes sont reciproques aux differences, par exemple comme $\sqrt{yy} = y$ et $\sqrt[3]{y^3} = y$ de même $\int d\bar{y} = y$ et $\int \int d d\bar{y} = y$. Par le moyen de ce calcul je me suis avisé de donner les touchantes et de resoudre des problemes *de maximis et minimis*, lors que les equations sont fort embarassées de racines et de fractions, sans que j'aye besoin de les oster, ce qui m'epargne souvent des grandissimes calculs. Par le même moyen je reduis à l'analyse les courbes que M. Des Cartes appelloit Mechaniques, comme par exemple les Cycloides, exprimant par une equation la relation entre x et y abscisse et ordonnée de la courbe." ["And as the roots are the reciprocal of the powers, likewise the [integral] sums are the reciprocal of the differences [differentials]; for example $\sqrt{yy} = y$ e $\sqrt[3]{y^3} = y$ likewise $\int d\bar{y} = y$ and $\int \int d d\bar{y} = y$. By means of this calculation, I have found the way to give the tangents and to solve problems of maxima and minima, when the equations are obstructed by roots and fractions, without the need to remove them. This often saves me a very large number of calculations. Likewise, I reduce to analysis the curves that Mr. Descartes calls mechanical, such as, for example, cycloids, expressing by means of an equation the relationship between x and y, the abscissa and the ordinate of the curve."][30]

He immediately showed that his mathematical procedure was faster, more automatic and more effective than the previous ones, like those followed by

[29]Leibniz 1962, VII, pp. 9–17. Leibniz 1960/65, VII, p. 32. Leibniz 1966, p. 30. Leibniz 1994, p. 38. Leibniz 2000, II, pp. 381–382. Walker 1972, pp. 295, 297–298 and 305–306. Tagliagambe 1980, "Chapter I".

[30]Huygens 1901, IX, pp. 450–451.

Huygens. The curves like the isochronous and the catenary ones were not among those included in an equation by Viète and Descartes, but they could not be ignored because "...nature often makes use of them". Hence the interest in the use of a method, "Analysis of the infinite", which went beyond both ordinary algebra and the indivisibles of Bonaventura Cavalieri (1598–1647) or the arithmetic of the infinite of Wallis. It did not go back either with geometrical figures or with numbers, but contemplated magnitudes in general. With his new analysis, Leibniz dealt with geometrical-physical problems, some of which were of practical interest for navigation, like the loxodromic curve,[31] for ballistics, for "the harmonic laws of planetary movements".[32]

"Car ce que j'aime le plus dans ce calcul, c'est qu'il nous donne le même avantage sur les anciens dans la Geometrie d'Archimede, que Viète et des Cartes nous ont donné dans la Geometrie d'Euclide ou d' Apollonius; en nous dispensant de travailler avec l'imagination." ["Because what I like most in this [infinitesimal] calculus is that it gives us the same advantage over the ancients in the geometry of Archimedes, as Viète and Descartes gave us in the geometry of Euclid and Apollonius; exempting us free working with our imagination."] "Quand je parle de la perfection de la Geometrie et de l'Arithmetique, je l'entends avec quelque latitude. Je crois qu'on pourroit parvenir à pouvoir donner tousjours la methode des solutions, ou à en demontrer l'impossibilité, mais ce ne sera pas tousjours par les meilleures voyes. [...] Je suis de vôtre sentiment, qu'il faudroit suivre les projets de Verulamius sur la physique en y joignant pourtant un certain art de diviner, car autrement on n'avancera gueres." ["When I speak of the perfection of geometry and of arithmetic, I mean it broadly speaking. I believe that we can always arrive at the identification of the method to find the solutions, or at the demonstration of their impossibility, but it will not always be by the best routes. [...] I am of your own [Huygens'] opinion that we should follow the projects of Verulamius [Francis Bacon] on physics, but adding a certain art of conjecture, otherwise not much progress will be made."][33]

Huygens was not to declare himself so optimistic about the power of infinitesimal calculus in which Leibniz believed. But the latter rejoined: "Cette reduction aux quadratures, que vous appellés impossibiles est ce que je souhaiterois de pouvoir tousjours obtenir, pour les problemes des tangentes renversées. Enfin je ne demand presque que cela pour la perfection de la plus importante partie de la Geometrie. [...] Il est vray comme vous dites, Monsieur, qu'il n'est pas assez de faciliter le calcul, il faut souvent quelqu'autre chose. Cela se voit dans l'Algebre même. [...] Il en est de même de mon calcul transcendant. Mais quand on a reduit les methodes à un simple calcul on s'avise plus aisément de ces adresses." ["This reduction to quadratures, which you [Huygens] call impossible, is what I would always like

[31] Loxodromics [oblique course] is the route in the sea that intersects the meridians at a constant angle.

[32] Leibniz 1962, V, pp. 226–233.

[33] Huygens 1905, X, pp. 227–228.

to be able to obtain for the inverse problems of tangents. Lastly, I hardly request anything other than this for the perfection of the most important part of geometry. [...] It is true, as you [Huygens] say, Sir, that it is not very much to facilitate the calculation, something else is often necessary. This can be seen in algebra itself. [...] The same thing happens with my transcendent [infinitesimal] calculus. But when the methods have been reduced to one simple calculation, it is easier to realise these capacities."][34]

In the *Journal des Savants* (in 1694) our German mathematician rendered justice to Newton, to whom geometry, optics and astronomy were greatly indebted, for "having had in his head something of the kind, according to what has subsequently become known". Then he added: "It is true that he makes use of other characters: but as characterisation itself is, so to speak, a large part of the act of invention, I believe that ours *donnent plus d'ouverture* [open up the field better]". By means of infinite series, it was possible to deal with also irrationals and transcendent irrationals, together with whole numbers and fractions. "In the end, our method, being precisely that part of general mathematics which deals with the infinite, represents exactly what we feel the real need for, because, applying mathematics to physics, the infinite character of its Author [God] enters in an ordinary manner into the operations of nature". Criticised specifically for his strange calculations performed with infinites and infinitesimals, too distant from the ancient methods, Leibniz defended himself by creating a kind of approximate calculus in which the terms were very large or very small compared with others. By so doing, he only differed from the methods of Archimedes in the symbolic expressions.[35]

More than Newton used his infinite series, our German mathematician put some differentials and some integrals into his considerations on dynamics, even if the ratio between formulas and text remained very low for the customs of modern physicists, as in Newton. Many of the signs we today use to write mathematical formulas and physical laws were introduced by Leibniz for algebra and for differential calculus: fractions, brackets, logarithms *log*, integrals $\int$.[36] Such a liking for new notations was, as we shall soon see, foreign to Newton. Today we understand relatively better the mathematical writings of the German, because his new notations were to meet with great success.

At the beginning of the new eighteenth century, in his comments on *An Essay Concerning Human Understanding* by John Locke (1632–1704), Leibniz honoured us with his observations on Chinese culture. "This has been very well understood also by geometers. They could easily have proved their propositions (at least the most important ones) by means of experience, and I have no doubt that the ancient

[34]Huygens 1905, X, p. 261. In these letters pp. 229–230, 263 and 285–286, Leibniz declared his interest in the division of the octave into 31 parts, and in publishing an article about music by Huygens in the journal *Acta Eruditorum*. But we already know that the Dutchman would not realise this.

[35]Leibniz 1962, V, pp. 306–308.

[36]Leibniz 1962, VII, pp. 218–223.

Egyptians and Chinese possessed such an experimental geometry. But the true geometers, above all the Greeks, desired to show the force of reason and the excellence of the science, demonstrating that in these subjects everything can be foreseen by means of internal light before experience. [...] And it is remarkable that the Greeks, who were so imprecise in other things, were so exact in geometry. I attribute this to Providence, and I believe that without this, we would hardly know what a demonstration is. Likewise I believe that it is above all in this sector that, up to today, we are superior to the Chinese." At this point, we therefore discover that the European scholars so eager of subtracting the demonstrations, described by me in Chap. 3, from the Chinese geometers, were not only questionable historians, but also modest imitators of Leibniz.

A few pages earlier, we saw that our German scholar distinguished the "characteristic" writing of logic from the Chinese characters: the former was more suitable for the instruction of the latter. For him, the Chinese language would be so complicated as to be reserved for few "wise men". He also knew that there had been a "... Chinese emperor [Qinshi Huangdi] who had assumed the task of destroying literati, because they were disturbers of the public order."[37]

Together with Huygens, among all the European natural philosophers and mathematicians, Leibniz was undoubtedly the one who had explicitly taken the most interest in China. But he received his information on the subject arrived through books and pamphlets written by missionaries, with the consequences that we shall see. In the *Acta Eruditorum* of 1697, the booklet *Novissima Sinica [The latest news from China]* was published by the Portuguese Jesuit Giuseppe Suario with the preface and comments by the German philosopher himself. From this, installed a curious tribunal of judgement, he deduced that the Chinese were inferior in the mathematical sciences, not knowing "... the art of demonstration, ... and [inferior] in the military art ... not so much out of ignorance as due to a deliberate choice of theirs, because they despise everything that causes, or fuels a certain ferocity in men, and they are contrary to wars, almost as if to imitate the highest Christian doctrine [...], but now we have arrived at the point where also good people have to cultivate the arts of causing harm, so that the evil will not take possession of absolute power. Thus we are superior in these arts." However, the German diplomat added: "... we are undoubtedly beaten (and I am somewhat ashamed to admit it) in practical philosophy, that is to say, in the principles of Ethics and Politics, ...".

In the Chinese, Leibniz praised their obedience to their superiors, and their respect for elderly people, especially towards their parents. Not that the worst vices did not exist also in China, because, as in "...*Arlecchino imperatore della Luna [Harlequin, emperor of the Moon]*, ...*c'est tout comme ici* [everything happens like here] ...". And yet the German philosopher praised the Chinese social customs, which were capable of controlling them better. With all due caution, the reigning Emperor Qing [Manchu] had opened up to European scholars like Ferdinand Verbiest (1623–1688), a Jesuit pupil of Johann Adam Schall von Bell,

[37]Leibniz 1967, II, 163 and 739.

thus favouring the European sciences.[38] Among alternating fortunes, successes and persecutions, the European missionaries had succeeded in remaining in China, also under the new Qing dynasty. Above all, Kangxi (1662–1722) gave them space for their activities. In this, the Manchu emperor had again assumed the classic Chinese Confucianism as the guidelines of administrative equilibria, but he also intelligently seemed to tolerate Buddhism, as well, together with other Western novelties, promoting a relatively enlightened culture. These Chinese events, however, initially so promising, were to prove to be too poor in optimism in the end, above all thinking of the philosopher of pre-established harmony. Curiously incapable of learning from its previous error, and less elastic than the Jesuits, the Roman Catholic Church re-created a contrast between the Imperial rites and its own version of Christianity, and thus not even the imperial Confucian *ren* [benevolence] could be enough for these foreigners to be seen in a positive light.[39]

But, like Matteo Ricci and other missionaries, Leibniz continued to believe in the power of geometry and its metaphysics as the "... way to the Christian religion ...". "... because virtue derives from wisdom, and in turn, the soul of wisdom is truth, and lastly, as those who have fully experimented geometrical demonstrations have a clear inclination towards the eternal truths and can discern the certain from the uncertain, while other mortals falter between conjectures, and do not know – almost as Pilate asked – what truth is, there is no doubt that the Emperor of the Chinese understood perfectly what Plato once tried to inculcate in the West, ...". If the Empire at the Centre had learnt the transcendent philosophies and the European sciences, he was afraid that "... we will soon be inferior to the Chinese in every field: ...". He hoped that in exchange, "Chinese Missionaries" would arrive in Europe to teach, in turn, "... the use of practical Philosophy, ... the use and the practice of natural theology, ...". The event that "... the divine gift of the Christian Religion" had been brought to China was a great merit recognized to the Jesuits by Leibniz, together with the role played in making peace between this country and Russia. For him, Antoine Arnauld, the Jansenist scholar who was a friend of Pascal,[40] had exaggerated in attacking them. *Natürlich*, the Protestant metaphysicist would have desired the realisation of a religious harmony in Europe between the Church of Rome and other Christians.[41]

Other Italian and French Jesuits were not so mean in supplying Leibniz with various summaries of Chinese culture, which he commented on, in his own way. The fundamental question was whether there were enough hints of transcendence in Confucian Chinese texts to allow the Christian revelation to pass. In this connection, our philosopher commented: "But Heaven and universal substance are not the same thing." Here, he seemed to understand the difference between the sky of the Chinese and the transcendent one, the dualist Heaven of his metaphysics. While the *Shangdi*

[38] Leibniz 1987, pp. 18–21.

[39] Leibniz 1987, pp. 182–183. Sabattini & Santangelo 1989, pp. 554–556.

[40] See above, Part II, Sect. 9.1.

[41] Leibniz 1987, pp. 21–25.

[emperor on high] was identified by the imperial men of letters with the sky, for the Italian Jesuit who succeeded Matteo Ricci at Peking, Nicola Longobardi (1565–1655), God created Heaven. At this point Leibniz commented by distinguishing in parallel the soul from the body. And yet, in judging that distant culture, the theo-rational philosopher appeared to be far more benevolent than the Jesuits he was reading. For the latter, the Chinese limited themselves to tangible appearances, and thus fell into atheism. For the former, "... words force them only not to advance anything on this subject which is without any foundation."[42]

The Jesuit interlocutor mistook the symbols of the *Yijing [Classic of Changes]* for those of astrology, gave a debatable interpretation of the *li* [reason for things], mixed up the five *xing* [phases] with the four Greek elements, and too late introduced the *qi*.[43] Thus it was confused with the *taiji* [extreme limit], first put aside, and then likened to the *li*. What can our German natural philosopher ever have understood of the *Yin* and the *Yang*? The *qi* was called "Being", a term used also for the *li*. And so on, everything was melted down and recast in current Western dualist moulds. Nevertheless, every now and then, Leibniz searched for harmony also in China, and took the summary in a "positive sense". Anyway, this Chinese cosmology was presented as proceeding "... by chance ... in a natural manner, and without any deliberation ...". This reminded Leibniz of Descartes. To cut a long story short, the judgement of the missionary in China was that their philosophy was materialistic, immanent and professed by atheist Confucian literati.[44]

On the other hand, a French Jesuit presented the 64 *gua* [hexagrams] of the *Yijing* as "... a metaphysics in numbers or a general method of sciences, ... [...] ... it consists of a double series of numbers, plane and solid, so closely connected together with one another by all the consonances of music and by a perpetual harmony, squaring exactly with the 64 characters and the 384 lines of the figure, in such a way that they represent the periods with all the harmony of the heavenly movements. As well as they show all the principles needed to explain nature and the properties of all things, the causes of their generation and of their corruption. They at the same time supply everything necessary not only to re-establish the original form of ancient Chinese music, which has been lost for at least fifteen or twenty centuries, but also to recover what we have lost of Greek music ...". However, without doubt worried he was giving China the superiority over Europe, this other missionary sustained that the inventor of the system, the mythical Fuxi, was not really Chinese, but had been confused with Zoroaster, Mercury Trismegistus and Enoch. The Chinese characters became Egyptian hieroglyphics with him.[45]

Leibniz showed a certain interest in the Chinese language, about which he posed questions to his correspondent from Peking. But he found only scarce information about dictionaries, unreliable etymologies and imaginative splitting

[42] Leibniz 1987, pp. 97–98.

[43] See above, Part I, Sect. 3.5.

[44] Leibniz 1987, pp. 56–61, *passim*, 98.

[45] Leibniz 1987, pp. 105, 115, 118.

up of the characters known as hieroglyphics. "…the character Taï [*tai*], *maximus* [greatest], is composed of two other characters, i.e. Ta [*da*], *magnus*, [great] and the dot (.) [sic!], *dominus* [lord], with the result that Taï-Ye [*taiyi*, the greatest one] analysed means exactly *unus magnus dominus*, a title which can be applied only to God, like *unus trinus maximus*. Hence it derives that to indicate the same idea with a single word, they have joined together the character indicating unity (–) with the character Ta, *magnus*, thus obtaining the character Tien [*tian*] which means *Heaven, the archetype* rather than the material sky, for which it is also used. Add to this the great relationship [? sic!] that exists, both as regards the concept and the sound, between the word Tien and the word *Theon, Theós* [God in Greek], both equally aspirated in their pronunciation. These words, which they often have in their mouth, *ye ta guei Tien* [? *yi ta wei tian* the great one becomes heaven], that is to say, the character Ye, *unus* [one], and the character Ta, *magnus* [great], form the character Tien, which is interpreted more naturally as applied to the Lord of Heaven than Heaven itself." Another, decisive, proof was supplied shortly afterwards. "…the second forms a cross, and is the number ten with the Chinese [*shi*, 10], as with the Romans and the Egyptians; this second part of the hieroglyphic means, for the Chinese, the totality of the things that the sky includes, as for Pythagoras, who represented all things by means of his famous quaternary 1.2.3.4, the sum of which is ten."

According to the French interlocutor, the imperial literati would have "…lost the hieroglyphic meaning of the characters, and the understanding of their ancient books, …" in which even the Christian revelation would be hidden, and which would be much older than the Chinese themselves. The Jesuit ventured to affirm that his analyses of the ancient texts, and of the "hieroglyphics" would bring to light "…the universal system of the ancient and divine magic, …". He also asked for lights about the "…new numerical calculus…" of Leibniz in relation to the hexagrams of the *Yijing* and about Kepler's correspondence with father Terentius.[46] He would have liked "…Chinese books could provide the wise men of Europe with material to fertilise the great design of perfection of the sciences, in which You [Leibniz] have so far played such a large part; …".[47]

These were the foundations on which Leibniz could construct his essay on philosophy and the science of the Chinese culture. The philosopher of harmony pre-established it in China, as well, interpreting with optimism even its incurably immanent aspects. Perhaps they considered "…God as the soul of the world and united Him with matter, …"? Then, discussing between *li* and *qi*, "Their disciples will more easily become convinced that God is *intelligentia supramundana* [intelligence above world], and above matter." Leibniz wanted to disprove those missionaries who denied that the Chinese admitted spiritual substances. As that culture moreover appeared to him to be the most ancient of all, he tried to "…give a good meaning to it." The *li* was thus separated from the material sky and became

[46] See above, Sect. 8.3.

[47] Leibniz 1987, pp. 121–122, 125–7 and 186.

transcendent, like his Christian God, being therefore able to represent a vehicle of conversion to the true religion. "*Li* . . . *Reason* or foundation of all nature. . . . *Li* is the law . . . they call it Being, Substance, Entity (par excellence). . . . is infinite, eternal, uncreated, . . . it is the *physical principle* of Heaven and Earth, . . . ; but also the *moral principle* of virtues, of habits and of the other spiritual things." Jumping with both feet together the *taiji*, the *li* assumed the figure of a ". . . Globe or Round. I believe that this refers to our ways of speech, when we say that God is a sphere or a circle the centre of which is everywhere, and the circumference of which is nowhere."[48]

But, in order to be able to identify the Chinese *li* with the Christian transcendent God, our theo-rational metaphysicist had to refute that Jesuit ". . . prejudiced against Chinese doctrine . . . who was left dazzled himself by the arguments of certain atheist mandarins, . . . ". In order to succeed in bringing the "air" [*qi*] closer to the European *pneuma Spiritus*, and also to his *Entelechia* [soul, final accomplished act by which form would be given to matter], the *li*, Leibniz made this last term ambiguous. The *li* would be defined as ". . . sometimes the supreme Spirit par excellence, sometimes also it stands in for every Spirit, perhaps because etymologically it means reason or rule." Certain philosophers, Chinese and European, could confuse the spirits with the organs of the body, simply because ". . . they had no knowledge of the pre-established harmony . . . ". Distinguishing the *li* from the *qi*, not even the Chinese men of letters would make the spirits material. And so on.

The renowned mathematician and jurist received an aphorism taken from the *Lunyu [Dialogues]* of Confucius (Chap. XV, 28) from another French Jesuit, his adversary. ". . . explaining the Li by the Tao (rule) [*dao* way], he says that the [*dao*] is unable to know man, but man is able to know it. It would be necessasry to have a very precise translation of this passage, to see whether Confucius is speaking about the first principle, or the Law, or the rule *in abstracto*; . . . ". At the only point, probably, where Leibniz encountered the *dao*, that is to say, the way, the process, the indescribable, ineffable, spontaneous becoming, it was mistaken by him for a rule or a law. Quite rightly, without doubt, he wondered about the translation, the crucial starting-point for the interpretation, but unfortunately, he had forgotten the ambiguity previously recognized, to appeal to a generally impossible precision. Hardly! It is out of question in the case of the *dao*, as could be understood by reading some classic of Taoism.[49] The most faithful translation of the passage would be: "Man can make the *dao* great, it is not the *dao* that makes man great"[50] Thus the faithful, historical meaning would have made the problem worse for Leibniz. Sure enough, it made more human, material and immanent the very principle that he would have desired to consider transcendent, in order to harmonise it with his philosophy, his science and his Christian religion. Anyway, he had understood that ". . . the classic Chinese authors . . . [interpreted] . . . these things without doubt *anthropopathos*, in the human manner, as they are in creatures." And

[48] Leibniz 1987, pp. 132–136.

[49] *Daodejing* 1973. *Zhuangzi* 1982.

[50] *Ren neng hong dao, fei dao hong ren*. Leibniz 1994, p. 90. Confucio 2000, p. 121.

yet he continued to think that all the attributes of the *li* sublimed it upwards: as certain mystics had denied for God "... a Being *Ens, ón* [to give Him, instead], more than Being, a *Super-Ens, uperesía*."[51]

Lacking the punitive will to the *li*, between good and evil, our theo-rational mathematician recovered the idea that, through the people, this 'reason' behaved *as if* it possessed it. He continued his optimistic interpretation also with the *qi*, the product of the *li*, choosing "... of the many possibilities, the most convenient one; ... all the rest came by natural disposition, ... as Mr. Descartes claims to make the present world system derive from a series of a few suppositions previously produced. And thus the Chinese, far from being blameworthy, deserve to be praised, because they make things derive from their natural dispositions, by means of a pre-established order. But here, *Chance* is not admitted, and does not seem to have any foundation in the Chinese passages." Leibniz granted his approval of the event that the Chinese did not recognize any separate immaterial substance "... apart from the Li, ...". He appreciated even more the Chinese world completely full, and wholly connected, ascribable to the *dayi* [great one], which he followed by his "one is everything". He had found in China his law of continuity, as we have seen above. Instead, identifying the *li* with the soul of the world, in the style of Baruch Spinoza (1632–1677), was rejected by our German metaphysicist, although some Chinese scholar might have made such an error.[52]

Leibniz could not hesitate on the identification of the *qi* as what particularly he was very keen on. "This Ki [*qi*] is called air and might be called *ether* by us, because originally, the matter is perfectly fluid, without any bond or hardness, without interruptions and without end which may distinguish the parts; in a word, it is the thinnest body that can be imagined. [...] As regards this production of first matter by means of the first principle, or of the primitive form, or the pure Act, or operation of God, Chinese philosophy is closer to Christian theology than the Philosophy of the ancient Greeks is: because they considered matter to be a principle parallel to God, which God does not produce, but only fashions." It is true that Leibniz did not accept the eternity of *qi*, but he justified it by the lack of Christian revelation. Failing precise information about the subject, the European scholar believed the Chinese texts had never written that *qi* had not had any beginning, like *li*. Even less, his interlocutors must have explained to him about the *taiji* which, in the deductive European spirit, he tried to make proceed in a linear fashion, from the *li* and from the *qi*. These notions, on the contrary, had been presented to him in confused circularities, which he judged to be tangled. He was partly right, in the face of these partial summaries, influenced by prejudices, but he was wrong to reject the circular arguments typical of Chinese sciences. The "... natural causes ... [are] machines prepared for this purpose, which are formed by the wisdom and the power of the supreme substance, which, together with them [the Chinese], we may call Li."[53]

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[51] Leibniz 1987, pp. 137–141. Leibniz 1967, I, p. 190–191.

[52] Leibniz 1987, pp. 143–145. Leibniz 1967, I, pp. 328–329. Leibniz 1967, II, p. 176.

[53] Leibniz 1987, pp. 146–147 and 161. Leibniz 1994, pp. 68 and 72.

For him, things could go better with the *shangdi* [Emperor on high], who could be more suitable to be identified with God, as Matteo Ricci had already done, at least in the expression *tianzhu* [principal heaven]. Ascended to Heaven, this Sino-Christian God made the *li* a *tianfa* [heavenly rule]. "This is pure Christianity, seeing that it renews the natural law engraved in our hearts, ... The ancient sages of China ... were more correct than they do knew. Actually, their astronomers discovered that Heaven is all the known universe and that Earth is only a subordinate star; and that there may be said to be as many systems in the world as there are fixed, or principal, stars; ...". If some Confucian literati identified the *shangdi* as heavenly matter, then according to Leibniz, he would have been expressing himself figuratively, or, like Plato, he would be pretending for the people. In the end, both the Christian God, and the Sino-Christian one, were bound to remain transcendent for ever. "... it is safer to consider the reason and harmony of doctrines than the wrapping of words." In the opposite case: "This tends only to destroy religion, as if it were only a political invention to maintain the people attached to its duty; ...". How could the hints of the Christian religion have arrived in the Country-at-the-centre, if not through the ancient Patriarchs of the *Bible*?[54]

Leibniz read a page dealing with birth and death. "When the composition of man takes place, and he comes into the world, in other words when Heaven and Earth unite, universal Nature does not come (because it is already present). When man dies, in other words when Heaven and Earth separate, universal Nature does not go away (because it is always everywhere). But the air [*qi*], which is the entity of Heaven, returns to Heaven, and the corporeal compound, which is the entity of Earth, returns to the Earth."[55] However, he did not have much doubt that the Chinese culture was not tormented by that dualism between the body and the soul, which he was able to close only by having recourse to divine harmony.[56] *Ti* means "body" in Chinese, and in its characters it includes the "root of man".

Though almost totally ignorant of the language, and knowing only a few pages from the at times unfaithful translations of others, the theo-rational philosophers did not hesitate to accuse even a Jesuit like father Longobardi, who, on the contrary, had spent his whole life in China. The latter would have given "... only external interpretations which force, or even destroy the text, and make it ridiculous, contradictory and fraudulent."[57] But for him, Chinese culture could only be capable confirming, at best, those universal, eternal truths which had already been invented in Europe, between Germany, Holland, France, England and Italy. The most interesting of these quite regarded the mathematical sciences, rather than the religious and philosophical diatribes in which we might have expected wider margins of discussion.

Inspired by that French Jesuit encountered above, Leibniz too interpreted the 64 hexagrams of the *Yijing* as combinations of 0 and 1: he transformed the broken

[54]Leibniz 1987, pp. 153–156.

[55]Leibniz 1987, p. 168.

[56]Leibniz 1967, I, pp. 189–199, 363, 493–494 and *passim*. Leibniz 1967, II, p. 709 and *passim*.

[57]Leibniz 1987, p. 170.

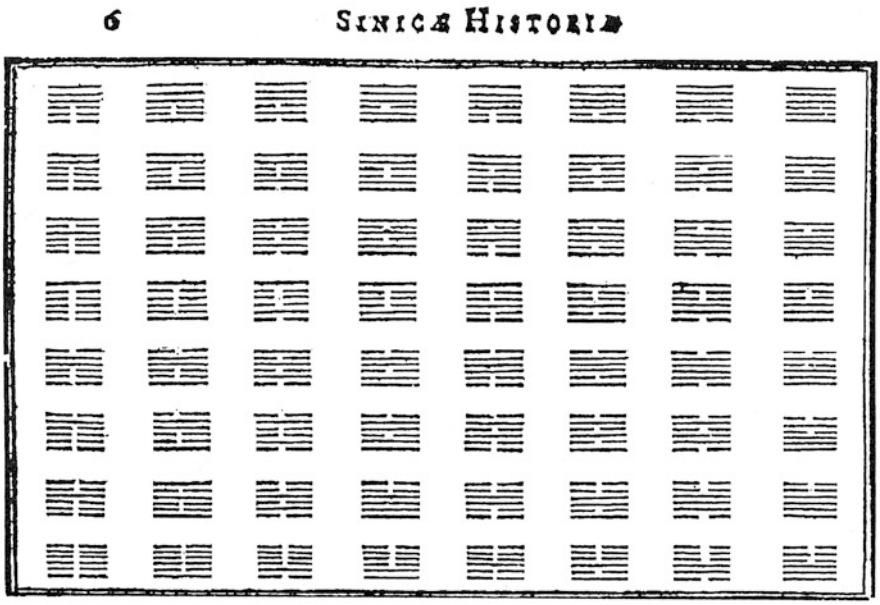

Fig. 10.1 Leibniz interpreted the 64 hexagrams of the *Yijing* as combinations of 0 and 1 (Leibniz, "Hexagrams", 1994)

line Yin and the continuous line Yang into those numbers. "...this is the binary arithmetic which the great legislator [Fuxi] seems to have possessed, and which I discovered a few thousand years later." (Fig. 10.1), The *Yijing* thus already would contain the art of "variations" to which at the beginning the German had devoted his *Dissertatio De Arte Combinatoria*. Today, in the era of computers, everybody would succeed in reducing many things, and not only numbers, to combinations of 0 and 1. On the base 2, for example, $3 = 1 \times 2^0 + 1 \times 2^1$ is indicated as 11, which are the coefficients of the powers of 2; or else, $8 = 0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$ is indicated as 1000, *und so weiter*.[58]

The invention of our mathematician and philosopher was enclosed in a letter of his semi-private communications, and was published in the journal of the Academie des Sciences. With the numbers written to the base 2, Leibniz was glad that it was possible to carry out the operations of arithmetic without knowing the tables by heart: "...sans calculer. Et une prolixité dans le commencement, qui donne ensuite le moyen d'epargner le calcul et d'aller à l'infini par règle, est infiniment avantageuse." ["...without calculating. And a prolixity at the beginning, which then gives us the possibility of saving calculations, and proceeding regularly to infinity, is infinitely advantageous."] Here we find that inclination to make everything simple

[58]Leibniz 1987, pp. 174–177. Binary calculations were also considered to be useful for music. Leibniz 1966, pp. 279–280. Luppi 1989, pp. 76 and 145. Walker 1972, p. 297.

and automatic which was the basis for our German mathematician's success in infinitesimal calculus. For the other kind of binary calculus, he would have to wait for three centuries. He, too, like Pascal, had once projected and got a calculator done. As we have seen above, he even thought of a proportional compass, in order to allow people who had practically no knowledge of music to compose it.

He would have liked "...déterrer le fondement de l'écriture Chinoise ...Cependant je ne sçai s'i y a jamais eu dans l'écriture Chinoise un avantage approchante de celui qui doit être necessairement dans une Caractéristique que je projette. C'est que tout raisonnement qu'on peut tirer des notions, pourroit être tiré de leurs Caractères par une manière de calcul, qui seroit un des plus importans moyens d'aider l'esprit humain." ["...to discover the basis of Chinese writing. ...However, I do not know if there has ever been an advantage in Chinese writing, close to that of necessity present in a Characteristic that I project. It is that every reasoning, derivable from notions, could be obtained from their Characters through a procedure of calculus, which is one of the most important means to help the human spirit."][59]

Our German metaphysicist and diplomat also included a bit of China in a manuscript on the novelties of geometry. "Graphicen qui imaginationi potius felici quam geometriae tribuit, videat quantum inter nostratum et Chinensium delineationes intersit. Chinensibus natura favens colores admirandos suppeditat, quorum gratiam facile nostri arte vincunt: arte, inquam, filia geometriae. Graeci non ante artes coluere, quam geometriam, et Arabes tum maxime in his studiis excelluere, cum potentia Asiam atque Africam complexi sine aemulis floruere; et non ante barbaries desiit in occidente, quam redux ab exilio Geometria, etiam architecturam et pictoriam et statuariam adduxit." ["In painting, which harmonises better with a happy imagination than with geometry, see what a distance exists between the drawings of our country and the Chinese. Nature favours the Chinese by providing colours to be admired, the grace of which our drawings easily surpass by means of art. By means of art, I mean, the daughter of geometry. The Greeks did not cultivate the arts before geometry, and then, it was above all the Arabs who excelled in these studies when, having conquered Asia and Africa by force, they flourished incomparably; and barbarism terminated in the West, only when geometry had returned from exile, bringing with it architecture, painting and sculpture."][60]

The discussion, whether the Chinese were atheists, who could be converted to Christianity, how ancient their culture was, and where it came from, went on also after the death of our philosopher: he soon received various refutations.[61] The reader should bear in mind our Chap. 3 to understand that it is impossible to identify the Yin and Yang, the broken line and the continuous line of the *Yijing*,

[59]Leibniz 1962, VII, pp. 223–227. Kreiling 1981, p. 149. Hofmann 1981, p. 161. Luppi 1989, p. 71. Walker 1972, pp. 296–297.

[60]Leibniz 1962, VII, p. 321. Here the philosopher of the sufficient reason exchanged the effects for the causes.

[61]Leibniz 1987, pp. 179–188. Gernet 1982. Leibniz 1994.

with the 0 and the 1 of the base 2 notation (Fig. 3.7). Otherwise, everything that the Chinese culture believes is continually transformed would be fixed in a transcendent, immobile dualism. But it was precisely this metaphysical pattern of Leibniz that was destined to have enormous success with the scientific community of the following generation. Unfortunately, he was also to have many, too many, followers in the work of simplification, corruption and distortion of the Chinese culture, to submit it to the Western scientific criteria.[62] While a part of his misinterpretation of the Country-at-the-centre depended on his interlocutors, another part could be attributed to his desire for a single universal science to which he was contributing.

Anywise, the interweaving of science and religion typical of this metaphysicist was also clear when he dedicated his attention to Europe. "Verum enim vero tenacior est varietatis suae pulcherrimae Natura rerum, aeternarum varietatum parens, vel potius Divina Mens, quam ut omnia sub unum genus compingi patiatur. Itaque elegans et mirabile effugium reperit in illo Analyseos miraculo, idealis mundo nostro, pene inter Ens et non-Ens Amphibio, quod radicem imaginariam appellamus." ["In truth, the nature of things, the mother of eternal varieties, or rather the Divine Mind, without doubt with its most beautiful variety, is more tenacious than it can suffer that everything is united under a single genre. And thus it finds a sublime and elegant way out in that miracle of analysis, in that prodigy of an ideal world, in that amphibian almost between Being and non-Being which we call the imaginary root of negative unity."][63]

In the British Isles, Newton and his pupils started to use, almost secretly, a calculus of theirs called that of fluxions; but timidly and full of caution in their publications, they let transpire almost only the traditional and complicated geometrical models of the cosmos. A different music of the spheres was played, instead, through the palaces and courts of the European continent. Here our character, bolder and not at all shy to present in public all his metaphysics, but unfortunately also highly verbose, led the mathematical sciences elsewhere. Leibniz called the new symbolism, which was to dominate the scene for the following two centuries, differential calculus or infinitesimal calculus.

However, even though the melodies of letters and symbols imagined by the German metaphysicist sounded very different from those of other scholars, he too, like Galileo Galilei, Descartes, Kepler, Christiaan Huygens or Newton, believed that it should transcend the approximate, impure earthly world. How, otherwise, would it be possible to obtain eternal, universal and perfect laws for the cosmos? Sure enough, our German wrote (in 1668) "... Because our bodies cannot have a determined magnitude, or figure, or movement without supposing an incorporeal being, it will clearly appear that this incorporeal being will be the same one for all things, for the harmony of all things among one another, ... that is to say, God". Thus he sought the reason for the physical phenomena in an "incorporeal" principle, and criticised (in 1670) the modern physicists who sought material causes, ignoring

[62] Tonietti 2006a.

[63] Leibniz 1962, V, p. 357.

"rational ones". "...the wisdom of the Author can be seen to shine principally in having constructed the clock of the world in such a way that everything derived from it, as by necessity, in view of the supreme harmony of the universe". He placed at the basis of his system the laws of mechanics, which, as he saw things (in 1680), depended "on the final causes, that is to say, on the will of God, decided to do what is most perfect".[64]

Leibniz knew Galileo Galilei, Kepler and Descartes; he recognized their merits and denounced their limits. The Frenchman had undoubtedly tried to give a more satisfactory explanation by connecting the heavenly bodies with a fluid material, but as regards geometry, he had developed an "unbearable, haughtiness unjustified", because not all problems could be reduced to algebraic equations. "...his great principle, that is to say, the idea that the same quantity of movement is maintained in the world is a mistake". For, on the basis of considerations about bodies that fall and rise up to various heights, in the absence of friction and other impediments, our German metaphysicist sustained that it was rather the "motive power" that was maintained. That is to say, he posited the embryo of kinetic energy and work. Like the Cartesians, he considered it right to explain natural phenomena by means of mechanics, but he wanted to derive it from "higher principles ...force or power of action which constitutes the passage from metaphysics to nature, from material to immaterial things ...That force possesses its own laws ...deduced not only from the principles of an absolute and brutal necessity, as in mathematics, but from perfect reasons ...the universal cause of all things *Deus ex machina* ...as mentioned by the author of the *Philosophia Mosaica [Mosaic Philosophy]*, with badly interpreted words from the Holy Scriptures".[65]

Also for Leibniz, all books seemed to have to be comprehended correctly, but unfortunately, they could be misunderstood, and thus needed the just interpretation. In this connection, our German philosopher had even devoted himself to the cabalistic procedures of gematria, and to the art of combining letters and numbers. More than with triangles, circles and other geometrical figures, this Great book appeared to Leibniz to be written in other alphabetic and numerical symbols: Latin, Gothic, Greek, Hebrew letters, new signs indicated for habitual operations (such as multiplication and equality) or also for wholly innovative, unknown procedures.[66]

Leibniz was looking for (in 1690) a new alphabet of thought, the letters of which one would be allowed to combine to obtain from the more particular inferior ideas more general ones. He thus arrived in embryo at a formal logic of his own, in which he considered all non-contradictory notions as possible. For an absolutely perfect being, then, possibility necessarily implied existence, otherwise it was necessary to construct the thing defined. "In abstract propositions, and in those that do not depend on the experiment, this is the only, and highest, criterion of truth: that they

[64]Leibniz 1967, II, pp. 32, 52–53 and 60.

[65]Leibniz 1967, II, pp. 58, 65 and 97. The *Philosophia Mosaica* had been written by Robert Fludd (1574–1637), Kepler's adversary, already encountered in Sect. 8.3.

[66]Leibniz 1967, II, pp. 116–131.

be identical or soluble into identical propositions. Hence it is possible to obtain the elements of eternal truth, as well as the method for all the sciences that are comprehended and demonstrated, together with the procedure typical of geometry". In God all things would be known a priori, down here where we are, instead, very few were. Where no metaphysical necessity existed, truth among phenomena could not have come about by chance, but required a cause . . . "we foretell that tomorrow the sun will rise, only because it has confirmed our faith so many times . . . ".

"A particular art" was needed to carry out, to order and to connect experiments. "At present, the human knowledge of nature seems to me to be similar to a storehouse full of goods of all kinds, but without order or an inventory . . . ". This interesting variation on Galileo's mercantile model betrayed the anxiety of Leibniz for the construction of a general abstract, complete system, dominated by the rules of logical reasoning. The hierarchy between things and disciplines saw the combinatorial art to subordinate algebra, because the latter made use of the general laws of the former . . . ". . . they are far more general and are valid not only for algebra, but also for the art of deciphering, for various games, and for geometry itself, treated linearly in the manner of the ancients . . . ". Together with the principle of contradiction, "A is A and non-A is non-A", our formalist also posited that of the third exclusion: "every enunciation is either true or false, and if the affirmation is true, its negation will be false".[67]

In this, he was different from Descartes and partly comparable to Newton: "I prefer a Leeuwenhoek [(1632–1723)] who tells me what he sees, to a Cartesian who tells me what he thinks".[68] The physics of our German metaphysicist contemplated (in 1694) a particular notion of force: ". . . to explain a particular science, *Dynamics* . . . ". ". . . although gravity and elastic force can and must be explained mechanically with the movement of the ether, nevertheless the ultimate reason for the movement of matter is the force impressed with the creation that is inherent to every body, but which, from the very conflict of bodies, is variously limited and restricted in nature". Once set in motion, the world clock of Leibniz would continue to move eternally, sustained by the law of conservation. As anyone can easily learn from any textbook of physics today, his law maintained, not the quantity of movement mv, as Descartes believed, but the *vis viva* [living force] mv^2, the first embryo of mechanical energy.

He admitted (in 1702) that nothing existed in the intellect apart from what derived from the senses. However, "being itself and truth cannot be learnt at all by means of the senses". Trusting the senses, for him, was like believing in the pure appearance of dreams. Thus he sought a criterion to distinguish the truth from appearance. "And if, during a dream, I found a proven mathematical truth, it would be equally true when I wake up". For him, mathematical truths did not need to be experimented. On

[67] Leibniz 1967, II, pp. 689–700.

[68] Descartes 1983, p. 44. The Dutch naturalist observed under the microscopy those spermatozoa and bacteria which confirmed the German's panvitalistic theories.

the contrary, "Induction will never give a perfect certainty". His beloved universal truths would never be reached by means of this route.[69]

"...I consider space as something purely relative, like time: it is an order of co-existences just like time, which is an order of successions. [...] ...space is nothing other than that order or relationship, and is quite nothing without bodies, ...[...] ...instants, outside the realm of things, are nothing and ...only consist of their successive order, ...". Through the interposed person of the Anglican theologian Samuel Clarke (1675–1729), our rational philosopher caused a dispute to develop against the *sensorium* of Newton, who sustained, on the contrary, that space and time were absolute. In concert, both of them involved their different models of God referring to the world of matter. Then the competition now became which of the two famous champions would succeed in maintaining himself further from an immanent divinity, as the soul of the world, in the style of Spinoza. Leibniz rejected that single spirit, which would have animated the whole world as the "...same breath of wind makes the different organ pipes play differently." Both would have liked to make their gods and their scientific laws absolutely transcendent. But how, then, would God have acted on the world which was often presented as, and regulated like, a mechanical clock? On the deep abyss created by such a dualism, now there flew the German angel, lifted up by the pre-established harmony, and the English one, sustained by an equally mysterious *sensorium*. "An infinite distance needs to be placed between the operation of God, which goes beyond the natural forces, and the operations of things which follow the laws that God has given them, and which He has made by nature suitable to follow, though with his assistance."[70]

Deriving them from the relative positions of bodies, Leibniz offered a non-absolute conception of space and time, which could be explained with greater clarity through music. Every composition creates its own time in the mode and in the order in which the notes are played, while the relative space can be read on the stave of the score where the sounds are written. Even though very probable in the style of the natural philosopher and lover of harmony, the comparison is not found (yet?) in the papers left at Hannover. Instead, in different places, and in abundance, how did he describe that even his God, though infinitely good, just, far-sighted and omnipotent, permitted evil in the world? "C'est à peu pres comme dans la musique et dans la peinture, car les ombres et les dissonances relevent tellement le reste; et le savant auteur de tels ouvrages tire un si grand avantage de ces imperfections particulieres pour la perfection totale de l'ouvrage qu'il vaut beaucoup mieux de leur donner place que de s'en vou-passer." ["It happens almost as in music and in painting, because the shadows and the dissonances put all the rest in such relief; and the expert author of these works obtains such as great advantage from these imperfections for

[69]Leibniz 1967, II, pp. 700–706. Leibniz 1967, I, pp. 364–365.

[70]Leibniz 1967, I, pp. 300–304, 312–313, 331–334, 347–357, 366–369 and *passim*. Leibniz 1967, II, pp. 300–304. Serres 1968, pp. 470–472.

the general perfection of the work, that it is far more valid to give them space rather than to do without it."][71]

He explained (in 1705) that as the souls followed their moral laws, swinging between good and evil, so bodies followed those of movement. And yet such different beings met and corresponded to each other like two pendulums constructed differently, but regulated on the same period. He called this accord "pre-established harmony". It supplied, without recurring to miracles to explain natural things, a new proof of the existence of God. "I do not mean to say at all that the corporeal world is a machine or a clock that works without the intervention of God; indeed, I affirm that his creatures need his continual influence. I add, however, that it is a clock that works without any need for correction, otherwise we would have to say that God changes his mind. God has foreseen everything, and has provided for everything since the beginning. In His works, there is a pre-established harmony and beauty." Leibniz did not posit any souls or miracles to maintain movement, as well he considered the collision of another body necessary to modify the movement. Therefore he considered action at a distance absurd.[72]

Our German philosopher explained his "pre-established harmony" to Antoine Arnauld by means of a musical example. "…c'est comme à l'égard de plusieurs différentes bandes de musiciens ou choeurs, jouant separément leurs parties et placés en sorte qu'il ne se voyent et même ne s'entendent point, qui peuvent neantmoins s'accorder parfaitement en suivant seulement leurs notes, chacun les siennes, de sorte que celuy qui les écoute tous, y trouve une harmonie merveilleuse et bien plus surprenante que s'il y auroit de la connexion entre eux." ["…it is like with many different choirs or musical bands, which play their parts separately, and are situated in such a way as not to see each other, or hear each other, but nevertheless can play in perfect agreement following their notes, each one his own, with the result that one who listens to all of them would find a wonderful harmony, and even more surprising, he would find that there was a connection between them."] *Cela va sans dire* [It goes without saying] that the invisible composer would be God. If the God of Leibniz above all had written music at the moment of creation, that of Newton appeared to be rather a kapellmeister who continually had to interpret it. In the face of an adversary of "pre-established harmony", he commented, again referring to music: "Ce chantre chante à livre ouvert, ses yeux sont dirigés par le livre et sa langue et son gosier sont dirigés par les yeux, mais son âme chante pour ainsi dire par mémoire, ou par quelque chose équivalente à la mémoire; car puisque le livre de Musique, les yeux et les oreilles ne sauraient influer sur l'âme, il faut qu'elle trouve par elle-même, …, ce que son cerveau et ses organes trouvent par l'aide du livre." ["This singer sings with his book open, with his book he directs his eyes; his tongue and his throat are directed by his eyes, but his soul sings, so to speak, by heart, or by something equivalent to the heart. Because, not

[71]Leibniz 1948, pp. 365–366. Serres 1968, pp. 461–462. Luppi 1989, pp. 105–112. Baas 1976, pp. 53–58.

[72]Leibniz 1967, I, p. 306. Leibniz 1967, II, pp. 707–714.

knowing the book of music, or how the eyes and the ears can influence the soul, it is necessary for her to find by herself, …, what the brain and the organs find with the help of the book."] Another time, the recurring musical comparison referred to the harpsichord.[73]

For all his life, Leibniz yearned to succeed in completing his *Scientia generalis*, an encyclopaedic system of which he only succeeded in writing fragments. He sang the praises of technical progress, from printing to the compass, from the telescope to the microscope. He was decidedly an optimist. "Take what you must, work as you must, and you will obtain what you desire."[74] "I will only add that the discovery of gunpowder seems to me a gift of heaven's goodness, for which our age should be thankful, rather than a sign of its anger, because apparently it was precisely gunpowder that contributed most of all to blocking the flood of Ottomans who were about to overrun our Europe". But to his favourite mystic of the seventeenth century, Jakob Boehme (1575–1624), who he quoted every now and then, that powder seemed to be generated by the wrath of God.[75]

Alluding to the invention of infinitesimal calculus, he retained that modern mathematicians had gone well beyond the ancients, discovering the secrets concealed by Archimedes. He praised the good monarch (Louis XIV or Frederick of Prussia) who "can give orders and make regulations to enable the sciences to make progress in a surprisingly short time". Among the Hannover's suite, who were now on the throne of Great Britain, he would have liked to assume the Presidency of the Royal Society. At Berlin, in 1700, he had founded the Academy of Sciences. He too, like Descartes, thought that the sciences advanced avoiding the useless disputes typical of philosophers. His *Scientia generalis [General Science]* should have served this purpose. According to him, everybody should have "imitated the scholars of geometry, for whom neither Euclidists nor Archimedists existed, because everybody was in favour of Euclid, and everyone was in favour of Archimedes, seeing that everyone was in favour of the common mistress, i.e. the divine truth". It was a pity that on our poor Earth, things were not so simple, to the point that Leibniz himself was to cross swords continually with a certain Newton, as regards the invention of infinitesimal calculus. Thus, we shall come across this other famous dispute further ahead.[76]

The sciences would have made progress more easily if a vast basic *corpus* had been accumulated, including all the knowledge acquired up to that moment. The situation for them was similar to that of commerce. "… for those who have already obtained large gains and considerable goods, it is much easier to go further and

[73]Leibniz 1960/65, II, p. 95, IV, p. 549, VI, p. 479. Luppi 1989, pp. 130–131 and 140. Serres 1968, p. 473. Baas 1976, pp. 59–60.

[74]Leibniz 1960/65, IV, p. 329. Leibniz 1987, pp. 5–8. Mittelstrass & Aiton 1981, pp. 153–155.

[75]Leibniz 1967, II, pp. 724–733. Leibniz 2000, II, p. 256. Koyré 1979. Tonietti 1999a, pp. 199–201. Tonietti 2004, Chap. 16. Walker 1972, pp. 299–300 and 306–307.

[76]Leibniz 1967, II, pp. 733–750. Hofmann 1981, pp. 164–165. Boyer 1990, pp. 456–475. Kline 1972, p. 380.

acquire great riches than it is for a man who does not possess anything to put together a small capital". Now that money had become the leading element, the great epoch of capitalism was putting evangelical message into practice: To him that has much, more will be given. The comparison seemed to have been copied from the similar saying of Descartes, who, among other things, was quite rich.

He would have liked, therefore, to compile a great encyclopaedia, but others were to succeed in this enterprise. Leibniz suggested certain precepts to scholars about how to behave: not to pretend originality, but to believe the most acient opinions; to avoid doubts, basing themselves on the truths of the axioms that nobody challenges; when it is possible, to demostrate the axioms as well. In this connection, he already succeeded in finding defects in Euclid. "It is even possible to advance a humorous, but true, paradox that there are no other authors whose manner of writing is more similar to the style of geometers, than that of the ancient Roman jurists …". Thus the mark given by Leibniz, to the laws of physics and to mathematical theorems, became taking the spirit of the divinity to infuse it into a formal structure similar to that of the juridical codes. Our German jurist aimed to discover the inventive principles of all the sciences, to arrive at a higher science, a general science from which to deduce all the rest. He subordinated all the sciences to mathematics, as the art of perspective, shadows, and the meridian depended on geometry.[77]

What instruments therefore were European scholars preparing in order to divide the True from the False, Good from Evil, the good from the wicked, friends from enemies, fellow-countrymen from foreigners? The laws. Western European culture squanders laws of all kinds. At first, they had religious origins, always in the *Bible*. Then came written codes for the Greek and Roman *lex*. Lastly, now, the scientific laws arrive, such as the laws of Kepler, the laws of movement, the law of gravity. Gottfried Wilhelm Leibniz was a jurist and diplomat, who substantially enriched the ongoing series of natural philosophers and mathematicians who had studied and also practised the codes of laws in force in their national States. He left us a "Meditatio juridico-mathematica de inter usurio simplice" ["Juridical and mathematical meditation on the rate over time of simple usury"].[78] Thus we discover that already in the seventeenth century, capitalism had established its positive premises which gladden our hearts still today.

With a full knowledge of the facts, therefore, the co-inventor of infinitesimal calculus could discern the similarity between the axioms of geometry and the formal structure of laws. Both the systems aimed to take away doubts in the process towards truth. As a better guarantee of his metaphysics, Leibniz posited his divinity that transcended the world. Thanks to this, (and to himself) the infinite could now at last enter fully into the mathematical sciences. From that time on, the term 'transcendent' was to assume in mathematics the technical significance to indicate that which surpasses the finite representation of algebraic equations. $\sqrt{2}$ is an algebraic number, π, instead, transcendent; the *parabola* is an algebraic curve,

[77]Leibniz 1967, II, pp. 735–736 and 744. Serres 1968, pp. 420–421.

[78]Leibniz 1962, VII, pp. 125–132.

the *sine* transcendent. Actually, to represent the latter elements, infinite symbols are necessarily used, calculating "seriebus infinitis" ["with infinite series"], the last term of which could never be written.

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \frac{x^{2n+1}}{(2n+1)!} \ldots$$

The paradox of Zeno from Elea, where the distance between the archer and the target, that is to say, the segment [0–1] had been divided in half infinite times, was now represented and solved by means of the infinite series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} + \ldots = 1$$

In the end, unfortunately, the arrow would kill the target. But now, the metaphysics of the infinite had succeeded in calculating the movements that could be observed on the Earth, that is to say, what the preceding Pythagoreans had not been able to do. By means of the infinite, our German philosopher had succeeded in filling up with irrational transcendent numbers any geometrical interval without leaving any gaps. In this way, he gave a greater mathematical substance to his principle of continuity. "Number in the broadest sense, including also the [whole] number, the broken [fraction], the surd [irrational algebraic], the transcendent [irrational like π] and everything that can be found between two whole numbers, is proportional to the line, and has no *minimum*, just as the *continuum* does not have one." *Natürlich*, "The truly infinite only exists in the *absolute* [God], who is anterior to every composition, and is not formed by the addition of parts." In order not to commit the sin of immanence, the theo-rationalist was careful not to extend it to space and time. "All these infinites, like their opposites, i.e. the infinitesimals, are found only in mathematical calculus like the imaginary roots of algebra."[79]

Precisely with the philosopher who sustained the metaphysics of the "sufficient reason", that is to say, that every effect must have a cause, could we not, or should we not notice a singular coincidence? Actually, with various theoretical justifications, and in practice, Leibniz admitted the equable temperament in music, while at the same time, in parallel and in concert, he granted the status of numbers, to all effects, also to those incommensurable *alogos* ratios called irrationals. Only a theo-rational philosopher like him could have taken the liberty of making irrational and transcendent numbers 'rational', by using such an 'irrationalistic', ineffable, obscure and divine notion as the infinite.

To the famous philosophical statement, the object of comments, rejection or approval for centuries, "Nihil est in intellectu quod non prius fuerit in sensu" ["There is nothing in the intellect which was not previously first in the senses"], he had

[79]Leibniz 1967, II, pp. 158 and 281–285. Mittelstrass & Aiton 1981, pp. 151–152 and 157–158. Hofmann 1981, p. 165. Boyer 1990, pp. 459–462.

added, as only he could do, "nisi ipse intellectus" ["except the intellect itself"]. In this way, he had created that transcendent, dualist soul which he would succeed in healing only by listening to the music of "pre-established harmony".

With his own words, we can seal that Leibniz had sought truth, by inventing transcendent, dualist laws, expressed in symbolic language. "... toute ma philosophie ... est fondée sur deux dictons aussi vulgaires que celui du théâtre italien, *que c'est ailleurs tout comme icy*, et cet autre du Tasse *che per variar natura è bella*, qui paroissent se contrarier, mais qu'il faut concilier en entendant l'un du fond des choses, l'autre des manières et apparences. Cela paroist passablement bon pour des personnes qui aiment la recherche de la verité, ..." ["... all my philosophy ... is based on two sayings as common as those of the Italian theatre, that *elsewhere everything is like here* and the other one of Tasso (Torquato, 1544–1595) *che per variar natura è bella [nature is beautiful in its variations]*, which seem to contradict each other, but which we have to reconcile: interpreting one for the foundation of things and the other for the modalities and appearances. This seems passably good for those people who love to search for the truth, ..."]. "... la maxime que *c'est tout comme icy* dans l'invisible comme dans le visible." ["... the maxim that *everything is like here* in the invisible as in the visible."] was insistently repeated as many as six times in the letter.

He took this line from *Arlecchino, Imperatore della Luna [Harlequin, Emperor of the Moon]*, a comedy of art in the Italian style, which was acted in the public squares and the courts of Europe. Coming back from our satellite to the Earth, the protagonist said in it: "Everything is like here." Our natural philosopher and metaphysicist must have liked this idea so much that he distributed it here and there throughout his manuscripts.[80]

> Et citharaedus ridetur chorda qui semper oberrat eadem.
> [And the citharist who always wanders up and down the same string is derided.]
> Horace

> ... [Newton] started from what he saw, with the aim of finding the cause
> Fontenelle

> Mystical dance, which yonder starry sphere
> Of planets and of fixed in all her wheels
> Resembles nearest, mazes intricate,
> Eccentric, intervolved, yet regular
> Then most, when most irregular they seem;
> And in their motions harmony divine
> So smooths her charming tones that God's own ear
> Listens delighted. ...
> John Milton, *Paradise Lost*, V 620–627

> ... the people of their island [of Laputa] had their ears adapted to hear "the music of the spheres, which always played at certain periods, and the court was now prepared to bear their part, in whatever instrument they most excelled." [...] I have not seen a more clumsy, awkward, and unhandy people, nor so slow and perplexed in their conceptions upon all

[80]Mittelstrass & Aiton 1981, p. 151. Leibniz 1704, pp. 345 and 348. Leibniz 2000, I, pp. 564–569. Leibniz 1967, I, p. 343. Leibniz 1967, II, p. 714. Bailhache 1995b, p. 13. Baas 1976, pp. 53–59.

> other subjects, except those of mathematics and music. They are very bad reasoners, and vehemently given to opposition, unless when they happen to be of the right opinion, which is seldom their case. Imagination, fancy, and invention, they are wholly strangers to, nor have any words in their language, by which those ideas can be expressed; the whole compass of their thoughts and mind being shut up within the two forementioned sciences. [...] He [Aristotle] predicted the same fate to *attraction*, whereof the present learned are such zealous asserters. He said, "that new systems of nature were but new fashions, which would vary in every age; and even those, who pretend to demonstrate them from mathematical principles, would flourish but a short period of time, and be out of vogue when that was determined."
>
> Sir Isaac Newton reported an assay taken at the Tower of Wood's metal, by which it appears, that Wood had in all respects performed his contract. His contract! With whom? Was it with the parliament or people of Ireland?
>
> Jonathan Swift

10.2 Sir Isaac Newton and Mr. Robert Hooke

Writing to Huygens, Leibniz expressed his judgement on Newton's *Philosophiae NaturalisPrincipia Mathematica [Mathematical Principles of Natural Philosophy]*: "Le livre de Mr. Neuton est un de ceux qui meritent le plus d'estre perfectionnés ... Je ne m'etonne pas si parmy tant de recherches difficiles, il s'y est glissé quelque faute de doctrine." ["This book by Mr. Neuton [sic!] is one of those which deserve to be improved ... I am not surprised that among so many difficult tasks of research, a few errors of doctrine have slipped in."][81] As he quoted him as "Neuton", may our German theo-mathematician thought of him as a *Neutöner*, that is to say, an exponent of the new music? The event that nobody perhaps had yet called the renowned Englishman New Tone, backs up the hypothesis that music had in general been amputated from the history of sciences. Such a simple and immediate etymology does not even come to the mind.[82] If I indicated him as only New Tone, I bet an euro that few would understand immediately and correctly whom I am referring to.

Leaving it to be decided how revolutionary or how traditional he was, Newton appears to me as New Tone, even if he was not to be New Notation, as we shall see. Also this Englishman, so careful to remain protected and safe in his roles, not only invented new music metaphorically, but actually wrote pages about music. He did so for that Proposition VIII of "Book III" where he gave the law of gravitational attraction. "...erit pondus globi alterutrius in alterum reciproce ut quadratum distantiae inter centra." ["...it will be the weight of one of the two globes on the other inversely proportional to the square of the distance between

[81] Huygens 1905, X, p. 261.

[82] Writing his name "Jsaacus Neuvtonus", without any false modesty, he had created the anagram of "Jeova sanctus unus" ["Jehovah holy and one"]. Westfall 1989, pp. 45 and 304.

the centres."][83] Between the edition of the *Principia* published in 1687 and that of 1713, Newton wrote a series of comments, the so-called classical *scholia*, which were undated, but can perhaps be placed around the 90s. However, in the end, these were not printed, but substituted by the famous general *scholium* which closed the treatise. In the integral original Latin version, they have only been published in recent times.[84] The scholium to Proposition VIII represented a real and proper musical model.

"On prop. VIII.

The ancients have not explained sufficiently in what proportion gravity decreases on moving away from the planets. However, they are believed to have veiled it, through the harmony of the heavenly spheres, indicating the Sun and the remaining six planets, Mercury, Venus, Earth, Mars, Jupiter, and Saturn by means of Apollo with his seven-stringed lyre, and by measuring the intervals of the spheres with the intervals of the notes. In this way they wanted the seven notes to be generated, the harmony of which they called *diapasôn* [octave], that Saturn was moved by the Doric, that is to say, deep, mode, [from *E* to *E*], while the remaining planets in the most acute modes, (as Pliny, lib. I c. 22 refers, on the basis of the thought of the Pythagoreans) and that the Sun played the strings. Whence Macrobius (lib. I, c. 19) affirms: 'Apollo's seven-stringed lyre shows many movements of the heavenly spheres for which nature is believed to provide the Sun as its guide'. And Proclus *In Timaeum* Plat. l. 3 p. 200: 'the septenari were consecrated to Apollo, as to the one who contains the universal songs in himself. For this reason, they called this god *Hebdomageta*, that is to say, the prince of the septenarius'. Likewise, from the oracle of Apollo at Eusebion *Praep. Evangel.* l. 5 c. 14, the Sun was called $\tau\tilde{\eta}\varsigma$ $'\varepsilon\pi\tau\alpha\varphi\vartheta\acute{o}\gamma\gamma o\upsilon$ $\beta\alpha\sigma\iota\lambda\varepsilon'\upsilon\varsigma$ 'the king of harmony with seven notes'. Furthermore, by this symbol, they meant that the Sun, with its strength, had an effect on the planets in that harmonic proportion of distances with which the force of the tension has an effect on the strings of different lengths, that is to say, inversely proportional to the square of the distance. [*... Solem vi sua agere in Planetas in ratione illa harmonica distantiarum qua vis tentionis agit in chordas diversarum longitudinum, hoc est in ratione duplicata distantiarum reciproce*.]

Actually, the force with which the same tension has an effect on the same string of different lengths is inversely proportional to the square of the length of the string. The same tension has an effect on the string half as long four times as strongly: because it generates the octave, and the octave is produced [also] by a force four times greater. If a string of a given length, stretched by a given weight generates a certain note, the same string stretched by a weight four times as heavy will generate the octave. And in a similar way, the same tension in a string one third as long will have an effect nine times greater. It surely will produce the twelfth [the fifth of the higher octave] and the string that generates a given note, stretched with a given weight, must be stretched with a weight nine times greater to produce the

[83] Newton 1687/1972, p. 577.

[84] Casini 1981 and 1984. Tonietti 2000b.

twelfth. And in general, if two strings of an equal thickness are stretched by weights attached to them, these strings will sound in unison when the weights are *inversely* [sic! Emphasis added] proportional to the squares of the lengths of the strings. To tell the truth, this reasoning is subtle, but it was known to the ancients. [*Et universaliter si chordae duae crassitudine aequales ponderibus appensis tendantur, hae chordae unisonae erunt ubi pondera sunt* ***reciproce*** *ut quadrata longitudinum chordarum. Subtilis quidem est haec argumentatio, sed veteribus tamen innotuit.*] In actual fact, as Macrobius confirms, Pythagoras tended the guts of sheep, or the nerves of oxen, attached to various weights, in order to work out the ratio of the heavenly harmony. By means of these experiments, therefore, he recognized that the weights which allow all the notes to be heard on equal strings, were inversely proportional to the squares of the lengths of string, with which the musical instrument emits those notes. As Macrobius attests, he applied to the heavens the proportion discovered by means of his experiments. And in this way, linking the weights to those of the planets, and the lengths of the strings to the distances of the planets, he understood, by means of the heavenly harmonies that the weights of the planets in the Sun were inversely proportional to the squares of their distances from the Sun.

On the other hand, philosophers loved to regulate their mystic arguments in such a way that, as a result of the derision, they out of turn manifested common things to the people, and concealed the truth beneath these kinds of arguments. In this sense, Pythagoras numbered his musical notes starting from the Earth, as if there were one tone from here to the Moon, then a semitone as far as Mercury, and then other musical intervals to the other planets. On the other hand, he taught that sounds were generated by means of movement. Also the friction of solid spheres, as if the largest sphere emitted the deepest sound, just like what happens to iron hammers when struck. Here, the Ptolemaic system of the solid spheres could be seen to be developing, while in the meantime, Pythagoras concealed his own system and the true heavenly harmony behind parables of the kind.

[Variant] As Macrobius tells us, while Pythagoras was passing in front of a smithy, he discovered by experience that the sounds produced by iron hammers were higher, or deeper, depending on the different weights of the hammers. Thus, by stretching sheep-guts or ox-nerves, tied to different weights, he deduced that the notes changed similarly with the weights attached. Therefore it is certain that through experiments of this kind, Pythagoras recognized the true proportion which shares an arcane secret. He even realized the numbers from which consonant notes are born. It is certain that Pythagoras, through those experiments, recognized the true relationship that exists between the notes of the string and the weights attached to them: that is to say, the weights which allow all the notes to be heard on equal strings, are inversely proportional to the squares of the lengths of string, with which the musical instrument emits those notes. Pythagoras indeed applied the ratios discovered, by means of his experiments, to the heavens, and thus deduced the harmony of the spheres. And thus, comparing those weights with the weights of the planets, and the intervals of sounds with the intervals of the spheres, and the lengths of the strings with the distances of planets from the centre of their orbits, he understood, through the harmony of the spheres, that the weights of the planets in

the Sun (to whose lyre everybody undoubtedly dances) were inversely proportional to the squares of their distances".

Therefore the English natural philosopher wanted to present himself as the legitimate heir of secret Pythagorean sects. And he went so far as to have them "veil" their own gravitation by means of the well-known theory of music. To what extent New Tone/Newton felt Pythagorean has already been underlined sufficiently by others.[85] He could perhaps conceal beneath the rules of the sect that had remained secret the relationship between the pitch of the sound and the weight that gives the tension to the string. In any case, it would have been set by Vincenzio Galilei and taken up by his son Galileo, together with Mersenne. But could he be so convinced that musical argument sustained his gravitational theory to the point of letting slip an error so serious as to undermine it at its roots? That is to say, he must have been so attached a priori to the Pythagorean heritage as to construct for it a musical model that did not work![86]

Without committing the sin of anachronism this time, and to illustrate it better, as an ironic quirk of history, we shall use those very symbols that he avoided, but which were introduced by Descartes and Leibniz. New Tone/Newton employed the inverse proportional relationship, which undoubtedly was really Pythagorean, between the pitch a of the sound, and the length l of the string. We shall write, then, as he did not write, $a \propto \frac{1}{l}$. He also made use of the direct relationship between the height of the sound and the square root of the weight p: $a \propto \sqrt{p}$. Even though it was difficult to make the latter to go back to Greek antiquity. Then, to obtain the higher octave $2a$ of a note, that is to say, twice its frequency, weight (or tension) being equal, he divided the sounding string into half its length, or the length being equal, he stretched it with a weight four times as heavy:

$$2a \propto \frac{1}{\frac{l}{2}} \qquad 2a \propto \sqrt{4p}.$$

But when he specified the ratio to obtain unison, our English natural philosopher wrote that the weights had to be "*inversely* proportional" to the squares of the lengths. This was not correct at all, because to compensate for the lowering of the string due to its lengthening, he should have increased the weight with the square of the length.

$$a \propto \frac{\sqrt{4p}}{2l} \propto \frac{\sqrt{9p}}{3l} \ldots$$

What was most difficult to realise with the Latin terminology (to the best of my knowledge, nobody realised this or even had a desire to do so) would become easier with the symbolism, already successfully used by some, to which we are already accustomed.

[85] McGuire & Rattansi 1966/1989. Gouk 1986 and 1999.

[86] Tonietti 2000b.

Gravity diminishes with the square of the distance of bodies. If we also made the weights for the sounding strings lighter by the same proportion, we might discover that, perhaps in the secret of their meetings, the Pythagoreans had already "veiled" Newton's law, but then the music of the spheres would not have sounded in unison. The planets further from the Sun would produce deeper and deeper notes, whereas the closer ones would produce acute notes. Not only Homer, but every now and then, also Newton must have dropped off. But why should the music generated by the planets in revolving around the Sun be in unison? Could it not be different? Or were the ears of New Tone as hard of hearing as those of Descartes? Typically of him, our English natural philosopher preferred to go back to the mythical (non-existent?) Pythagoras, rather than recognize the merits of other Italians, Frenchmen, Germans or Dutchmen, whether Catholics of Protestants.

Failing sure experimental proof regarding the music of Saturn, New Tone/Newton made his hypothesis about unison which we feel to be incompatible with the origin of earthly sounds. And yet, if he did not realise this, he must have had a blind (and deaf) faith in his ability to unite the heavenly harmony with the earthly one. Colin Mac Laurin (1698–1746) traced a summary of Newton's discoveries. Quite rightly, and unlike his master, he referred that the tensions had to *increase* with the square of the length of the string in order to maintain the same note, whereas the gravity of planets *decreased* with the square of the distance. And yet he, too, concluded: "If, therefore, we were to imagine the musical strings extended from the Sun to every planet, for all these strings to sound in *unison*, it would be necessary to increase or decrease their tensions *in the same proportions* [emphasis my own] sufficient to make the gravity of the planets equal. And from the analogy of these proportions, it would appear that the famous doctrine of the harmony of the spheres took its origin".[87] Having become powerful and authoritative, Newton's Sun was by now capable of dazzling even the most competent followers.

But would it be so difficult to construct a simple mathematical model which, by combining heavenly and earthly harmony, would justify the law of gravity? It was enough to abandon the hypothesis of unison in order to admit a more varied music, as Kepler had already done, for example, and as Newton himself had suggested at the beginning of his scholium. As Saturn moved, for the Pythagoreans, with a deeper note, it was sufficient to imagine, by analogy, that as they moved away from the Sun, the planets diminished the pitch of their notes, to obtain the same decrease of gravity with the distance. If Newton had had a better ear to hear the music of the spheres, he would have realised that the height of the notes produced by the planets decreased with the inverse of the square of the distance from the Sun. Consequently, by analogy, or better, by means of an elementary algebraic substitution,

$a \propto \frac{1}{l}$, and $a \propto \sqrt{p}$: therefore $\sqrt{p} \propto \frac{1}{l}$,

he would thus have obtained, at last, his gravity

$p \propto \frac{1}{l^2}$.

[87] McGuire & Rattansi 1966/1989, pp. 83–84.

Furthermore, he would have succeeded in remaining more faithful to the Pythagorean tradition which he loved, as transpired from the whole of the *scholium* considered. But perhaps, like the Pythagoreans, Newton, too, did not want to make himself understood clearly, or to be understood by everybody. Without doubt, not all historians were to succeed in doing so.[88] Leaving this question open for our readers to answer, we may, however, conclude that we have now perceived better how deeply Newton believed in the music of the spheres. In any case, he had searched for a *unique* theory which accounted both for the planetary movements and for sounds on the earth. New Tone/Newton offered a kind of gravitational theory unified with music, which modelled force on the tension of strings.

As we did not, previously, underestimate metaphysics in Leibniz, now we can definitely not do so with the intelligence of Newton. Let us search for other reasons for this. If the Englishman had made his music of the spheres polyphonic, would he not have conceded too much to that other German astronomer, who had dedicated himself much more than him to this subject? In his *Harmonices Mundi Libri Quinque*, Kepler had made faraway planets like Saturn and Jupiter produce deep notes, and the closer ones acute notes. For him, the planets formed a complex polyphony, and certainly not unison, precisely because their orbits were elliptic. Could Newton risk having the laws of gravity "veiled" not only by the faraway ancient Greeks, but also by that German astronomer to whom he already owed so much? That *lapsus*, then, saved the faith of Newton in the music of the spheres, without having to concede too much, as usual, to Kepler.

It might also be sustained, however, that the classical *scholia* were not published by Newton and thus they cannot have counted very much for him, compared with his other works. We can easily concede this. However, we must remember, for our justification, that he was very loath to publish, and that their exclusion, in the end, from the editions of the *Principia* printed in 1713 and in 1726, and their substitution with the general *scholium* might be attributed to other reasons. What led Newton to give up his classical scholia may not have been so much the lack of conviction or interest in them, because after all, he really wrote them. He may have been led to this decision by his well-known attitudes towards his rivals, and his art of avoiding disputes at all costs. This led him to adopt forms of self-censure and to ignore his adversaries, thus avoiding quoting them. As possible shock-absorbers, he used the secretary of the Royal Society Henry Oldenburg (1615?–1677) and others, like Clarke.[89]

In any case, the *scholium* to Proposition VIII of the *Principia* was not the only place where Newton dealt with music. How well did he know it? And how interested was he in it? It seems that he did not like listening to it very much, and, like a Puritan of his times, he even considered it a dangerous source of temptation. Yet as a student of Isaac Barrow (1630–1677) at Cambridge, he must necessarily have studied the theory of music that was currently believed at that

[88]McGuire & Rattansi 1966/1989. Casini 1981 and 1984. Gouk 1986 and 1999. Tonietti 2000b.

[89]Casini 1981. Barone 1989. Westfall 1989, pp. 272–279. Newton 2006, pp. 215–285.

time. It is interesting that this master of his felt that it was rather subordinate to geometry than to arithmetic, as in the tradition.[90] New Tone/Newton followed the activities of two professional musicians: Thomas Mace (1613–1706) composed a *Musick's Monument* (1676) and proposed a "hearing trumpet" of his to the Royal Society. By Thomas Salmon (1648–1706), our English natural philosopher knew the *Proposal to Perform Music in Perfect and Mathematical Proportions* (1688) and a manuscript on the "Division of a Monochord".[91]

But his notes on music, left during his student days, are more interesting. In that period, he had read the *Compendium* by Descartes, translated into English, taking his inspiration for a figure of the intervals from it.[92] From the manuscripts, one of which was entitled "On Musick", we learn how he considered the Greek modes, and how he intended to divide the octave. He probably agreed with the criticism brought against Descartes by the First President of the Royal Society, William Brouncker (1620–1684), in his *Des-Cartes Excellent Compendium of Musick and Animadversions of the Author* (1653), where geometry was preferred to arithmetic, and equal semitones were calculated using logarithms.[93]

In his notes, New Tone/Newton constructed, by means of whole numbers and fractions, his symmetrical variant of the Zarlinian scale. He compared this with the other numerous possible divisions of the monochord. A table of numbers, from *g* to *g* [from *sol* to *sol*], was called "A string divided by a Musical progression", juxtaposing it to "A string divided in a Geometrical progression". In the second table, he included calculations of $\sqrt[12]{2}$, $\sqrt{2}$ and the other roots necessary to divide the octave into 12 equal semitones. The numbers obtained were compared with those of the fractions with which Zarlino had constructed his scale, to verify how far the intervals of the minor second, ..., the fifth, or the fourth, were higher or deep in the two cases. In another table, he used logarithms. He listed scales divided into innumerable parts: 12, ..., 29, ..., 41, 51, 53, 100, 120, 612. From the one divided into 53 micro-intervals, he obtained the best approximation to the so-called "just" semitone. Thus his preference went to the latter, now broken down into basic elementary micro-intervals, like species of sounding atoms. Our English natural philosopher ignored all considerations about which scale was the most suitable one to compose and play music, in accordance with the practice of musicians around him. In "On Musick", he devoted himself to combining tones and semitones variously, in order to reconstruct the Greek modes, inspired, perhaps, by Marin Mersenne, seeing that it is unlikely that he knew the manuscript of Maurolico.[94]

Above all in his *Opticks* of 1704, Newton developed a synaesthesia between light and sound, arriving at the point of creating a close analogy between the colours that were created by a prism and the notes of the musical scale. First of all, there had

[90] Gouk 1999, pp. 224–228.

[91] Gouk 1999, pp. 153 and 230–233.

[92] Gouk 1999, p. 140.

[93] Gouk 1999, pp. 142–143.

[94] Jeans 1980. Gouk 1986, pp. 41–44. Gouk 1999, pp. 233–237. Tonietti 2006b.

to be seven colours, like the number of notes: violet, indigo, blue, green, yellow, orange and red. In book I, part II, prop. III, problem I, the colours were distributed along a line divided in accordance with the harmonic proportions of a musical scale. "... in proportion to one another, as the Numbers

$$1, \frac{8}{9}, \frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{9}{16}, \frac{1}{2}.$$

and so to represent the Chords of the Key, and of a tone, a minor third, a fourth, a fifth, a major sixth, a seventh and an eighth above that Key: ...".

The intervals corresponded to the succession from violet to red. Synaesthesia returned in proposition VI, problem II. This time the colours were distributed along a circumference divided into seven parts "... proportional to the seven Musical Tones or Intervals of the eight Sounds, $Sol, la, fa, sol, la, mi, fa, sol$, contained in an eight, that is, proportional to the Number $\frac{1}{9}, \frac{1}{16}, \frac{1}{10}, \frac{1}{9}, \frac{1}{16}, \frac{1}{16}, \frac{1}{9}$." The representation in the circle of the musical monochord recalled both the *Compendium Musicae* by René Descartes, and Kepler's *Harmonices Mundi* .... The analogy between colours and notes was re-proposed by Newton in book II at the beginning of the second part, by means of a model and more complex calculations. Lastly, in *Query* 14 of book III part I, our English natural philosopher wondered: "May not the harmony and discord of Colours arise from the proportions of the Vibrations propagated through the Fibres of the optick Nerves into the Brain, as the harmony and discord of Sounds arise from the proportions of the Vibrations of the Air? For some Colours, if they be view'd together, are agreeable to one another, as those of Gold and Indigo, and others disagree."[95]

It is to be noted that by shortening the string, the height of the sound increases, whereas the colours, according to Newton pass from violet to red. But this should not come as a surprise to us, because, as is known, he sustained a 'corpuscular', and not an 'undulatory' theory of light. The contents of the *Opticks*, had already been partly expounded by Newton in his letter to Oldenburg of December 7th, 1675. In his *Optical Lectures [Lessons of optics]* of 1672, published only posthumously, Newton developed an analogy between the spectrum of light and the equable temperament, admitting that the results were equally good compared to the scale divided by rational numbers.[96] But then in the *Opticks*, the reference sound spectrum remained the one divided only by the latter. In the dispute against Hooke and his undulatory hypothesis for light, Newton made the image of light emerge as vibrations of the ether, similar to air waves. Colours would be created "... in a similar manner to the way in which bodies, ..., stimulate vibrations of various widths in the air, which, in conformity to the widths, produce the different tonalities in sound." But here the

[95]Newton 1704/1952, pp. 126–128, 225–226 and 346.

[96]Gouk 1999, pp. 242–244. The research of Penelope Gouk was used as a basis by Benjamin Wardhaugh, adding some extra details. However, it sounds very curious to me that the latter completely ignores the musical scholium written by Newton for proposition VIII of the *Principia*; see the previous pages. Wardhaugh 2008, pp. 53–56 and 120–125.

English natural philosopher, despite Mersenne, seemed to confuse the number of the vibrations in time, that is to say, the frequency, with the amplitude of the wave.[97]

Asked for a comment on *A Philosophical Essay of Music*, published in 1677 by Francis North (1637–1685), Newton refused in his letter to give an explanation for consonances based on the coincidence of strokes. "For though ye pulses of ye strings should reduce one another to a synchronism, yet those of a string will strike ye ear sooner or later than those of ye other, according as ye ear is more or less near to one string than another. [...] ... it follows ... that concords arise not from ye coincidence of pulses at the ear nor have any dependence on such coincidences, ...".[98]

More than other supporters of the traditional theories, New Tone/Newton had at least opened up to the calculation of the equal semitones of the equable temperament using irrational numbers based on the roots of 2. He also appears to have been one of the first to have raised physical objections to the theory of coincidences then in fashion. It remains to wonder, however, how far he was stimulated in this by a desire to distinguish himself from the philosophy of Descartes which he disliked. Furthermore, it is significant that he took the liberty of expressing criticism above all in private, whereas in official publications, he returned to being a more orthodox follower of the current musical theories. Anyway, he was more at his ease when he modelled sounds as periodic undulatory movements of the air. Then, he reduced them to particles which mathematical oscillated in an elastic (harmonic) way, like several tiny pendulums. Having made mathematical the idea, which as we shall see was proposed by Hooke, he succeeded in deriving, from his famous principles of mechanics, the speed of sound. He thought of it as a product of the frequency by the wavelength, and compared it with certain experimental data, such as those of Mersenne, albeit with some difficulty.[99]

In about 1664, the young Newton started to read *La Géométrie* by Descartes, as well as the texts of Wallis, Viète, and Barrow. But for our English natural philosopher, who did not possess any background knowledge yet in classical geometry, the algebraic equation became at that moment more important than the curve designed, whereas for the Frenchman, it remained only a fast aid for calculation. Essentially self-taught, New Tone/Newton worked to obtain the values

[97]Newton 2006, pp. 250, 254 and 258. Continuing the series of mutilations, this recent anthology of Newton's writings about optics, has ignored the main analogies created with music. The omission of these passages appears to be all the more curious because in the texts, our Englishman also tried to give "mathematical explanations" of the luminous phenomena. But together with the well-known descriptions of the experiments with prisms, the numerical ratios of the intervals between notes were the most mathematical things that he could offer, apart from the geometry of rays.

[98]Gouk 1999, pp. 246–248. Kassler 2004. Wardhaugh 2008, pp. 125–133.

[99]Newton 1687/1972, pp. 510–534. Koyré 1972, pp. 87–126 and 214–217. Dostrovsky 1975, pp. 209–218. Westfall 1989, pp. 315–318, 394–397, 414, 417, 775–777 and *passim*. Gouk 1986, p. 85. Gouk 1999, pp. 248–251.

of roots, of π; he squared curves (that is to say, he calculated their areas) and he studied methods to compute logarithms rapidly. Having obtained an approximate value of the desired magnitude, he went further to arrive increasingly close to the truth, but in certain cases it seemed to escape him, unreachable, inexpressible, without any *logos*, surd and irrational. What could he do to come gradually closer to it, while knowing that the elementary operations of algebra would not be sufficient? The equations of Descartes contained a finite number of terms, those that Newton now manipulated possessed an infinite number. Without worrying too much about how to justify it, he added up infinite figures, and multiplied an infinite number of factors, obtaining finite values for the desired unknown quantities. That is to say, he added up what we would call today a series of powers in the unknown element. He proceeded by means of increasingly precise approximations, until "at the limit" he reached the exact value. Thus by generalising algebra with faith and nonchalant, our English natural philosopher reached infinity, as we have seen his antagonist, Leibniz, did.

Every little passage, every less than modest addition, brought him closer and closer, but he had to consider an infinite number of such tiny steps of progress towards the solution. And yet, for him, these numbers so close to '*0*' were not indivisible, that is to say, he did not consider them to be infinitesimals, to be treated as numerical atoms for calculation, as they were, instead, for Wallis or Bonaventura Cavalieri (1598?–1647). In the world of Newton, points moved in space, generating geometric curves with their movement, in the style of Barrow. Thus he arrived at the values of magnitudes, not by means of static operations, like counting an infinite number of beads, but in continual movement until he reached his goal by means of infinite values of the variable. The mathematical model of the Englishman did not classify the qualities, it was not immobile like that of Descartes, but dynamic, because it included time. A manuscript of November 13th, 1665 was entitled "How to find the speeds of bodies by means of the lines that they describe". His mathematics became the science of bodies in movement. As time flowed in a uniform manner, bodies moved in space and flowed in it with a continuity, describing curves called "fluents". To find the speed at a given moment, it was necessary to calculate its tangent at a given point.

Descartes calculated the tangent, searching for the value for which the intersections of the straight line with the curve coincided at a single point, that is to say, the point at which the relative equation admitted coinciding roots. Newton, instead, imagined a process of successive approximations, which, at the limit of smaller and smaller intervals of time and space, confused the curve with the chord: he called this the calculation of the "fluxions". He also found that the calculation of the tangent to the trajectory, as a calculation of the speed of the body in movement, represented in a certain sense, the inverse operation to the calculation of the area lying under the curve. That is to say, he, too, invented what today is called the fundamental theorem of integral calculus.

At that time, around the *annus mirabilis* 1666, Newton was totally obscure, known only to Barrow. He should have been living at Trinity College, Cambridge. But, due to a terrible plague in those wonderful, yet tragic, years, he mainly lived in his mother's house. As confirmed first of all by the people who were closest to him, who were probably referring to a late memory of his, subsequently described by Voltaire, it was here that the famous episode of the apple took place.

The cause of bodies dropping to the ground thus seemed to him the same as that of bodies revolving in the sky. The idea of that single universal reason had probably come to him via Kepler, who had already pointed his finger at the only link existing between the planets. Newton appears to have found the quantitative law of attraction by means of the Third Law. Sure enough, having established (as Huygens had done) that the centripetal force of a circular movement is proportional to the square of the velocity and inversely proportional to the radius, that is to say, $F \propto v^2 \frac{1}{R}$, the velocity being proportional to the radius, and inversely proportional to the time taken to cover the distance, $v \propto \frac{R}{T}$, we would obtain that $F \propto \frac{R^2}{T^2} \frac{1}{R}$, from which, seeing that $T^2 \propto R^3$, by Kepler's Third Law, we would get, in the end, $F \propto \frac{R^2}{R^3} \frac{1}{R} \propto \frac{1}{R^2}$. Thus the attraction had to be inversely proportional to the square of the radius. As if this were not sufficient, at that time our English scholar was decomposing sunlight with a prism into a continuous spectrum of colours similar to the rainbow, and was formulating a highly original corpuscular theory. But in those years, whether wonderful, fortunate, or just turbulent, everything, from the calculation of fluxions, to gravity, to optics, remained outlined and incomplete.

Subsequently, Newton wrote few articles about infinite series (1669), about fluxions (1671, our derivatives), about quadratures (1676, our integrals) and he did not publish anything immediately. All his inventions were to wait for the arrival of the new century. At last, solicited and financed by the astronomer, Edmund Halley (1656?–1743), the *Philosofiae Naturalis Principia Mathematica* was published in 1687. This book immediately met with great success, in spite of its particular difficulty, and was to see two more editions during the author's lifetime. Here, the natural philosopher could not appear to be more different from Galileo Galilei, Kepler and Descartes. In his text, he left a short, inexorable succession of definitions, axioms, laws, propositions, and theorems, all numbered with great cure. Just as others had enjoyed discussing with words of a general nature, so he limited himself to what was strictly necessary for his argument, which was always of a mathematical nature. With this style, he appeared to be the most modern figure in the art of concealing hypotheses, presuppositions, his own personal philosophical and religious convictions, as if they were irrelevant. From the *Principia*, our English physicist hardly allowed anything to transpire of the route followed to obtain those results; it was sufficient for him to justify them.

After him, separating the context of the discovery from that of the justification, as some philosopher of science might have said in recent times, that is to say, demarcating himself with respect to philosophy, religion and politics, was practically to become the general rule. The distance of the *Principia* from Kepler's *Harmonices Mundi Libri Quinque*, with all his acute and bizarre digressions, with his continual

shameless manifestation of all his convictions, could not seem greater, although they were both written in Latin. For this reason, Kepler always risked, ingenuously and sincerely, losing out. For too many of us, today, he would lay himself open to ridicule, entangled in beliefs which the scientific community, with time, has abandoned. Newton, instead, did not want to run any danger, and protected himself completely against criticism, behind the most unassailable arguments possible for the period.

Although he had sometimes used them to find theorems, and they were his most natural (secret) language, actually the new procedures for the infinite calculation of series, of fluxions and of quadratures hardly ever appeared openly in the *Principia*. Here, he continued to use the traditional geometrical techniques, considered safer and not subject to controversy. Our English natural philosopher rarely used even finite algebraic expressions, as if they, too, did not possess sufficient authority to sustain the extraordinary system that he had set up. Any physicist or mathematician of our great epoch would not find any of the formulas to which our university textbooks have accustomed him. Mathematical analysis and differential calculus, which we are accustomed to using to express laws of dynamics similar to those we owe to Newton, needed another symbolic, transcendent language, as we know. But New Tone/Newton did not have the slightest inclination to invent it, because he preferred to justify himself in the calm river-bed of the geometrical tradition.

In spite of all the concerns and the precautions adopted, even our modern English physicist was forced to leave some rare comments here and there, to add some notes to the text to explain himself a bit better, or to punish some adversary who sustained different theories, without naming him, of course. Thus he wrote the *scholia*, from one of which we started this section. By means of them, come out into the open his faith in the harmony of the cosmos, to a level which would have been difficult to suspect in Newton.

Clearly, however, the relative mathematical model, though similar in its pattern to those of Galileo Galilei, Kepler and Descartes, produced its own particular melodies. "... the most modern ones, having abandoned the substantial forms and the covert qualities, tried to reduce the phenomena of nature to mathematical laws, it seemed appropriate in this treatise to cultivate mathematics for that part which is connected with philosophy" .... "Geometry is based on the practice of mechanics and is nothing other than that part of universal mechanics which proposes and demonstrates the art of measuring extremely accurately". Thus we would have the same ideal programme found in Galileo Galilei, Descartes or Leibniz. But that of Newton was specific, because the "rational mechanics will be the science of movements that derive from any forces and the science of the forces that are required by any movements, expounded and demonstrated exactly". Having demonstrated the general propositions mathematically in the first two books, in book III the forces of gravity were "derived by means of which bodies tend towards the Sun, and the single planets ... Would to God that it were possible to deduce the remaining phenomena of nature from the principles of mechanics with the same kind of reasoning". Thus he reasoned by means of forces.

After the definitions of mass, quantity of movement, inertia as a disposition to resist, force as a variation of movement, centripetal force, quantity of acceleration, our English scholar added a *scholium*. He defined time: "... absolute, true, mathematical time, in itself and by nature, without relationships or anything external, flows uniformly ...", whereas "apparent, vulgar, relative time" would be "... the hour, the day, the month, the year". He defined space: "... absolute space ... always the same and immobile", whereas relative space would be "in relation to bodies". Lastly: "... the parts of immobile space, where bodies truly move, do not fall under the senses". These were the notions which we have seen Leibniz reject. Having assumed into the heaven of ideas and subtracted from the earth two such delicate notions as space and time, Newton gave the laws or axioms of movement: the law of inertia, according to which a body perseveres in its state of movement; the law of dynamics, according to which the change of movement, acceleration, is proportional to the moving force impressed; the law of action and reaction. Then, among other things, he gave the rule for the parallelogram by means of which several forces are reduced to a single force.

In one *scholium*, he admitted: "By the method of the indivisibles, demonstrations are made shorter. But as the hypothesis of the indivisibles is arduous, and as that method is considered less geometrical, I have preferred to reduce the demonstrations of the following things to the first and last sums, and evanescent and nascent ratios of quantity, that is to say, the limits of sums and ratios". Trying to explain better what he meant by "limit", Newton wrote that if "the final relationship of evanescent quantity" $\frac{a \mapsto 0}{b \mapsto 0} = c$, had not existed, the "ultimate velocity of a body where movement finishes" would not have existed, either. For him, therefore, the physical evidence of the existence of a velocity justified a calculus procedure so new and so uncertain at that time in its foundations. And was this not an act of faith in the harmony between mathematics and nature? Newton demonstrated that if the orbits were elliptic, and the force was directed towards the focus, then it must be inversely proportional to the square of the distance (Sect. III Prop. XI). In the case of this force, he derived the laws of Kepler, but he did not recall him at all. Previously, on the other hand, he had quoted Galileo Galilei. In the *scholium* to Sect. XI of the 1st book, he explained what attraction was: "any tendency of bodies to draw close to one another ... a mutual searching for one another ... spirits emitted which move each other reciprocally, ... an action of the ether or the air, ... any corporeal or incorporeal means which in any way propels the bodies that rotate inside it one towards the other".

In the second book, our English physicist descended from Heaven to the Earth, to study the movements of bodies in a means with friction proportional to the speed or to its square. He also dealt, therefore, with the movement of bullets. In Sect. II, Lemma II gave the rules for the calculation of fluxions (derivatives), and in the *scholium* relative to the first two editions, he also quoted Leibniz, whereas in the *scholium* of the third edition he cancelled this. Between the two, surely there had been a heated controversy in those years about who had invented the new method

for the calculation of tangents, and for maxima and minima. Newton did not speak about it at all, and did not leave any trace of it here.[100]

No better treatment was reserved to Descartes, whose planetary model of vortices was liquidated. Newton tried to demonstrate that the Sun, revolving in a viscous means, would have made it revolve with a period proportional to the square of the distance $T \propto R^2$ while the laws of Kepler gave a sesquialtera proportion $T \propto R^{3/2}$. Those who believed in the theory of vortices were therefore invited to reflect on this. However, Descartes did not deserve to get a mention, either. To be precise, our Englishman tried also with the model of a rotating cylinder, finding that now the period depended linearly on the radius: $T \propto R$. And we may also wonder whether Newton was really convinced that he had excluded the possibility also of an intermediate model between the two. Then may he have known the law of vortices that are formed in water in certain conditions without any need for particular rotating bodies? This appeared to have been known for a long time to that other observer of water in movement, Leonardo da Vinci. The product of the velocity by the radius is constant in the vortex, it recited, and thus it was equivalent to the first model of Newton. $v \times R = \text{constant}$, from which $\frac{R}{T} \times R = \text{constant}$. But which model was to be assumed, then, for earthly vortices? And which one for the heavenly ones of a fluid whose properties did not seem to be directly measurable?

In his third book, entitled "World System", the renowned Englishman expounded the rules of philosophizing, that is to say, his method. The causes necessary for the explanation were to be sought, and nothing else. "Hypoth. I. Causas rerum naturalium non plures admitti debere, quam quae & verae sint & earum Phaenomenis explicandis sufficiunt. Natura enim simplex est & rerum causis superfluis non luxuriat. Hypoth. II. Ideoque effectuum naturalium eiusdem generis eaedem sunt causae." ["First hypothesis. No more causes of natural things must be admitted than those that are true and sufficient to explain their phenomena. Nature actually is simple and does not abound in superfluous causes for them. Second hypothesis. Hence the causes of natural effects of the same kind are the same."] Then he declared his faith in the inductive method, because the truth obtained by induction could not be disproved by hypothesis.[101] The astronomic observations confirmed the laws of Kepler, which he brilliantly recalculated from his law of universal gravity, and here, at last, his name was mentioned.[102] All bodies, in a void, fall in equal times.

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[100]On fluxions, Newton wrote to Oldenburg a letter, so that it were forwarded to Leibniz, and it finally arrived only in 1677. But the fundamental theorem was encoded. "6accdae13eff7i319n404qrr4s8t12ux. ... Data aequatione quotcunque fluentes quantitates involvente, fluxiones invenire; et viceversa." ["... Given an equation including any number of fluent quantities, find the fluxions; and vice versa."] Newton 1687/1972. Westfall 1989, "Chapter four", "Chapter five", pp. 272–280, 416–417, 441–448, 539–546, 550 and 736–825 and *passim*. Cf. Koyré 1972, pp. 92–98 and 114–120.

[101]Cf. Koyré 1972, pp. 108–114 and 291–303. Cf. Westfall 1989, pp. 475–479. Newton 1687/1972, p. 550.

[102]The German astronomer was treated better by Halley, who wrote that Newton's propositions were "... found to agree with the *Phenomena* of the Celestial motions, as discovered by the great Sagacity and Diligence of Kepler." Gingerich 1970, p. 308.

Weights do not depend on shape. The force of gravity cannot be the same as the magnetic force, because the latter was indicated as $\frac{1}{R^3}$ (magnets attract one another like dipoles!). Tides were created by the united action of attraction of the Moon and the Sun.

Until the end, Newton tried to avoid disputes and controversies, as far as possible, continuing to reason by means of fine geometrical designs, hardly ever using algebraic formulas or the calculation of fluxions. Without doubt, his "General *scholium*" at the end of his *Principia mathematica* must have seemed an incontrovertible, unquestionable conclusion. Unfortunately, the finale was influenced, despite everything, by the context, much more than the rest. It may seem to us that all the efforts to maintain himself 'objective' and 'rational' – as was subsequently often to be said – were seriously compromised. After recalling again the difficulties of the theory of vortices, our great modern physicist transported us into the Heaven of ideas. Here, as could be verified on Earth with the help of Boyle's machine to produce a void, a feather and solid gold fell at the same speed. Here, the planets, the comets, all bodies moved freely "in perpetuo" [for ever], led by the law of gravity.

And yet, in the heights of the Heavens, Newton included another Being, whose properties now he finally gave. "This most elegant system of the Sun, the planets and the comets could not be born without the design and the power of an intelligent and powerful Being ... subject to the dominion of the *One*... He governs all things, not as the soul of the world, but as the Lord of the universe. And in view of His dominion He is usually called the Lord God Pantocrator ... The Supreme God is an eternal, infinite being, absolutely perfect; but a being that is in all ways perfect would not be the Lord God without dominion. He is eternal and infinite, omnipotent, omniscient. He is not eternity or infinity, but He is eternal and infinite; He is not duration or space, but He endures and is present. He endures for ever and exists everywhere and existing for ever and everywhere, he constitutes duration and space. Being *always* every particle of space, and *everywhere* every indivisible moment of duration, undoubtedly the creator of all things and Lord, it will not be true that He exists *in no time* and *in no place*". Having guaranteed his model of space and time by means of this God, our English theologian described Him as without any body, thus attacking idolaters. "... He cannot be seen, heard, touched, nor must He be venerated in the form of anything corporeal. From a blind metaphysical necessity ... no variety of things is born. The entire variety of the things created ... could only be born from the ideas and from the will of a necessarily existing being".

After invoking God as a seal to the *Principia*, thus putting all his adversaries to silence, Newton added a surprising sentence in his second edition (1713). "And this as regards God, about Whom it is thoroughly pertinent for experimental philosophy to debate, starting from phenomena". In the third edition of 1726, "experimental philosophy" became a less demanding "natural philosophy". But in all cases, the mathematical model of the cosmos proposed by our English scientist was guaranteed by an incorporeal God, one, eternal, ... about Whom, "starting from phenomena" natural philosophy absolutely had to have its word say, and it must not remain silent at all.

Only after this specification did our natural philosopher pen the sentence which has been quoted so often and misunderstood too many times as well. "So far, I have expounded the phenomena of the heavens and of our sea through the force of gravity, but I have not yet assigned the cause of gravity" . . . "To tell the truth, I have not yet succeeded in deducing so far, on the basis of the phenomena, the reason for these properties of gravity and I do not pretend hypotheses [*hypotheses non fingo*]. Anything that cannot be deduced from phenomena must be called a *hypothesis*; and in *experimental philosophy* there is no place for hypotheses, whether metaphysical, or physical, or with covert qualities, or with mechanical qualities. In this philosophy, the propositions are deduced from phenomena and are generalised by induction". He was thus polemical both against mechanical hypotheses like those of Hooke, and against metaphysics (above all that of Leibniz, covertly).[103] But for him, God would not be a pure hypothesis, as he had explained above. On the other hand, he tried to describe, in the end, "an extremely thin spirit" which pervaded all bodies, "by means of the force and the actions of which the particles of bodies are mutually attracted at small distances". If he could not describe its laws accurately, it was because a sufficient number of experiments had not been conducted yet.

Newton read and interpreted by means of geometry (the new mathematical analysis) what Galileo Galilei had baptised, with an effective metaphor, the book of nature. He had revealed some of its difficult secrets with originality. But this was not the only book that interested him; he was particularly keen also on another text, the Book *par excellence*, that is to say, the *Bible*. He attempted to create a rigorous dating system by analysing the chronology of the Kings. He interpreted the prophecies of Daniel and John. He practically wrote a treatise on the *Apocalypse [Revelation]*: "Having searched, & by the grace of God obtained, knowledge in ye prophetique scriptures, I have thought my self bound to communicate it for the benefit of others, remembring ye judgment of him who hid his talent in a napkin."[104]

"Truth is ever to be found in simplicity, & not in ye multiplicity & confusion of things. As ye world, which to ye naked eye exhibits the greatest variety of objects, appears very simple in its internal constitution when surveyed by a philosophic understanding, & so much ye simpler by how much the better it is understood, so it is in these visions. It is ye perfections of all God's works that they are all done with ye greatest simplicity. He is ye God of order & not of confusion. And therefore as they that would understand ye frame of ye world must endeavour to reduce their knowledge to all possible simplicity, so it must be in seeking to understand these visions [of the *Apocalypse* . . .]. In construing ye Apocalyps to have little or no regard to arguments drawn from events of things; . . . because there can scarce be any certainty in historical interpretations . . . unless ye construction be first determined."[105]

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[103]Newton 2006, pp. 216, 220, 232, 237–238, 244–245, 248–249 and *passim*. Westfall 1989, pp. 254–265 and 791–793. Mamiani 1994a, pp. xxvi–xxviii. Koyré 1972, pp. 31–43.

[104]Newton 1994, p. 2.

[105]Newton 1994, p. 28.

It is interesting that the co-founder of modern physics, rightly or wrongly considered to be the main one, left one of his very few references to his method when he was taking the place of the prophets of the *Bible*. And yet it is just here that we can read the most important characteristics of the future European sciences. We may discuss how capable he was of interpreting a fairly controversial Christian text, above all in his age of bloodthirsty religious conflicts, but less about what his future physicist colleagues would believe better. In the following centuries, actually, the "truth" would be sought by attempting to cancel variety from the world, to reduce it to the order of a few simple elements. But to obtain them, this "vision", similar to that of *Apocalypse*, had to be constructed [with] "little or no regard to arguments drawn from events of things". It would not be the least point that this "truth", about *Apocalypse* [and the world], should avoid "historical interpretations" that were far from being certainties. For him, even these should be "constructed" in anticipation of the events, as any prophet should be capable of doing provided he were true, because he would be in direct contact with God. In the text, by means of an explicit comparison with the interpretations of the *Apocalypse*, the mathematical sciences became transcendent, and outside the history of earthly events. "That [construction of the *Apocalypse [Revelation]* or of the world] must not be strained to fit history but such things chosen out of history as are most suitable to that."[106]

The renowned natural philosopher listed his figurative interpretations of the words used in the holy texts to prophesy events of religious history. Among these, we may read: "Eyes being ye most chief organs by which a man acquires knowledg, are ye most proper emblem thereof." But as regards the ears, he remained silent. And yet, "This ye very soundings of ye Trumpets imply, as being so many Alarms to war."[107]

In one part, Newton argued his allegories by basing himself on the interpretation of dreams left by the Arab, Achmet. He would have been geographically and linguistically close to the ancient Jews, and therefore, according to Newton, he would be relatively more reliable than other exegetes. Thanks to him, our scholar expanded the Orient, confusing together India, Persia and Egypt. Not that the Englishman drew directly on these sources, but he read about them through the Cambridge Professor, Joseph Mede (1586–1638), whose *Clavis Apocalyptica [Key to Revelation]* had been among his sources of inspiration and of comparison. Achmet, or maybe Ahmed, was a doctor who lived at the court of the famous Abbasid Mamun in the ninth century. Newton probably knew a Greek edition of his works, as well as the Latin translation *Apomasaris Apotelesmata, sive de significatis et eventis somniorum, ex Indorum, Persarum, Aegyptiorumque disciplina [The results of Apomazar, or on the meanings and the events of dreams, according to*

[106]Newton 1994, p. 30.

[107]Newton 1994, pp. 80 and 110.

the doctrine of the Indians, the Persians and the Egyptians.] published at Frankfurt in 1577.[108]

As he was doing with natural philosophy, Newton also looked for coherence and consistency in sacred texts. He omitted to comment on the cabalistic interpretations of the famous 666 attributed to the Beast. Instead, he preferred "…to make the description of the Beast harmonize with his number 666. For 'tis not probable that they should be introduced for no end, & I cannot yet see any unless to make up this harmony, which consists in ye tripple repetition of his description by the Trumpets, Vials & Thunders, under the first six of which his reign is comprehended."[109]

In his comment, music only played negative roles, and did not represent the harmony of the cosmos here. At the sound of Biblical trumpets, wars and disasters were heralded. To quote the prophet Isaiah, (*Isaia*, **23** 15–16), "…shall Tyre sing as an Harlot. Take an harp, go about the City thou Harlot that hast been forgotten, make sweet melody, sing many songs that thou mayest be remembered."[110] The natural philosopher argued in favour of the identification of Tyre as the prostitute of Babylon, a symbol for him of a church corrupted by idolatry, not only that of Rome, but also the Anglican church.

To look for the "truth" in Biblical prophecies, Newton had given himself rules similar to those followed for optics and mechanics in the same period. He presented a summary of these also in the first edition of the *Principia*, when he still called them "hypotheses", as we have seen above.[111] At that time, it took very little to end up at the stake, or on trial, to be persecuted or sent into exile. Our English theologian-cum-physicist, as always, moved cautiously, preferring the direction of the Royal Society or the Mint, to the trouble that awaited heretics. A part of his Biblical and religious writings was published in the eighteenth century, only after his death. The majority of his writings, like the alchemical documents, ended up in a chest which scholars have only recently started to read and publish. We must note a singular consistency between his notion of God and the attitude that he followed in the construction of his mathematical models. That God without any body, infinite, abstract, *unique* gave rise to the suspicion of the Arian heresy, that is to say, the limitation of the divinity of the Christ who had become visible and incarnate, thus denying the Trinity. Therefore, if the Messiah became only a man inspired by the *logos* and a prophet, then God did not have to descend into the sensible worlds, not could he be represented by means of images. In order to find Him again and to be able to reach the truth, it was necessary to detach himself from the Earth. Inasmuch as they were dominated by the God of Newton, the laws of the cosmos could now be known only through a procedure which removed the flesh from appearances, to restore the ideal skeleton: mathematics thus became the method for transcendence.

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[108]Newton 1994, pp. 51, 67 and 73. Mamiani 1994a, pp. xiv, xxxvi–xxxix and 256. Westfall 1989, pp. 341–342.

[109]Newton 1994, pp. 120–122.

[110]Newton 1994, p. 218.

[111]Mamiani 1994a, pp. xxxi–xxxv.

He wrote in a letter: "... I am inclined to believe some general laws of the Creator prevailed with respect to the agreeable or unpleasing affections of all our senses; ...".[112] We may feel that the pleasures of music are also devalued here. The aim to be reached remained the same as that of Leibniz, even though the Englishman used very different means.[113]

Others have dealt with Newton, the alchemist, or the commentator of *Apocalypse [Revelation]*. What has interested us here is above all the New Tone/Newton who saw the mathematical formulation of natural laws in the music of the spheres. Without doubt, the numerical regularities, used in dividing the monochord, in order to obtain consonant notes, were as ancient as the regularities of the cosmic movements, and thus the one and only God of creation must have put in both cases His unique law. Hence the need to discover the relationships between them. His faith in the music of the spheres seems to have above all a maieutic function for him. Even though, in his cautious style, he was prudent not to make a flag out of it, to wave under his castle walls. Unfortunately, the musical interests of New Tone/Newton were practically ignored by historians of the sciences until recently.

We have already met Robert Hooke (1635–1703) as the scholar closest to Newton in all senses, including universal gravitation, so close as to become his worst rival, even receiving accusations of plagiarism.[114] In proportion, music seemed to weigh on the latter's scientific activity less than we are about to feel its pressure on the former. In his *Micrographia [Illustrations Under the Microscope]*, the properties of matter were described as follows. "... particles that are all *similar*, will, like so many *equal musical strings equally stretch't*, vibrate together in a kind of Harmony or unison; ...".[115] Comparisons with music like this were widespread in his work, and not only in the studies on sound. However, the only – almost – article dedicated by Hooke expressly to music, "A Curious Dissertation concerning the Causes of Power & Effects of Musick" remained a manuscript and was to be read, without success, at the Royal Society only posthumously: by a singular coincidence precisely in the year 1727, when Newton died. It was the last sign of the different destiny reserved to the two English adversaries, and of the shadow in which the latter was to keep the former, both during his lifetime and afterwards.

From the manuscript, we learn that the musical dance-cure for the bite of the tarantula (which originated in the South of Italy) had emerged at the Royal Society, probably by way of Kircher's *Musurgia Universalis* or the works of Marsilio Ficino. If music controlled the dance of the world, Hooke was interested in the relative vibrations induced by the notes in strings and glasses, without touching them. "... to examine from wt causes they proceed." The vibrations of bodies were transmitted to the air, and these arrive at the ear. "... admirable Organ: how rarely Nature has in this play'd the part both of an excellent Geometrician & Mechanick, ...". The

[112] Gouk 1986, p. 53. Westfall 1989, pp. 324–348, 359–371 and 848–873.

[113] Mamiani 1994b.

[114] Westfall 1981. Westfall 1989, pp. 254–265, 285–288, 397–407, 419–... and *passim*.

[115] Gouk 1980, p. 586.

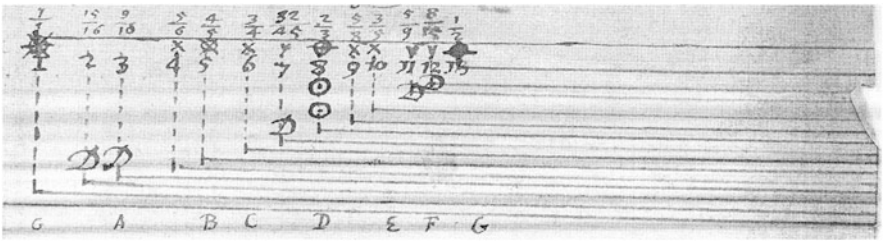

Fig. 10.2 How Hooke divided the octave (Hooke, in *Musick Scripts*, MS O IIa 1 11–12; Trinity College Library)

variety of music was appreciated, and explained, calculating the combinations of notes, referring the enormous number of factorial 17, but mistaking it for 16! It is clear that this other English natural philosopher preferred mechanics. As a matter of fact, the traditional numerical ratios were missing from the manuscript, which insisted on tracing back the causes to the working of the ear. He wrote about the "Acustick faculty" and "Acustick spirits". "This is that which does *demulcere aures* [caress ears], affording the acustick faculty the most pleasant titillation & most ravishing pleasure & delight of all. And the reason of it is clear: for the Eare, being tun'd to either of the notes struck together in concord, is likewise regularly moved wth a pleasing variety, there being on all harmonious notes a coincidence of the Vibrations."[116]

To explain consonances, Hooke transformed the theory of coinciding strokes into the coincidence of "vibrations". To others, from Maurolico to Descartes, it had seemed to corroborate the traditional numbers of Pythagoras and Zarlino. Now that it had changed with the vibrations, the English rival of Newton considered that the theory, on the contrary, appeared to sustain a mechanical model of sounds inspired rather by the experimental philosophy of Francis Bacon. And yet also Hooke's "vibrations" were solved in successive strokes. "... my notion of sound, that it was nothing but strokes within a Determinate degree of velocity ... there was no vibration in a puls of sound ... the sound in all bodys was the striking of the parts one against the other and not the vibration of the whole ...".[117]

In one note, he left a figure (Fig. 10.2) where he designed strings of a length proportional to the rational intervals in the style of Zarlino. But he did not write anything more elaborate or meditated. He only believed that, arriving at the end in the soul, the harmonious movements could "... be computed and adjusted and reduced to number measure and proportion geometrically, and soe the grounds causes and reasons of each of them can be most plainly evidenced and most distinctly proved." The evident tone similar to that of Leibniz was reinforced with

[116]Gouk 1980, pp. 601 and 605.

[117]Gouk 1980, p. 583.

the idea that God had constructed body and soul in harmony in accordance with the rules of geometry, as Pythagoras and Plato had sustained.[118]

Rather than such metaphysical-theological meditations, however, Hooke seemed to prefer to continue to measure the duration of the notes by means of a pendulum. He had proposed to his colleagues of the Royal Society even to change the notation, but without any consequences. From an extant diary of his, he thought that he could improve some musical instruments like harpsichords and organs. This corresponded well to his multiform manual ability with clocks, but also in this case, nothing was to come of it, or at least nothing remained of it.[119]

Before the onerous commitments with the Royal Society experiments took up his time, some traces remained of some ancient organ lesson of his and the engagement as a member of a choir at Oxford. But before the physics of sound officially became acoustics, while this word, if used, had a much wider spectrum of meanings than today, we can notice that music returned in many of Hooke's activities. Already when he was an assistant to Robert Boyle (1627–1692), he exploited his ability with air pumps, inventing the modern model, to verify the transmission of sounds in the necessary means: he thought of it as an elastic form of compression and rarefaction. In the style of Mersenne, but without quoting him, with brass wires, having fixed the measures, our skilful English experimentalist succeeded in counting the frequency of sounds. One note, "*G.Sol.Re.Ut*" emitted by a pipe tuned like that of the string, was measured to vibrate 272 times a second. Vibrating strings were compared to pendulums of the same frequency. With these, experiments were conducted even on gravity. He invented cog wheels which revolved, emitting notes of a given, comparable frequency. This could be calculated with greater precision by the number of cogs, and by the number of revolutions performed by the wheel in one second. We have already encountered a similar system with Huygens (Fig. 9.12.)[120]

Inspired by Francis Bacon, and perhaps by Galileo Galilei, Hooke also set water in vibration inside glasses, descriving the figures that were formed on the surface. The natural philosopher who was to rebuild London after the Great Fire of 1666, Christopher Wren (1632–1723), improved the experiment using a violin bow. Perhaps he even succeeded in verifying the nodes of vibration in a piece of string, as did the two musicians presented by Wallis. In the *Lectures De Potentia restitutiva [Lectures on the force that returns]*, after considering a series of strings that entered into vibration by sympathy, provided that the ones were perfectly tuned with the others, Hooke went on with music. "Now as we find that musical strings will be moved by Unisons and Eighths, and other harmonious chords, though not in the same degree; so do I suppose that *the particles of matter* will be moved principally by such motions as are Unisons, as I may call them, or of equal Velocity with their motions, and by other harmonious motions in a less degree . . . ".

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[118]Gouk 1999, pp. 201–202 and 212–213.

[119]Gouk 1999, pp. 202–207, 136.

[120]Aubrey 1950, p. 165. Gouk 1980, p. 578. Gouk 1999, pp. 199 and 208. See below, Sect. 12.2. Boyle's musical interests are described in Wardhaugh 2008, pp. 119–120.

In the formulation that has remained under his name, as Hooke's Law for Elasticity, that is "ut tensio, sic vis" ["like the tension, so the force"], elastic bodies were controlled by forces proportional to the undergone displacements and started to vibrate, too, like strings, or to oscillate like pendulums. "... it is very evident that the Rule or Law of Nature in every springing body is that force or power thereof to restore itself to its natural position is always proportionate to the Distance or space it is removed therefrom ... From the same also it will be easy to give the reason of the *Isochrone* motion of a Spring or extended string, and of the uniform sound produced by those whose Vibrations are quick enough to produce an audible sound, ...". "All solids have a tremulous Motion like Bells ...".[121]

These periodic movements became his general model for every phenomenon of the world, even though, due to the limits recognized for the human ear, not everything could be perceived as a harmonic sound or a noise. Consequently, Hooke developed the analogy with the vibrations of light. But his luminous oscillations took place in the ether, as those of sounds took place in the air. Therefore they were waves, and not particles, as Newton thought. From this famous adversary, the distance increased when he sustained that also the force of gravity had the mechanical nature of very fast vibrations which propagated everywhere. He had enounced this before Newton, and he had made it decrease with the inverse of the square of the distance. Hooke perceived an analogy between the soul and the Sun: "... which as it were a Representation of the Soul of the World." With his ether, was the English experimentalist not similar to Descartes and Leibniz? Not exactly, because he believed in the void, which he was able to produce with Boyle's pumps, and also no vortices existed in that ether of his, but only light waves. Thanks to it, also the action of gravity at a distance would become a mechanical process.[122]

Like Huygens, Hooke really constructed, and did not only theoretically hypothesise mechanical gadgets, whether pendulums, pumps, barometers, microscopes, sounding tubes or clocks, which, like the Dutch man, he knew to be indispensable for navigation, and for the calculation of the longitude on the open sea. However, he inclined for spring clocks. Our experimental philosopher had the idea of a harmony common to all of that universal elastic vibrating world of his. Unluckily he failed in expressing it through the language that had become necessary in that epoch. Among his many, excessive engagements, including the rebuilding of London after 1666, in spite of the event that he had also been appointed Professor of Geometry, he had not taken pains to find a transcendent symbolism, like his adversaries, who, in one way or another, also wrote about mathematics. For this reason, Hooke would not have all the success that he deserved thanks to his experimental results. The extremely heavy Sun of Newton kept him in the shade. The former was very good at measuring, and at constructing mechanical models, the latter was an unparalleled mathematician of the whole universe; the one was Curator of Experiments and then a simple,

[121] Gouk 1980, pp. 586–587 and 589. The experiments with musical sound performed by Hooke are narrated with interesting details in Wardhaugh 2008, pp. 98–118.

[122] Aubrey 1950, pp. 166–167. Westfall 1981, pp. 484–486. Gouk 1999, p. 216 and *passim*.

it seems ineffective secretary, the other became President of the Royal Society. Incapable of obtaining recognition or credit for his achievements, Hooke concluded bitterly: "a posse ad esse non valet consequentia" ["from being able to being, the consequence is not strong enough"]. In the best of the possible worlds, he had personally experienced how easily the whip of arrogant dominators could overcome any claim of his, however modest or well founded. *The Virtuoso*, a satirical comedy about the members of the Royal Society and their sexual hypocrisies, took him as its main target, thus arousing his reaction. "Dammd Doggs. Vindica me Deus [May God avenge me], people almost pointed."[123]

The patient reader will understand, but perhaps he will not be ready to forgive me, if I am thinking of other than the "vis prope divina" ["almost divine force"] shown by Newton. Beyond the undeniable capacities of the character as a mathematician and a natural philosopher, anyone can judge by himself to what extent his success at Cambridge and in London derived from his ability to move adroitly among the political and religious conflicts of the period. He had taken sides with the winners, maintaining his heretical Arian faith well concealed. Having been appointed, first governor, and then president of the Mint, he directed it firmly, or rather implacably I would say, dominating everyone and everything, going well beyond his official prerogatives. He continued to do the same for 20 years, until the end of his mandate with the Royal Society. One of the few who succeeded in withstanding him was the astronomer John Flamsteed (1646–1719).[124]

While the author of the *Principia* had touched on the Arab world in his Biblical exegesis, we have a direct testimony of the interest shown by Hooke in China. To learn anything about the faraway Empire-at-the-centre, even these Anglicans and Puritans were forced to rely on our well-known Jesuits. The experimental philosopher translated two letters of father Ferdinand Verbiest (1623–1688) from French.[125] From these, it was possible to obtain some acquaintance of the "... great use of *Mathematical* Knowledge ..." as an effective means to spread the Christian religion. But according to the Englishman, the letters should have also contained some curious observations about mathematics and physics.

The exponent of the Royal Society expected benefits for Europe from this. Indeed, he called China the "*Empire* of learning". "This will discover a new *Indian* mine and Treasure, and make a new trade to bring it hither, which may be not unworthy the consideration of our *Honourable East-India-Company*, As well as of several learned Men, who have leisure, and other opportunities to make further progress in this Inquiry; ..." Hooke's interests extended to the language and the abacus of the Chinese, which was compared with the "... *Roman Abacus*, which questionless was the Ground of the Signification of figures according to places."[126]

[123] Westfall 1981, 483. Westfall 1989, pp. 466–473, 480–483, 486–489 and 536. Shadwell 1927.

[124] Westfall 1989, pp. 496–515, 629–635, "Chapter thirteen", 857–858.

[125] Needham 1959, III, pp. 450–454. Martzloff 1988, pp. 25–26.

[126] Hooke 1686, p. 37.

In reality, hardly anything had been written about mathematical sciences, whether Chinese or of western origin, by the Jesuit on a mission to the Chinese emperor. Instead, his pages described north-eastern and western China, with a few observations about customs. Their aim seemed to be that of favouring the penetration of the Country-at-the-centre for religious, commercial and military purposes. The exponent of the Royal Society added a comment of his own, not about the sciences, but about the Chinese language. Also the figure of the abacus seemed to be approximate, because in the rows at the top of the figure, there should be two beads for each position, and not one, as designed.[127]

Hooke wrote the numbers in Chinese characters, together with the date "MDCLXXXVI annus V mensis XIII die" ["the year 1686, the fifth month, and the thirteenth day"]. Among the few other characters mentioned by him, a few lines were missing from two of them: *wan* [ten thousand] and *tang* [soup].[128] He listed those that he considered to be the fundamental signs of written Chinese, because he would have liked to break it down into a few basic elements to re-combine. He wondered whether there were like the figures used in Europe for numbers. He had doubts about those who spoke of Chinese as composed of scribbled characters to be learnt by heart, because he, on the contrary, had noted order and regularity. In brief, our experimental philosopher tried to characterise the Chinese strokes with the letters or syllables of other languages, like Greek, Arabic or Sanskrit.

He even advanced the hypothesis that a very long time ago, at the beginning, Chinese had a writing that was developed linearly and horizontally from left to right, and not from the top to the bottom, starting from the right, as in the books that he knew. All this could be demonstrated by the book *Ye-kim [Yijing, The Classic of Changes]*, which others, both Chinese and Europeans, had mistaken for a "... Conjuring Book or a Book to tell Fortunes ... whereas by the small Specimen I have seen of it, I conceive it to contain the whole Ground, Rule or Grammar, of their Character, Language, and Philosophy, ...". From a calculation of syllables, $8 \times 8 \times 8 = 512$, another source of his information emerged. "... what I have learnt from the Book of *Fohi* [*Fuxi*] I shall give the next opportunity; which will explain the reason of the multiplication of the 8."[129] There is little doubt that here, also Hooke, like Leibniz years later, combined Yin and Yang three by three: $2^3 = 8$.

It was less surprising that our English experimentalist should attribute the European art of printing to a German merchant returning from China or that he discovered the musical pronunciations of their syllables, to multiply their meanings. He also realised that in Europe, in the opposite way to Latin letters, numbers were written from right to left, in the same way that the Arabs compiled their books, and thus the figures had arrived here brought by them.[130] We may conclude that among

[127] Hooke 1686, p. 79.

[128] Hooke 1686, p. 80.

[129] Hooke 1686, pp. 73–75.

[130] Hooke 1686, pp. 65 and 67.

the many interests of Hooke, though theology was not included, exotic languages were, for better or worse.

Under the microscope, he designed fleas or lice, and examined fossils, hypothesising both movements of the earth's crust and mutations of the living species, in defiance of the *Bible*. What was this Hooke, then? A geologist, a biologist, an architect, a physicist, a geometer, a polyglot? He could not incarnate all these professions, while the tensions between academic disciplines were progressively separating them. Thus, like a Leonardo da Vinci two centuries later, this multiform English experimental philosopher was downgraded, by a host of specialised philo-Newtonian historians, to become an inventor of extraordinary mechanical contraptions: he may have had some good intuitions, but he was incapable of developing them properly, "…if we judge him by the standards of a Newton". If, on the contrary, we tried to evaluate Newton by Hooke's standards, we should remember that his prejudice against his rival's mechanisms in favour of a transcendent astronomy at least delayed the assignment of the prize destined to whoever first succeeded in calculating the longitude at sea. When the artisan John Harrison (1693–1776), clearly not a rational scholar of mechanics, succeeded in constructing such a precise chronometer that calculated the longitude within half a degree, he was awarded only half the sum allocated, and he had to spend the rest of his life trying, with difficulty, to obtain justice.[131] The poor, sickly and weak Hooke, to cap it all, was bowled over by a love affair with his niece, Grace. Whereas, apart from an episode in his youth, perhaps, the powerful, inflexible, rigorous, lofty president of the Royal Society was never to suffer such weaknesses. With him, it was forbidden to joke about women, especially if they were nuns, above all because people might have talked about his beautiful niece, Catherine, who was the object of gossip. The time would have come, therefore, to right the balance, both with Hooke, and also with Huygens.[132]

Among the famous pages that Alexandre Koyré (1892–1964) dedicated to Newton, there is no trace of music, and we can easily understand why. This scholar maintained that during the seventeenth century, the cosmos as a finite, orderly, hierarchical system had disappeared, becoming the open, indefinite and infinite (at least until Albert Einstein) universe. In a highly suggestive image, he described the passage by the expression: "the explosion of the sphere". Thus, for this European historian, every harmonic cosmos had disappeared. But then, why did almost all the figures of the so-called scientific revolution continue to occupy themselves also with musical theories, like before? So much so, that the list of those who did not leave anything written on the subject would be shorter than the complementary one.

Perhaps Koyré did not have either enough breath to blow up and explode the sphere that dominates Kepler's books, or the necessary attention for the pages about music written by New Tone/Newton from which we started. When this author

[131] Westfall 1989, pp. 877–880. *The Oxford Companion To Ships and the Sea* 1979, p. 376.

[132] Westfall 1981. Noble 1994, p. 287. Westfall 1989, pp. 62, 201, 360–361, 561 e 630–634. Cf. Koyré 1972, pp. 198–203 and 245–289. Arnold 1996.

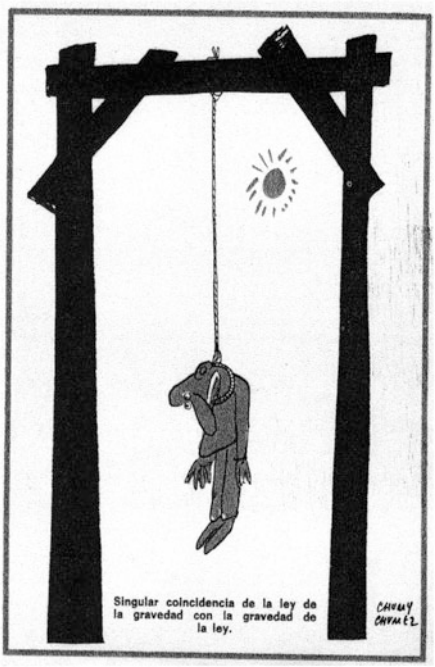

Fig. 10.3 A singular coincidence between the law of gravity and the gravity of law; Chumy Chumez in *Hermano lobo* (m. 7 24 Junio 1972 p. 11)

claimed to uphold "…the disappearance or the violent expulsion from scientific thought of all reasoning based on values, on perfection, on harmony, on significance, …", he did so at the cost of mutilating the evolution of the sciences of all the numerous pages presented here. As a result, shortly afterwards, Koyré, too, ended up by describing Newtonianism as the "order and harmony" of nature. Even, when he was commenting on, and criticising, the "harmonic" circulation of the velocity as inversely proportional to the radius, in the *Tentamen de motuum coelestium causis* written by Leibniz, he omitted (deliberately or out of ignorance?) that the term "harmonic" had been chosen by the ancient Greeks to describe the ratios of music (Fig. 10.3).[133]

[133]Koyré 1972, pp. 7–8 and 138–147. See here, Part I, Sect. 2.2 and Part II Sect. 10.1. The Englishman Benjamin Wardhaugh is to be recalled, not only for some interesting details about the context of the United Kingdom in the seventeenth century, but also, unfortunately, for some modest omissions. Among other things, these would concern even his famous fellow-countryman, Newton, who had left us the *scholium* on music, written, in fact, for the *Principia*; Wardhaugh 2008, pp. 122–125. Cf. Casini 1981; 1984; 1994; Tonietti 2000b. He finally records, it is true, Newton's "…erroneous statement of the relationship between the tension of a string and its pitch." But he has forgotten to reveal in which text it is found. Nor could the reader, worried about this singular *lapsus* that afflicted this gigantic English figure, satisfy his curiosity by searching in the secondary literature quoted by him. Thus Newton has unintentionally presented us with another Pythagorean mystery.

> Sir *Nicholas Gimcrack*. . . . 'Tis below a *Virtuoso*, to trouble himself with Men and Manners. I study Insects; and I have observ'd the *Tarantula*, does infinitely delight in Musick, which is the reason of its poison being drawn out by it. Ther's your Phaenomenon of Sympathy!
> *Longvil*. Does a *Tarantula* delight so in Musick?
> Sir *Nicholas Gimcrack*. Oh extravagantly. There are three sorts, Black, Grey, and Red, that delight in three several sorts and modes of Musick.
> *Bruce*. That was a curious Inquisition; how did you make it?
> Sir *Nicholas Gimcrack*. Why, I put them upon three several Chips in water, then caused a Musician to play, first a grave Pavin, or Almain, at which the black *Tarantula* onely mov'd; it danc'd to it with a kind of grave motion, much like the Benchers at the Revels.
>
> Thomas Shadwell, *The Virtuoso*, act III.

> Fluxions and gravitation would not have been of any use without a pretty niece.
>
> Voltaire

> Newton was not the first of the age of reason: he was the last of the magicians, the last of the Babylonians and the Sumeri, . . . [. . .] Why do I call him a magician? Because he saw the universe and all that is in it *as an enigma*, a secret that can be deciphered only by applying pure thought to testimonies, to mystic keys imposed by God on the world to enable the brotherhood of the esoterics a kind of hunt for the philosopher's stone.
>
> John Maynard Keynes

> I will embrace the true religion of Christ with all my soul; . . . I will make theology the object of my studies and I will take holy orders when the time fixed by the statutes comes, or else I will leave the college.
>
> Oath taken by Isaac Newton

10.3 Symbolism and Transcendence

The event that today, we can understand a little more easily the symbolic formulas of Leibniz than the geometrical arguments of Newton is already a part of the following history. That history was to pass through the Bernoulli family of Basle, which was rich in physicists and mathematicians, like Jacob Bernoulli (1657–1705), Johann Bernoulli (1667–1748), Daniel Bernoulli (1700–1782), through the Swiss scholar Leonhard Euler (1707–1783), through the Frenchman Jean–Baptist le Rond d' Alembert (1717–1783), through Giuseppe Luigi Lagrange (1736–1813) from Turin, and through the Frenchman Pierre Simon de Laplace (1749–1827). Not without further contrasts of tone, the differential calculus of Leibniz then met with Newton's *Principia*, forming the mathematical models of rational mechanics and analytical mechanics. This is the history that we shall meet in the following chapters. The different kinds of music of the spheres that we have heard resounding in the XVII and eighteenth centuries now slowly began to disappear in the distance. It would be curious if, in the infinite mass of papers left by the Bernoullis, which have been 95 % sifted, no traces were found regarding music, in spite of the event that they had worked on every argument of physics and mathematics.[134] And yet clear echoes of

[134]Private communication of Patricia Radelet de Grave, 29.09.2009. But the opinion of this editress is denied by Daniel Bernoulli in the following Sect. 11.1.

the musical theories remained in the terminology used, although by and large the memory of their origin was lost.

The periodic movement (like the pendulum) produced by a force proportional to the distance was called *harmonic*: it could be obtained also by projecting the uniform circular movement on the diameter. *Harmonic* was the name given to the series formed with the reciprocals of the terms of an arithmetic progression $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ The potential function of the Newtonian gravitational force $\frac{1}{R}$ was called *harmonic*, as also the potential of the velocity for an incompressible fluid in irrotational movement. This also gave birth to *harmonic* analysis and the spherical *harmonics* which generalised the circular functions like the *sine* and the *cosine*, through which it was possible to represent circular movement and the group of rotations. In the infinite series of *sines* and *cosines*, introduced by the Frenchman Jean–Baptiste Fourier (1768–1830), every sound of the Earth would thus be broken down into the pure notes emitted by the heavens in their circular movements, as we shall see in Chap. 12.

In projective geometry, between four notes on a straight line, the first of which was posited as the origin, and the others having the abscissae a, b, c, the *harmonic* relationship existed, defined as

$$\frac{b}{b-a} = -\frac{c}{c-a}$$

from which, inverting, it was possible to obtain $\frac{2}{a} = \frac{1}{b} + \frac{1}{c}$. Thus the inverses $a' = \frac{1}{a}, b' = \frac{1}{b}, c' = \frac{1}{c}$, because they satisfied the relationship $2a' = b' + c'$, were in an arithmetic succession. In particular, abscissa points $1, \frac{1}{2}, \frac{1}{3}$, stood in a *harmonic* relationship to one another. And this recalled the harmonic, geometrical and arithmetic means narrated in Chap. 2 of Part I.

Was this profound transformation in the European world, which some called the scientific revolution of the seventeenth century, putting music aside? Yet without a fundamental conviction as regards the agreement together of the relationships between the bodies in movement in heaven, the famous models proposed would not find any justification with their authors themselves. After assimilating, in the nineteenth century, the myth of progress and misplaced hopes in positive philosophies, we understand that in the twenty-first century, the mathematical sciences are pushing in completely different directions. But at the time, when the dynamic laws which guide the movements of the cosmos were invented, the references to harmony, or better still to music of the spheres, appeared to be evident, and fully explicit.

Without the faith in harmony, first during the Renaissance and then the triumphant affirmation in the seventeenth century, it would not be possible to understand the reappearance of mathematics as the language essential to express the new sciences. Having lost the memory, or the faith in harmony, having lost the ear to agreements between mathematics and the world, the books of historians, mathematicians and physicists centuries later would be full of questions about

"The Unreasonable Effectiveness of Mathematics in the natural Sciences".[135] For Galileo Galilei, for Kepler, for Descartes, for Leibniz and for New Tone/Newton, the problem itself would have seemed absurd, and not worth discussing. The question had become rather *how* to interpret the musical language of the spheres, and not *whether* it came from (some) God used in creating the world. And as regards the *how*, we have seen how profound the differences were. Otherwise the cosmos would have proved to be a chaos that was impossible to know and to study. The numbers of the Greeks, the Latins, the Indians, the Arabs and the Jews, the geometrical figures, the letters of algebra and of infinitesimal calculus, appeared to be symbols introduced with the aim of coming closer to the primeval language, to decipher its the enigmas.

Neither Leibniz, nor Newton would have totally concealed the idea that those transcendent languages of theirs, obscure for the non-initiated, but necessary to invent the desired mathematical laws, had been expressed by certain Christian divinities of theirs, imagined outside the sensible world. And yet, to allow the new physico-mathematical sciences to circulate freely, all this metaphysics and theology, variously linked with music, was raising obstacles that were too big. Above all, Newton's astronomy was proving to be too useful for navigation, thanks to the calculation of the tides and the ephemerides. Thus, what had proved to be clearly present and stimulating in the context of the discovery became useless heavy dead weight in the context of the justification, which, instead, eagerly required practical results for the wars and the voyages of conquest planned. Some would consequently have tried to get rid of it, maintaining the rest, where possible. For this purpose, the historians of the sciences were to prove extremely useful, if not indispensable, to sterilise and clean up the events under examination. However, in the train of controversies between British and Continentals regarding priorities, merits or errors, the religion professed by the contenders also happened to emerge, as did also music.

The Irish bishop, George Berkeley (1685–1753) attacked the new analytical methods, accusing them of inconsistency on the logical level, and obscurity: first, the increase in the abscissa and in the ordinate of the curve were considered as small as may be desired, but different from *0*, then at the end, it was made to be equal to *0*. The rules of algebra judged this to be as absurd as dividing by *0*. The rules of geometry, which this philosopher preferred, made it clear why errors compensated one another inexplicable proving to be "true". But how could a similar method expect to be considered scientific? "Because proceeding blindly and arriving at the truth without knowing how, and by what means we proceed, cannot be called science." Analysts, as they were to be called from now on, applied rules that they did not understand, as sailors used the rules of astronomy. Alluding to gravitation, which was considered to be a occult property of matter, the bishop accused the new mathematicians: "... while they express their disappointment at the enunciation of any mystery, as far as they are concerned, they do not create any difficulties, as they are capable of straining out a midge and swallowing a camel." As if he were a

[135] Wigner 1967.

follower of Descartes, Berkeley would rather have claimed clear, distinct principles from Leibniz and Newton.

Even if he was not thinking of Huygens, perhaps, who had been capable, as we know, of solving the many new mathematical problems only by means of geometry, the philosopher of the saying *esse est percipi* ["being means being perceived"] devalued the fluxions and the infinitesimals in the face of the traditional methods inspired by Archimedes. And yet, as Newton had done in the *Opticks*, among the final rhetorical queries, he seemed to touch upon the principal characteristic of mathematical analysis. "33. I ask whether it would not be more correct to approximate well rather than to tend to accuracy by means of sophisms. [...] 53. I ask whether, admitting that the purpose of geometry is practice, and that practice consists of measuring, and bearing in mind that measurement refers only to an extension that can be assigned, unlimited approximations correspond wholly to the purposes of geometry."[136]

Effectively, the tangent was capable of coming as close as desired to the relative curve, whether algebraic or transcendent, and calculation with tangents symbolised by differentials or by fluxions was much easier to learn than any complex geometrical procedure. Furthermore, the laws of the natural world that all of them sought, inasmuch as they were perfect and transcended the contingent phenomena which depended on unreliable senses, seemed unreachable: because they were infinitely far away, and out of the reach of imperfect men. But then they could be approximated by a procedure of calculation which made it possible to contemplate large and small infinites without being dazzled. With these means, for example, velocities and accelerations of bodies in movement could be represented well, or maxima and minima could be calculated.

As presented by the two leading contenders, Leibniz and Newton, Berkeley did not fail to underline that these physical-mathematical problems concerning infinity were in reality soaked of religious metaphysics which were difficult to eliminate. "And no geometer has the right to apply the rules of the great author [Newton], without having first considered the metaphysical notions from which they are derived." As such, he judged them by the standards of his theology, and preferred to admit faith to accept the mysteries of Christianity, rather than to understand the inconsistent obscurities of analysis. He did not want to eliminate metaphysics, but he criticised the one which was to his eyes incomprehensible and bad. He attacked Newton for his conceptions, both of absolute space, and of his God in relation to it. "14. I ask whether extension can be thought of as an attribute of an unchanging, eternal being. [...] 49. I ask whether we should not admit the existence of a *philosophia prima* [first philosophy] and of a certain transcendent science, superior in dignity and more universal than mathematics, which deserves rather to be understood than despised by our modern analysts. [...] 58. I ask whether it depends on critical reflection that the same man who admires the great author [Newton] for his fluxions derides him for his religion. [...] 64. I ask whether mathematicians,

[136]Berkeley 1971, pp. 77, 81, 85, 87, 97 and 99.

who are so demanding in questions of religion, are rigorously scrupulous in their own science. Is it not true that they submit to authority, that they have a tendency to trust, that they believe in inconceivable things? Is it not true that they, too, have their mysteries, and, worse still, that they add inconsistencies and contradictions to these?"[137]

It did not escape the attention of the Irish bishop that what was at stake largely depended on the symbolism proposed. "Whether you argue by means of symbols or words, the laws of argument remain the same. Nor can it be granted that you claim the privilege of being exempted from these laws in mathematics. [...] 37. Nothing is easier than assigning names, signs or expressions to these fluxions: and it is not difficult to operate or compute by means of these signs. But it is much more difficult to omit those signs and nevertheless retain in one's mind what those signs should mean. [...] 25. I ask whether finding appropriate notations or symbols for quantities is not the most general character or tendency of mathematics. And whether it is the arithmetic operation that limits and defines their use."[138]

Together with his pupils who defended him, continuing to elaborate fluxions and series, there were in Great Britain those like Berkeley who attacked Newtonianism. The followers of Leibniz, like the Bernoullis, purified his gravitation, transforming it and recasting it by means of their infinitesimal calculus.[139] Vice versa on the European continent, others could be found who sang its praises, presenting it as the philosophy of progress. The most famous of these appeared to be Voltaire (1694–1778), who encouraged his young friend Gabrielle-Émilie, marquise of Châtelet (1706–1749) to translate, with great success, the *Principia* into French as the *Principes mathématiques de la philosophie naturelle* (1759). In the preface written on the occasion of its publication, the philosopher of the Enlightenment liquidated as "foolish" the follower of vortices formed by the "thin matter" of Descartes and Leibniz. For him, only a follower of Newton could be called a "physicist". "But Newtonians do not exist, just as do not exist Euclideans. It is a privilege only of error to give its name to a sect."[140] This, which today has remained the current money (perhaps a little devalued) in physics departments, could be coined only by misinterpreting and recasting a fictitious, convenient version of Newton.

The great Danish calculator of logarithms and Newtonians series, Nicolaus Mercator (Kauffman 1619–1687) left us something else about music in a manuscript of his quoted by the *Treatise ... of Harmony* by William Holder. Dividing the octave into 53 parts, he sought a measure that was common for all the intervals.[141] When he decided to make a clear distinction between rational and irrational numbers, he used the following comparison: the former were like the notes that give harmony,

[137]Berkeley 1971, pp. 68, 93–95, 99 and 100.

[138]Berkeley 1971, pp. 73, 89 and 96.

[139]Boyer 1990, pp. 477–506.

[140]Koyré 1972, pp. 68–70.

[141]Barbour 1951, p. 125. Wardhaugh 2008, pp. 47-, 152–154.

with respect to dissonances.[142] We have already seen above that he had developed the analogy between music and logarithms.[143]

On the other hand, the Frenchman Joseph Sauveur (1653–1716), whom we have already met together with Leibniz, devoted himself almost exclusively to the art of sounds. He, too, was educated by the Jesuits of La Fléche, and continued the investigations of Mersenne: he proposed his division of the octave now into 43 equal degrees, each of which, in turn, was separated into seven parts. He compared it with many others, including a division into 17 intervals with the thirds of a tone that are typical of an Arabic scale.[144] However, he seems to have had a poor ear and he had to have recourse to musicians. Actually, he proved to be more a scholar of sounds in themselves than of musical notes. Thus he succeeded in measuring the frequencies of notes better than we saw Mersenne or Huygens did. He was the first to use beats, that is to say, the periodic oscillation of the intensity when two notes differ slightly in frequency and their air waves interfere with one another. The frequency of the new oscillation is the difference between those of the two notes. Working on organ pipes, he succeeded in measuring a note with about 100 cycles a second. The great Newton was subsequently to verify it with the wavelength, and, as we know, by means of his estimate for the speed of sound: the wavelength was twice the length of the pipe.[145]

Sauveur also studied the vibrations of a string, comparing them to the oscillations of a pendulum. He imagined them as multiple complex movements which presented interesting phenomena. As it was not only one note (the fundamental one) that was heard, but also others of an increasing height, the Frenchman argued that the string did not only vibrate as a whole, but also in equal, shorter parts, separated by immobile parts called "nodes". He called this succession of notes: "sons harmoniques [harmonic sounds]", the upper harmonics of a note. When he presented his theory to the Academie des Sciences in Paris, in 1701, someone recalled that Wallis and others had already sustained something similar in the *Philosophical Transactions of the Royal Society*.[146] But these were without doubt investigations carried out independently.

Anyway, the physicist from Paris was the first person to associate the harmonics of a sound with its characteristic timbre. "Organists begin by knowing their organ, that is, the stops and the particular effect of the mixtures of those stops; because, although mixtures of the same stops produce almost the same effect, there is always some difference that challenges the organist to mix them almost the way the painters mix colours, and each has his own taste. However, there are general rules that determine these mixtures. The first is that in all these mixtures the sounds of organ pipes activated by the same key are harmonic; …". Hence we discover that

[142] Whiteside 1981, p. 310.

[143] Section 9.3.

[144] Barbour 1951, pp. 114, 122 and 125–128.

[145] Dostrovsky 1975, pp. 201–204 and 209–218.

[146] See above, Sect. 9.2.

organ-makers had already gained an empirical knowledge of the art of combining harmonics of a sound together, to obtain timbre effects. This was what the French scholar built his general theory on.[147]

As many of these phenomena are still present today in current textbooks of physics, few have resisted the temptation to make modern acoustics start with him. Effectively, like Francis Bacon, before him, and others, he introduced the term "acoustique" which was destined to enjoy success in the scientific community. But this would be a hasty conclusion. Despite his problems with his ear, and his vocal defects, Sauveur had still devoted himself sufficiently to intervals between musical notes, by recurring to professional musicians and instrument-makers. Furthermore, we shall see, in Chap. 11 below, that music continued to awaken the interests of two of the most important physicist-cum-mathematicians of the eighteenth century. Just as one swallow does not make a summer, so one word like "acoustics" could not inaugurate the autumn of our history.

But music was not to be ignored by all historians of the sciences. Whether inspired or not by the article of Paul Tannery seen above,[148] some scholars wondered what part the art of sounds may have played in the scientific dramas of the seventeenth century. Rarely, sufficiently competent or ready to investigate some pages of music directly, these scholars often fell back on the specialist secondary literature which had already studied the history of how to divide the octave. Here, they could learn, if they so desired, how lengthy, complex and controversial the events connected with that problem had been, or with how to tune instruments in order to be able to play the notes of some composition.

Extracted by Murray Barbour, the 178 different scales for the octave clearly show that tower of Babel which had been built up for the theory of music over the centuries. This scholar from the United States made an effort to classify these innumerable divisions into large classes, including: the so-called 'just' tuning (like that of Ptolemy and Zarlino), the mean tone temperament (with the major thirds in the ratio 5:4 and the fifths lower with respect to 3:2) and the equable temperament (all the intervals equal). But as 68 cases were left out, he had to call them "irregular". All the scales were compared with the Pythagorean model, on the one hand, and were measured exactly, on the other, with the late-nineteenth century system of the *cents* (one hundredth of the equable semitone) commonly adopted still today. Among the innumerable texts consulted, I will not hold it against him that he missed out that of Giovan Battista Benedetti or the inclinations towards the equable temperament of Leibniz and Newton.

It sounds much more debatable to me that Barbour took the octave for granted also in the cases of Chinese and Indian theoreticians. He read it in an encyclopaedia of music, written by Maurice Courant to the beginning of the twentieth century; I have denied it in the appropriate chapters. Rather, it is a more serious charge that this scholar seems to see the equable temperament as the final scientific limit of his

[147] Dostrovsky 1975, pp. 204–209; Dostrovsky 1981a, pp. 127–129.

[148] Section 9.3. Tannery 1915.

history, and the culminating point where the Tower of Babel touches the heaven of truth, or at least of what is better in practice. The gamuts chosen by theoreticians should rather all be considered arbitrary and all possible, depending on the means available to construct them: the ratios and the *mesolabium* of the ancient Greeks, the roots and the logarithms of the Europeans, rational and irrational numbers, or the ear and the brain. Certain periods preferred to play music on the Pythagorean octave (perhaps; when?), others on the 'just' tuning. On some instruments, like the lute with frets, a musician like Vincenzio Galilei and others preferred the equable temperament. This contended the keyboards of organs, harpsichords and virginals for a few decades with the mean tone.

Subsequently, until today, the equable temperament has prevailed. Why? In the last chapter of Barbour's book, it emerged that, in the dispute between the more or less scientific theoreticians and the musicians guided by the ear or by their taste, the latter overcame. Among the leading exponents, the list of those who had adopted the equable temperament included such famous characters as Vincenzio Galilei, Tarquinio Merola (sixteenth century), John Bull (1562–1628) only sporadically, Girolamo Frescobaldi (1583–1643). "There is ample evidence that in Italy during the first half of the eighteenth century, equal temperament or its equivalent was being practised." It was practised by Domenico Zipoli (1688–1726) and Domenico Scarlatti (1685–1757). The very famous *Das wohltemperierte Clavier* by Johann Sebastian Bach (1685–1750) should not be translated directly and hastily as the *equally tempered Harpsichord*, but rather as the *well tempered harpsichord*. A German theoretician wrote about "die gleichschwebende Temperatur" to mean "equable temperament". But Bach was not a theoretician, and could equally well include in his title equally tempered octaves. He did that above all because his music, extremely rich in modulations and transpositions into near and far tonalities at either end of the keyboard, would have found difficulty in being tuned to the mean tone.

Also vocal music, though doubtlessly variable in intonation, seeing that it needs to be accompanied, at times more, at times less, by some instrument, when rich in chromatisms, sounded much better in the equable temperament. Cyprien de Rore, the correspondent of Giovan Battista Benedetti, Luca Marenzio (1553–1599) and above all Carlo Gesualdo, prince of Venosa (1560–1613) wrote music of this type. Lastly, I will point out an imprecision: when the theoretician from the United States counted nine diatonic commas in the Pythagorean tone, seeing that even Boethius wrote that it contained more than 8 and less than 9.[149] In a particularly synthetic and clear manner, Rudolf Rasch explains the problems created by tuning musical instruments in accordance with the Pythagorean doctrines: both in the classic style and in the Ptolemaic- Zarlinian variant, called the "just intonation" in the Anglo-Saxon world.[150] It remains to answer the question: is "just intonation" really just? Why? Musical practice, historical evolution and the differences with non-Western cultures indicate that the term "just" attributed to the Ptolemaic-Zarlinian tuning

[149]Barbour 1951, pp. 193, 123–124 and *passim*. Tonietti 2006b.

[150]Rasch 2004.

should be accompanied by the qualification "would-be" or "self-styled". History demonstrates to what extent the ears and the sensibilities of those who listen to music are moulded by their relative cultures.

When Alistair Cameron Crombie was planning a study on Mersenne, of which, however, I have not succeeded in finding the conclusions, he was forced to take a minimum of interest in music. "If we count modern science as one product of Western Europe unmatched elsewhere, another is surely the music developed during the same period." We would expect some reason to justify this juxtaposition. But the English scholar did not go beyond a rapid review of well-known names. On Vincenzio Galilei, he repeated, as we have seen too many times, the usual mistaken judgement, probably copied from Walker. "His ratio for organ pipes was less happy." Whereas we have seen above that the ratios used by the musician from Florence to tune organs, as well, had led him to find proportions in harmony with his ear, and distant from the Pythagorean ones.

That common (ignorant?) Eurocentrism which emerged above returned here and there. Thus non-Europeans, like the Chinese and the Indians, "... seem to have involved no central conception of mathematical proof, ...". With these prejudices in mind, Crombie believed that in Avicenne (Ibn Sina), music aimed "... to bring the soul of the listener into relation with the harmony of the spheres." Evidently, he had never read the musical chapters of the *Book of Healing*, where the Aristotelian Arabo-Persian had rejected precisely the Pythagorean music of the spheres. Nor can he have taken much pains in consulting Boethius or Guido D'Arezzo, if this great English historian attributed medieval European music to the Arabs. As if that were not sufficient, Crombie confused the theory of *ictus* [strokes] with modern wave theories of sound.[151]

We have already seen above that another English historian, Stillman Drake, had exploited the experiences with sounds of a musician like Vincenzio Galilei. Quite rightly. But why should the role of music in the scientific revolution of the seventeenth century be distorted by that debatable thesis which reduced it to experimentalism? Sure enough, the musical theory associated with his father, Vincenzio, inspired by Aristoxenus, sounded very different from that of his son Galileo, who remained within the Pythagorean mainstream.[152]

When studying the early theories of vibrations, Sigalia Dostrovsky encountered many of our characters and consequently absorbed a good dose of music, seeing that the physical phenomena that she studied took their origin here. She gave many reasons to see crucial notions for the physics of sound arise from the theories of music, such as frequency, harmonics and wavelength. We have already encountered the events recalled also by her in our history. In some cases, however, we differ. I, too, doubt whether Galileo Galilei had really been able to observe the division of the wave in the water inside the chalice at the sound of the higher octave. At least, because the Florentine astronomer should be considered a little less of an

[151] Crombie 1990, pp. 365, 368, 8, 113, 330. Here, Part I, Sect. 6.7, Chaps. 3–4 and Sect. 5.2.

[152] See Part II, Sect. 9.2, at the end.

experimentalist than he is generally represented. His father, the musician, Vincenzio Galilei was much more of an experimentalist, in that he followed his ear, but he was overlooked by Ms. Dostrovsky. Actually, she did not realise that he had given the rule to tune open organ pipes by making up for the *end-effect*.

I moreover find it difficult to consider the theories of strokes, transported by sound to the ear, a good prelude for counting its frequency. If for no other reason, because the former gave a discrete image, whereas the latter had a meaning within the continuous undulatory model. The strokes would be whole numbers, on the contrary, the frequencies were also irrational numbers. Lastly, Ms. Dostrovsky put too much faith in the first article on gravitation, as commented by Newton in the *scholium* through the music of the spheres.[153] Actually Mersenne's law for sounds could not coincide with Newton's model for gravitation, because the former implied a planetary polyphony *à la* Kepler, whereas Newton had rejected this, in order to listen to the planets revolving in unison.[154]

Even though the limited space of the article did not concede her all the sufficient details, the Australian Jamie Croy Kassler was animated by problems partly similar to my own. But she totally overlooked Vincenzio Galilei, although, in contrast with the reigning Pythagoreanism, she attributed its due value to the position of Aristoxenus. In this connection, among his few followers, she pulled out Nicolas Oresme (1323–1382) whose consistency she observed between the geometry of incommensurables and practical music. This ancient French natural philosopher preferred the variety aided by the ear of Aristoxenians. Surely in the fourteenth and fifteenth centuries, the compositions of *Ars nova* were bringing fresh air into palaces and churches. The music of Guillaume de Machault (1300–1377) and Guillaume Dufay (1400–1474) had found its reasons in the theoretical books written by Johannes de Muris (c. 1290–c. 1360) and by Philippe de Vitry (1291–1361).

As in the best Platonic tradition, a papal bull condemned all new rhythmic and melodic licences, which often became licentiousness in the minds of the censors: "…we are prepared to take effective action to prohibit, cast out, and banish such things from the church of God". We know that, unfortunately, it was not rare to choose to transform the conflict between different conceptions of music, of sciences and of the world into an attempt to eliminate those considered not to be orthodox, because they were critical of the vested values. I would invite history colleagues to answer the final question posed by our Australian scholar, "why would an investigation of the history of the musical model lead to a better understanding of the history of science?", only after reading this book of mine.[155]

Paolo Gozza was looking explicitly for something comparable in the essay placed at the beginning of an anthology of texts dedicated to the: "…ruolo della musica nello sviluppo della scienza moderna e ruolo della scienza moderna nell'elaborazione della teoria musicale …" ["…role of music in the development

[153] Dostrovsky 1975, pp. 180, 211. McGuire & Rattansi 1966.

[154] See above, Sect. 10.2.

[155] Kassler 1982, pp. 139, 124 and *passim*.

of modern science and the role of modern science in the elaboration of musical theory ..."]. But he remained a bit too subservient to the secondary literature that came from the Warburg Institute in London. In actual fact, he propagated the error of Walker regarding Vincenzio Galilei and how to tune organ pipes. The scholar from Bologna only added in a note the correct defence of the Florentine musician, offered by Palisca. We have already pointed out repeatedly that the modern acoustic theory of the *end effect* is equivalent to the cubic numbers followed by Vincenzio Galilei. Gozza also made a blunder in Mersenne's formula $\nu \propto \frac{1}{l}\sqrt{\frac{t}{\sigma}}$. While he wrote about a proportionality between the frequency ν and the *square* of the tension t, and about an inverse proportionality with the *square* of the section $sigma$. To obtain the frequency sought, it is without doubt necessary to take the *square* of the tension and the section, precisely because they appear in the formula as a *square root*.

At this point, we could not expect our scholar from Bologna to perceive, in McGuire & Rattansi, Newton's error from which we started in this chapter. Furthermore, he unfortunately completely ignored the extent to which the problem of incommensurable irrationals was linked with the equable temperament of Aristoxenus, Vincenzio Galilei and Simon Stevin. The final sentence, "Since then [XVIII cent.] music has no longer been a part of scientific culture, to whose development it had contributed since the times of Pythagoras", sounds to me like too clear-cut and definitive a judgement.[156] We shall find in the following chapters still more music, and not only mathematics or physics. In this discipline, the acoustics section was largely created only in the nineteenth century. Did the entry "acoustique" exist in the *Encyclopédie* of Diderot & D'Alembert? Even at the beginning of the twentieth century, we shall still find famous physicists interested in music.

Among the few historians not indifferent to the art of sounds, the prevailing tendency has been to narrate to what extent modern post-Galileian sciences changed the theories about it. Faithful to their prejudices about the "nature" and the "purity" of the scales considered to be "just", there were those who condemned the equable temperament of Stevin and Vincenzio Galilei, also continuing to propagate the latter's presumed 'error' with cubic numbers for organ pipes. Thus the Florentine musician would become more of a "sceptic" than an experimenter possessing good ears. When exploring the modalities that the so-called scientific revolution of the seventeenth century had followed, no great role was found for music, which, at most, might exemplify certain aspects. According to this picture, in the theories of music of the period, there would be more metaphysics than science.[157]

The best-known historians of the sciences have ignored music. Only Thomas Kuhn realised how serious this omission was.[158] The philosophers of sciences who also deal with the musical pages written by Descartes, Galileo Galilei or by other

[156] Gozza 1989, pp. 5–6, 21, 34, 40, 64 and *passim*.

[157] Cohen 1984, *passim*. Coelho 1992.

[158] Kuhn 1985, pp. 42–43.

famous characters are rare. When music was promoted to the status of a "science", this would be offered as a current, stereotyped image, as in some textbook of progress.[159]

How far music was capable of changing the representation of certain famous natural philosophers would still be remained to be discovered. This is the direction toward which works Natacha Fabbri, with alternating results, not always of the same value. As regards Mersenne, whose music can hardly be ignored, she correctly underlines that the art of sounds had become the main way for the Minim friar to reach his transcendent God, avoiding the dangers of scepticism. "La science des sons approche davantage de cette racine, qu'elle est plus propre pour nous faire cognoistre l'autheur de l'univers, que les autres sciences, …" ["The science of sounds reaches this root [God] better, with the result that it is more suitable than the other sciences to make the author of the universe known, …"]. Harmony would elevate towards divine things, broadening the function that Christopher Clavius had assigned to mathematics in his translation of Euclid inspired by Plato. Defending the dogma of the Trinity against the various anti-Catholic Arian heresies, Mersenne had represented the unity of the divine nature in unison.[160]

The Italian philosopher does not seem to be so able at placing the *Compendium Musicae* within the work of Descartes. Finding in that text some of the subjects which will subsequently acquire much more importance, it has not perceived its distance, however, with respect to the more mature *Geometry*. Here, at the beginning, the philosopher of Method accepted the extraction of roots among the legitimate arithmetic operations, and thus destroyed the classic *Quadrivium*, with its contrast between discret sciences and continuous sciences. As this is one of the fundamental dualisms on which he constructed his whole world as a *continuum* full of all possible good things, we must understand better how Descartes situated the division of the octave. As he accepted for this purpose the neo-Pythagorean ratios of Ptolemy and Zarlino, in spite of some geometrical figure in the shape of a circle, the *Compendium musicae* ended up by remaining in the traditional river-bed of arithmetic, avoiding irrational numbers like roots. Also the French philosopher used the theory of strokes, *ictus* or *battemens*, which should not be translated by "battimenti" [beats], to avoid, at least, ambiguity with the different well-known sonorous phenomenon seen above.

In the text written during 1618, therefore, geometry had not played any decisive function, as he himself wrote. "Illa proportio Arithmetica esse debet, non Geometria" ["That [musical] ratio must be arithmetic and not geometry."] The geometrical mean actually would have led him to the irrational ratios which were believed to generate unbearable dissonances. Nor, even less, did the frequencies of notes appear here, destined perhaps, to enter as "tremblement" ["vibration"] only later in the correspondence with Mersenne. In his first work on music, admitting some hints to geometry, musical scale was to remain too incomplete with only the rational

[159] Asinari 1997.

[160] Fabbri 2008, pp. 27, 51, 64 and *passim*.

numbers, to allow us to think of the future *continuum*. Of it, surely, there is less here than in Kepler's *Harmonices Mundi Libri Quinque*. Introducing too much geometry already into the *Compendium musicae*, Ms. Fabbri is committing just that sin of anachronism, which she declares she wishes to avoid. Furthermore, Descartes rejected the equable temperament and was to admit only later, with Mersenne, the ear as a criterion, capable of judging the "pleasant" intervals in accordance with taste.[161]

Even though both of them prefer art to some presumed natural quality in the tuning of instruments, Natacha Fabbri recognizes a certain difference between Galileo Galilei and his father Vincenzio. The former sought abstract representations, whereas the latter followed the sense of hearing. But she does not succeed in grasping that for music, they had arrived at even more distant results. The astronomer followed the Zarlinian variant of the Pythagorean tradition, whereas the musician followed the equable temperament of the irrational numbers. As a result, the former, in his *Dialogo sopra i due massimi sistemi del mondo [Dialogue On the Two Principal Systems of the World]*, distinguished his Pythagoreanism from the "idiocies" about numbers generally believed by populace. Galileo Galilei actually continued to think that incommensurables constituted disorder in the desired harmonic cosmos. Evidently, it was not enough for the Italian philosopher to know the studies carried out by Palisca, who appreciated in Vincenzio Galilei the direct experiences on organs and lutes.

Ms. Fabbri appears to move among categories that are a bit too general, like "nature", "art", "experiments", "mathematics", "reason", "order", ... In this way she ends up by obscuring music and making it confused: like the important relationship between the oscillation of the pendulum in Galileo Galilei and that of the sounding strings. Both the oscillations, sure enough, take place in times or periods T inversely proportional to the frequency ν, $T \propto \frac{1}{\nu}$ and thus the periods T are proportional to the length l of the string, both for the pendulum and for the sound. But for the pendulum the proportionality is with the square root of the length and does *not* depend on the weight p, $T \propto \sqrt{l}$, whereas for the vibrating string, it is directly proportional to the length and inversely proportional to the square root of the weight, $T \propto \frac{l}{\sqrt{p}}$.[162]

As others had already started to maintain in the faraway 1995,[163] also the Italian philosopher underlined that the idea of harmony went through all these Kepler, Galileo Galilei, Mersenne, Descartes. She succeeds in demonstrating very clearly that the harmony of music was the model of a hoped-for unity, or a "discordia concors" ["concordant discord"] between our cultural subjects. "De mundi concordia, atque discordia plura leges, ubi Deum esse postmodum ex Musica ostendemus." ["As regards the agreement and the discord in the world, we shall show many laws where God is present, which come from music."] Only, at this moment

[161] Fabbri 2008, pp. 86–87, 91, 100–103, 116, 123, 156 and *passim*. See here, Sect. 9.2.

[162] Palisca 1989. Palisca 1989a. Palisca 2000. Fabbri 2008, pp. 170–173, 198–199, 220, 222 and *passim*.

[163] Tonietti 1999a.

in the field of religion, Mersenne was unfortunately thinking of perfect unison, and not appear to be ready to enrich his Christianity with heretical dissonances. Thus the results, as we already know, were not to be particularly peaceful. The more they had harmony on their mind, the more the reality of their epoch abounded in hard, bitter, unbearable dissonances. Above all, the conflicts of a religious nature tormented them, and were made explicit in their writings, even if the subjects were sounds, astronomy, the fall of bodies, the geometry of curves or optics. Sometimes in Mersenne the reasons of phenomena became instrumental with respect to his religious purpose.[164]

Thus, rather than music, to which subject she adds little or nothing to what we already know, Natacha Fabbri's essay ends up by appearing to be generally centred around the various images of God, and His possible intervention in the natural and civil world by means of the various religious denominations. This seems to me her main merit in a panorama of studies on the philosophy of science, which is above all embarrassed by themes of the kind, and largely engaged in hiding beneath the wonderful carpet of rationality everything that appears to be incompatible with a presumed progress of the sciences.

Among the rare scholars who do not simplify this aspect of scientific evolution for their own convenience, we may include also Amos Funkenstein.[165] However, this scholar with his varied German-Hebrew-from-the-United-States cultural background, revealed his great erudition, also in the linguistic field, more to veil the interpretation of philosophical and theological texts rather than in offering wholly convincing meanings. He gave us good evidence of the event, when he wrote that there had been "... in the XVII century such a degree of fusion between theology and physics as had never been seen before (and there was never to be seen subsequently). The theological arguments and the physical ones became practically indistinguishable, with advantages and disadvantages, both for theology and for physics." While he supplied the God of Descartes with all the transcendence difficult to hide, he also admitted an incorporeal God of Leibniz made to exist through the principles of non-contradiction and sufficient reason. Albeit with the exception of Pascal, he curiously maintained that divinity, without doubt transcendent in the Middle Ages, had, however, reacquired a body in the following centuries. But the only philosopher for whom this was undoubtedly true was Baruch Spinoza (1632–1677) with his famous "Deus sive natura" ["God as nature"]. This dangerous Dutch pantheist, however, was neither Catholic, nor Protestant, nor Anglican, nor Calvinist, and he was considered a heretic, even by Rabbis. Above all, he did not leave any trace of writings about the mathematical sciences. Consistently with the preceding chapters, and in the opposite sense, therefore, the many papers exhibited by Funkenstein will shortly be seen to demonstrate the religious transcendence of the new sciences, and not the immanence induced by these on the models of God in the seventeenth century.[166]

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[164]Fabbri 2008, pp. 256–257 and *passim*.

[165]Funkenstein 1996.

[166]Funkenstein 1996, p. 86 and *passim*.

After comparing our last physicists and mathematicians with medieval theologians and philosophers like John Duns Scoto (c. 1266–1308) and William of Ockham (c. 1295–c. 1350) or Thomas Aquinas (1227–1274), he concluded that "God, who was the source of every contingency in the Middle Ages, became the source of all rationality: a great methodologist of the complete intelligibility of nature." Funkenstein did not fail to stress that the ideal experiments of Galileo Galilei were similar to certain medieval controversies on the worldly omnipotence of God. In the seventeenth century, also the criteria had changed for the writing of history, giving importance for the events above all to the context, without which they lost all sense. Good. They also tried to avoid the error of anachronism. Better. But why did this German-Hebrew-from-the-United-States philosopher not notice that the mathematical sciences of the period were trying, thanks to God, to make themselves totally independent of every human connotation?

In the *Bible*, the simple language of everyday man had been used, in order to be understood by them, with all the ambiguities and the limitations of the case. But now, in the sciences, our natural philosophers tried to work back to what God really meant to communicate to us in his creation. On this point, Leibniz and Newton had been explicit. However, Funkenstein unfortunately forgot to underline that they had used, and invented, that transcendent mathematical symbolism of the seventeenth century reserved to the initiated. Although he, too, had found in Leibniz, without drawing the due consequences, sentences like "Quid tum? cogitationes fieri possunt sine vocabulis. At non sine aliis signis. Tenta quaeso an ullum Arithmeticum calculum instituere possis sine signis numerabilibus." ["What can we say, then? Thoughts can occur without the use of words. But not without other signs. Try and see, I beg you, if you can carry out an arithmetic calculation without numerable signs."][167]

The polyglot-philosopher of science and religion affirmed that "... starting from the seventeenth century, the political theories methodologically took natural sciences as their model." And yet, going by his numerous references extracted from an intricate network of various concepts, he shows us equally clearly that in Moses Maimonides (1135–1204) the term "law" had possessed meanings that were accepted in two different environments. "... Maimonides creates a close analogy between laws of nature and social laws. ... No natural law is completely deterministic, even in the mind of God." As a result, by the grace of God, this law became contingent for the philosopher from Cordova.

We find the same concept in Thomas Hobbes (1588–1679). "The new laws of movement had, starting from the XVII century, two paradigmatic functions. Hobbes' theory of science as a systematization of mental constructions corresponds directly to Hobbes' theory of society as an autonomous human construction (even though necessarily required); ... At the same time, the laws of movement can be considered as laws to which social phenomena are, ultimately, reduced, ...; or at least, it is possible to assume them as a metaphor or a material paradigm of the

[167] Funkenstein 1996, pp. 239, 379 and *passim*.

laws that govern society." The English philosopher, both of science and of politics, actually had used in them the same method of investigation, probably learnt from Francis Bacon, who divided and broke things down in order to know them. "... in a clock, or another analogous mechanism, it is possible to know the material, the form and the movement of the workings, only if these are separated and considered separately, ...". The clock became, for him, a model, both of nature and of his absolute social State. Thanks to an omnipotent God, even Newton had appeared he could see some crack in the immutability of laws. "... it may even be admitted, therefore, that God ..., because He is such, may vary the laws of nature and create world of different kinds in different parts of the universe. ... in all this I do not see anything contradictory." Even if reserved to the power of the Most High, then, the natural laws brought again into play the will, and not only the intellect.[168]

If, in following medieval theology up to the sixteenth and seventeenth centuries, it is possible to uncover a connection between the sciences and those theocratic societies, it remains, however, to be understood which of the two forces exerted greater pressure on the other. Funkenstein tried to convince us that it was the new modern sciences that secularised the debates about God, and mechanised the absolutist State of the period. But why not to see that for centuries, from Moses to Luther, a huge pressure had made the current of its influence flow above all in the opposite direction, making the mathematical sciences divine and transcendent?

Our German-Hebrew-from-the-United-States philosopher ignored precisely what we here have placed at the centre of the investigation, perhaps because he preferred to study classical philology rather than the relative mathematics. As regards Euclid, he surely made a bad mistake when he sustained that the demonstration written by the Alexandrian to obtain an increasingly long sequence of prime numbers was *not* constructive, whereas it can easily be seen that it makes it possible to calculate with precision a new prime number.[169] Furthermore, he underestimated the problem of incommensurables and of musical harmonies, which would have succeeded in clarifying the picture more. For Agustine of Hippo (354–430), God would have perfected "... universi saeculi pulchritudo, ... velut magnum carmen cuiusdam ineffabilis modulatoris excurrat." ["... beauty of the secular universe, ..., as the great song of the same ineffable player concludes."] Another theologian had compared the possibility that the proportions of universal order became corrupted to "... una chorda plus debito intenderetur ..." ["... a single string that is tended more than it should be ..."].[170]

Lastly Funkenstein betrayed religious-Mediterranean-Euro-centric prejudices when he measured a presumed "... superiority of European civilisation over other civilisations ..." through mechanical clocks, accusing the Chinese of being incapable even of repairing them. From a book by Antoine Arnauld about the logic of

[168] Funkenstein 1996, pp. 409, 273, 403–404, 230 e *passim*.

[169] See Part I, Sect. 2.4.

[170] Funkenstein 1996, pp. 359–361, 267, 169, and *passim*. However, he admits: "I do not possess the competence."

Port Royal, he also extracted for it the idea that these poor Chinese had attributed the ticking of the clock to its "sonorific quality". Civilisation, for the well-known reasons, would not have been able to do without the Jews, the Arabs, the Muslims or the Turks, but for him, the Far East could have undoubtedly been left out. He actually roundly scolded a colleague of his who was guilty of sustaining that "... without a 'rational theism' ... Oriental cultures have never succeeded in creating a technological society, or a unified body of sciences; ...". But unfortunately he attacked him, only because the model was too deterministic. To our philosopher of religion and science, on the contrary, also scepticism, pantheism, and atheism seemed equally necessary for "scientific progress", as long as they proceeded in the "ways of the Lord". That is to say, that they proceeded with the help of that 'invisible hand', or of that 'providence' which certain German philosophers of history would have pompously called 'heterogenesis of ends'. Sure enough he concluded: "I do not know if a non-theistic society would be capable or not of creating a technological-rational society similar to ours; ... I only know, with certainty, that they have not done so."[171] Since the beginning of this work, I have shown, on the contrary, that Chinese, Indians and Arabs created 'other' sciences suitable for their different cultures.

Thus I find Amos Funkenstein more convincing when he repeats insistently: "From source of all contingencies, as He was in the Middle Ages, God became the guarantor of the absolute rationality of the world." or "... the scientific interests were expressed in theological terms. Theology and the other sciences became a single science." Even better, "Newton and Leibniz solved the dilemma, each in his own way, but still theologically. Common to both is the transformation of God into a methodological guarantor of the rationality and intelligibility of the world."[172] Compared with this latter philosopher, whose competence extends to many languages and religions, but who forgets some heretics, I will shortly show what deep theistic traces the illustrious natural philosophers studied here have left us in their way of inventing the most modern sciences.

The scientific revolution of the seventeenth century was an event full of such great significance as to stimulate the most varied flights of imagination and points of view. Among all these cowls, habits, scapulars, tonsures, beards, masses, sermons, liturgical functions, litanies and profound sacred songs, it should have been noted that there were too great a lack of skirts, long hair and female voices to be a coincidence. David Noble has taken up the previous religious arguments, shifting the attention from the books to the environment where studies were carried out, so as to throw light on the event that the characters were almost all male. "It is here, in the struggle of the Latin clergy to interpose itself between God and the rest of humanity, that the singular culture took form, which in turn generated Western science. It is here actually – where orthodoxy invented heresy, thus identifying it with women – that a world without women emerged: ...".

[171] Funkenstein 1996, pp. 387–388, 433–434 and *passim*.

[172] Funkenstein 1996, pp. 181, 415 and 229.

The priests decided orthodox culture, even the scientific one. Thus the historian from the United States overturned the misleading commonplace, which today contrasts science with religion, to show us their interdependence: "…the history of Western science must be understood in the context of the history of Latin Christianity, with its tenacious tension between orthodoxy and religious revival. Essentially a sacred activity, science took form in the framework of an epic social struggle for access to divine knowledge." "Though some historians may characterise it, retrospectively, as a lay enterprise, Western science has always been, fundamentally, a religious profession: more a continuation of the Christian tradition than a deviation from its course."[173]

In the Christian tradition, which had been consolidated in centuries of wars and theological debates, we shall find some of the main characteristics present in European sciences or absent in the other cultures, already studied in the initial chapters of Part I. Considering the subject, Noble has reviewed here a secondary literature prevalently concentrating on the dualism between the female contrasted with the male. It arrives as a burning humiliation, not very metaphorical, that the religious authorities and imperial-royal had pushed people to transcend the sexes in God through all possible ways. Thus we arrived at the culminating moments of the self-eviration of a founding father of the Church like Origen (125–254) and the enforced castration of the Aristotelian scholar who was to found the University of Paris, Pierre Abelard (1079–1142), to put a definite end to his famous love story with Eloise (1101–1164).[174]

With the abstractness of his mathematics, and thanks to the disembodied transcendence of his Apocalyptic God, Newton was later to succeed in atrophying the organs of his body, making all that shedding of blood useless for him. "…the essence of the new philosophy was transcendence. …the mechanisation of nature, the abstraction of the mind from matter and the mathematical reconstruction of natural phenomena disqualified all the other ways of thinking as inadequate (and perhaps dangerous), and offered the means to obtain the desired emancipation from the disorderly particulars of space and time, of the person and of experience: a depoliticised, impersonal, disembodied, universal, absolute and authoritative knowledge (similar to God), perfectly appropriate for the ascetic-clerical finalities."[175]

In the historical alliances of the Roman Emperors Constantine (274–337) and Charlemagne (742–814) with Christianity, the wars had been declared "Holy" and the clergymen had become "milites Christi" ["soldiers of Christ"]. "…in the performance of their liturgical function, also the monks had been enrolled in the

[173]Noble 1994, pp. 12–13, 205 and *passim*. Donini 1990.

[174]Noble 1994, pp. 35–36, 181–185, 287–298 and *passim*. Abelardo & Eloisa 2008.

[175]Noble 1994, pp. 297–298. Though Noble's was an essay that hardly dealt with the mathematical sciences at all, and too little with natural philosophers, but succeeded well in communicating to us the religious cultural context, an attribute less than unappropriate regarding Newton was inadvertently expressed here: "implicit naturalistic deism". Instead, the English astronomer had been a heretic Arian theologian, who had self-censured himself out of convenience, and the last of the Christian prophets, as Noble, too, wrote. Noble 1994, pp. 335 and 290.

king's armies. They were not simply like warriors, …; they were real and proper warriors, only their wars were not battles against earthly enemies, but battles (equally important) against heavenly enemies." "…there where the swords have opened up a pathway for you, you will fight with a sharp tongue. There where war had produced a massacre, you will serve with a peaceful [sic!] doctrine. There where the forest of heretical depravity has scattered thorns all around, the cedar of the Catholic faith will be uplifted by you to the stars. You, soldiers of philosophy, are probably more able to fight with Mercury's art, the arms of Phoebus, the spear of Minerva …".[176]

Suckling from similar roots, the relative fruits were bound to grow. We have already seen that some of the natural philosophers had not concealed his own military interests, indeed, he even used them to justify himself. These became increasingly evident with the scientific revolution of the seventeenth century. Mercator (Gerhard Kremer 1512–1594) had invented the fundamental geometrical instrument for every wartime achievement on land or by sea: a reliable map, more precise than the ancient portolanos. After just escaping from a long imprisonment, or worse, charged, as usual, with heresy for his theological ideas, this Fleming took refuge in Germany and designed a plane nautical map easier to use than others. It was sufficient to keep the route with the compass, cutting the meridians at a constant angle.

Projecting the points of the terrestrial sphere on the cylinder tangential to the equator which contained it, the meridian circles became parallel straight lines: Mercator's projection. On the maps easily obtained by developing the cylinder of the plane, after cutting it in the sense of its length, it constructs a good approximation of the figures on the sphere for latitudes that are not too high. Coming close to the poles, the meridians converge, finally meeting. Using this, together with the compass, the sextant, the ephemerides calculated by Newtonian astronomers, Harrison's clocks to measure the longitude, and the forecasts of the tides, the Europeans were to become masters of the whole Earth in the following centuries. The famous map-maker constructed globes, designed and etched maps of Palestine, Europe and the British Isles. These must have been so numerous that they weighed a lot, in all senses, and from then on, deserved the name of the mythical bearer of the world: Atlas.[177]

Therefore the historical context, under consideration here for its influence on the evolution of the mathematical sciences, would reveal not only religious aspects. But unfortunately, also these other stimuli, deriving from requirements imposed by the military and commercial expansion of Europe, have generally been underestimated and concealed. So much so, that presenting Newton's *Principia*, trying to show his "Social and Economic Roots" aroused, at that time in 1931, surprise, curiosity and ample scandal among numerous historians of the sciences. Boris Hessen maintained that this English natural philosopher had not been a fruit of divine providence, as

[176]Noble 1994, pp. 74–76, 130, 188 and *passim*.

[177]Kish 1981.

he himself had been inclined to believe, or guided by pure personal genius. This Russian scholar and exponent, at the time, of the Soviet nomenclature, underlined aspects of a letter written by Newton to Francis Aston (1645–1715), which Westfall, on the contrary, has overlooked.[178]

Aston had been a fellow-student at Trinity College, Cambridge, and was to be the future secretary of the Royal Society, with whom the famous English natural philosopher was to maintain contact and a correspondence. In his first long letter to him, dated May 18th 1669,[179] suggesting what to look for during a trip to Europe, Newton had undoubtedly also revealed his own interests. Among these, he had included, methods of navigation, war techniques, the art of metals, mines, gold, how to grind glass, pendulum clocks, perhaps useful to calculate the longitude, and of course, the alchemistic art of transforming metals. Yet as regards these subjects, there is less in Hessen's article than I have distributed in the preceding chapters. He surely remembered the artillery problems dealt with in the arsenals at the time of Galileo Galilei, the parabolic trajectories of shells, the pumps to be used in mines, instruments for navigation. However, the Soviet Russian underestimated the event that it was not the movements of the Moon, but the chronometers of Harrison (contrasted by Newton) which had in practice solved the problem of the longitude.[180]

In reality, the greatest worries of the man who was to direct the Moscow Institute of Physics, appeared to be quite different, apart from an extreme poverty in historical and documentary details. As he had offered more material about Newton's God than the relative socio-economic motivations, Hessen contrasted him with the presumed materialism of a Descartes, as if the French philosopher (apart from the *cogito*) had not invoked a God himself, to give the initial impulse to the vortex of the world. The Soviet Russian ended up by excusing the Englishman, in whose case he could not conceal the clear recourse to theology, in view of his own physical system, only as "... the typical representative of the rising bourgeoisie, ... [...] ... a child of his class." Boris Hessen would like to be able to exhibit "... the complete coincidence of the physical thematics of the period, which arose out of the needs of economics and technique, with the main contents of the *Principia*, ...". But he did not succeed in going beyond some vague mention of Newton's capacities in casting gold and silver at the English Mint.[181]

The Soviet scholar, instead, invoked at every step the "dialectic materialism" in the version of Friederich Engels (1820–1895), to confirm his hypothesis. Thus his faith was mainly in him, and in the event that a "... genuine scientific knowledge of the laws of the historical process leads with irrefutable iron necessity to the conclusion that the change from one social system to another is inevitable." He was therefore bound to arrive at a grotesque deformation of the events under

[178] Westfall 1989, pp. 201–202 and 422.

[179] Newton 1959, v. I, pp. 9–11.

[180] Hessen 1971, pp. 151, 171–176 and *passim*. See above, Sect. 10.2.

[181] Hessen 1971, pp. 188, 182, 191, 176 and *passim*.

examination, to make them fit into his a priori, anachronistic model. "Here the connection between technology and science, between the investigation of the general laws of physics and the technical problems raised by economic development is established with extraordinary clarity."[182]

However, the result that the first steam engines used in English mines would supply the impulse for the invention of thermodynamics was not sufficient to replace the creative circumstances experienced by Newton in the same general model of social class struggles. A century was to pass between them. Furthermore, the general law establishing the conservation of energy, in whatever form, mechanical or not, was not enunciated by the Officer of the French Military Engineers Corps, Sadi Carnot (1796–1832), or by the philosopher Engels, but by the German doctor Robert Julius von Mayer (1814–1878) in the mid-nineteenth century. Apart, perhaps, from a generic bourgeois class, the historical context was different.

Hessen's psalm finished in the glory of the scientific progress realised by the (unreal) socialism, which would be capable of removing all the obstacles interposed, so far, by the other classes. Whether we believe or not in progress or class struggle, we all know today how things were destined to go with the latest great scientific inventions. Nuclear bombs, computers and genetic manipulations came to light, thanks to wars and economic crises in the capitalistic West, but this is another story.

We, too, have not cancelled, but rather underlined, when they appeared under our noses, the explicit relationships exhibited by scientific innovations with the art of war and navigation. However, vice versa, they are obscured and destroyed, all the same, both by the faith in some providential divinity invoked to guarantee truth and harmony, and by the faith in the presumed inexorable laws of History. For this latter philosophy, in every case, all events proceeds towards the good, and victory, because its rhythm is marked by the "... development of productive forces [sciences, technologies] and production [of commodities] relationships [among people divided between social classes]", in accordance with the idea proposed by Karl Marx (1818–1883).[183] Not having found any trace of such imaginary laws of history, I prefer to continue to narrate the personal and collective affairs of our natural philosophers, without simplifying them or reducing them to, or deducing them from, some general theological or philosophical principle. The presence of such principles can only be found in the desires, the academic interests and the vanity of some individuals.

Joseph Needham (1900–1995), whom we have already encountered in Chap. 3, took part in the 1931 Congress where Boris Hessen spoke, and stated that he was influenced by his contribution, as also by those of the Soviet Russian delegation. In the language of the 1970s of the twentieth century, as a prisoner of the dichotomy between externalism and internalism for sciences, this English scholar who dedicated his studies to China tried to solve a question that he had: why had the Country-at-the-Centre and India not produced modern sciences as in Europe. He thus suggested that the causes should be sought outside the sciences, in the

[182]Hessen 1971, pp. 211, 200 and *passim*.

[183]Hessen 1971, p. 203.

different social and economic conditions offered by these countries. "Some of these idea-systems, in fact, such as Taoism and Neo-Confucianism, would seem to have been much more congruent with modern science than any of the European ones were: notably Christian theology."[184]

This Cambridge Professor and Fellow of the Royal Society would thus seem never to have realised to what extent his idea of a single universal science was indebted to Newton's faith in his one and only God. Nor would dialectic materialism, flaunted by Hessen as the universal method for nature and for history, help him at all in understanding how much the modern sciences, for better or worse, were indebted, in the same way, even to the theology of Christianity, and had not only been influenced by the socio-economic structure, garnished with various class struggles. With more modesty, and no faith at all in universal progress, I prefer to continue to think that Chinese scholars had fully developed their sciences, which were largely different, until the Jesuits arrived with their cannons, to make them follow the path of European sciences.

No faith at all in Soviet socialism transpired from Arthur Koestler (1905–1983) who did not concede too much awareness, in his *Sleepwalkers*, to the natural philosophers of the scientific revolution about what they were inventing. But then, did he, too, believe in a sort of providence which would lead us, in all cases, to the truth, albeit *obtorto collo* [despite ourselves]? Thinking about the way he would narrate the trial of Galileo Galilei, this Hungarian with the background of many cultures (who in the end chose the English language) cannot have explored in depth the theological aspects of the dispute with the Catholic Holy Office. He seemed to make it a question, mainly, of academic rivalry, pride, imprudent rhetoric, or psychology overwhelmed by passion. And yet *patet* [it is evident] that they had discussed absolute truths, and the God who could guarantee them. There is little doubt that Koestler preferred the truths of the sciences to the rival 'truths' proclaimed by the (self-styled) Communist Central Committees or by the Holy Office.[185]

In the affairs of Galileo Galilei, Koestler saw, in the end, "…that the divorce between science and faith came to a head…". We know very well that this commonplace, which is current among philosophers, theologians, modern scientists or historians of rational progress, does not correspond to the writings left by many natural philosophers. In this connection, regarding Newton, our skilful Hungarian divulger in search of calm places without too much censure, wrote that we know "…very little …". At this point, on the contrary, we know enough details about the Arian theology he used, to maintain the balance in a cosmos otherwise uncreated and unstable. Only in the Epilogue does Koestler readjust the argument a little, admitting that the protagonists were "…religious thinkers …", but some, like Galileo Galilei, offered God only a "…formal homage …", another, like Newton,

[184]Hessen 1971, pp. viii–ix.

[185]Koestler 1991, pp. 417–493, 479 and *passim*. In a hastily added note, the polyglot novelist and essayist reported that the Jesuits had brought "…a Copernican astronomy …" to China. We know from Sect. 8.2 that things went very differently.

was "... a confused metaphysicist ... an extravagant theologian ...". Clearly, the essayist and multilingual novelist did not read with care the famous letter written by Galileo Galilei to Cristina di Lorena, or Newton's many religious writings, and therefore he did not understand the consistent relationship with their scientific positions. After all, for Koestler, "The mechanical universe could not accept any transcendent factor" and would naturally have become "... deaf to the harmony of the spheres."[186] Instead, not only was it forced to receive its part of transcendence, but it even maintained it until our days. Only these characters concealed it from us, transforming it in other guises: those of the transcendent symbolism of the mathematical sciences, invented just by theologians disguised in other clothes like Newton and Leibniz, who merrily continued to believe that they could understand the music of the spheres.

A priori ideas, equally different from those of Marxism-Leninism, more sensitive to philosophical and religious problems, though partial, if we want to follow the evolution of the mathematical sciences closely, returned with Arthur Oncken Lovejoy (1873–1962). To cut a long story short, however, we would not be very interested here in his philosophy of science and of history, among many others, except as an example of the constant presence of the term "Being" in the debates among our European natural philosophers. *The Great Chain of Being* had been a common way for centuries in Europe of understanding the whole universe. This professor from the United States proposed some "unit-ideas" as guides for Western collective thought. Among these, he included that "ultramundane quality" which I prefer to call transcendence here, partly to underline its religious characteristics. At the top of the chain, which is a hierarchic scale, there is absolute Being, that is to say, God. He read it in the verses of Alexander Pope.

> Vast chain of being! which from God began,
> Natures aethereal, human, angel, man,
> Beast, bird, fish, insect, what no eye can see,
> No glass can reach; from Infinite to thee,
> From thee to nothing. ...[187]

The cult of Being and of transcendence are rediscovered by Lovejoy not only in Nicolas de Cuse (1401–1464), but even in the less suspectable Giordano Bruno (1548–1600) with his regular and necessary theological justifications. Could traces of all this be missing from the divulging works of Bernard le Bovier de Fontenelle (1657–1757), the secretary of the Academie des Sciences in Paris, where the absolute Being populated the whole universe with far more relative beings? Even Immanuel Kant (1724–1804) was to deal with an infinite Being and the scale of being.[188]

Of course, this "great chain of Being" above all held Leibniz prisoner with its steel links. He had written: "All the different classes of beings, which, taken all

[186]Koestler 1991, pp. 493, 521, 523–525, 537 and *passim*.

[187]Lovejoy 1966, p. 66 e *passim*.

[188]Lovejoy 1966, pp. 118–127, 137–140, 148, 207, and *passim*.

together, make up the universe, are, in the ideas of God, …, essential, as well orderly as on a single curve, and thus so closely united that it would be impossible to include others …, given that it would imply disorder and imperfection. […] …it is necessary that all the orders of natural beings should form only one chain, …". In the German philosopher of pre-established harmony, as also in his English Newtonian adversary Samuel Clarke, the discussion was about the Being of beings, in the scale which terminated for both of them with the "Most simple, absolutely eternal, infinite, original and independent Being."[189]

Beyond all these arguments of theirs, which appear to be variants of the ontological proof offered at his time by Anselmo d'Aosta to bring God into existence, we must note that here, certain profound characteristics of the European sciences came to join together. The languages that could predicate properties by means of the verb "to be" were now using the same word "being" as a synonym of an animate or inanimate thing existing in the world: "beings". Finally, they arrived at the desire to predicate properties of a "Being" superior on the scale or in every sense to all the other "beings", which was written, thanks to the letters of Latin, with a capital letter: our old acquaintance called God. In this case, languages, realities of the world and religious transcendences came to coincide in the terms *esse, essere, être, to be, sein, ser, zijn,* …, used by a great variety of Latin, Italian, French, English, German, Spanish, and Flemish scholars, … or at least scholars who expressed themselves in these languages.

Lovejoy explained: "…a rigorously rational world must be …a pattern of things determined wholly and once and for all by 'necessary truths' …in this way, all things are so closely linked with the existence of the necessary Being, and that Being, in turn, implies the existence of all the rest, that the totality does not allow possible additions, omissions or alterations." Until William James (1842–1911), all the history of philosophy was seen by the philosopher of ideas from the United States as a series of comments on Plato. Even if, at the end, he concluded that "…it is the history of an unsuccessful attempt …", he did not give up the idea of celebrating it as: "…the erroneous hypotheses are often ways to the truth. …the idea of the Chain of Being, …, has had many happy consequences in the history of Western thought."[190] Without doubt, the theological and naturalistic intertwined in it, providing impulses and bends in the evolution of the mathematical sciences (which Lovejoy did not choose to criticise). But this represents only the advertising aspect, even becoming a caricature, obtained by choosing arbitrarily and amplifying only certain characteristics. However, it here plays the useful function of showing us the commonplaces followed by the orthodox people. From time to time, heretics have represented incompatible or antagonistic criticism, and not simple variants of the same general model with a change of sign.

The examined scholar reviewed only his favourite "unit-ideas", ignoring the others. Among the former, he greatly insisted on the notions of "ultramundane

[189]Lovejoy 1966, pp. 153, 156–160, and *passim*.

[190]Lovejoy 1966, pp. 351–352, 356.

quality", of "being", of "law" and of "truth". We, too, have often encountered them in our history, sometimes explicitly, and at other times in implicit forms. But among them, he failed to point out that "dualism' which is typical of European-Western thought, which tried, in a dialectic manner, to submit "becoming" to "being", the "error" to "truth", "immanence" to "transcendence", and the unstable "variety" of the world to the single "law". In a profoundly different way, on the contrary, I have shown in the preceding chapters to what extent these notions depended on the capacity of the European-Western historical context to select and construct dominant cultural positions by its own means. Among those the languages used in discourse are not to be underestimated. Leibniz also wrote: "... les langues sont le meilleur miroir de l'esprit humain, ..." ["... languages are the best mirror of the human spirit, ..."][191]

Having arrived at this point, we might re-read the preceding pages, looking for the elements that can be grouped together under Lovejoy's "idea-units". We will leave this to the more willing readers, who, however, will at the same time be careful to remember that they were not present in the dominant Chinese culture, and were only partly originating in the Indian and Arabic cultures. Here, I offer only those few examples which seem particularly convincing to me. In the end, we shall have to realise that also our philosopher from the United States, in spite of all his interest in poetry, from which he has succeeded in extracting his paradigms, completely ignored music, even when gleaned from the texts of Leibniz.

As regards transcendence and discussions about God, there is no use in adding much. Let us simply refresh our memories with a few more affirmations. Leibniz proposed: "10. God is neither a 'Worldly intelligence', nor an 'Ultramundane intelligence', but He is rather an omnipresent intelligence, both inside and outside the world. He is in everything, everywhere, just as He is above everything."[192] Newton told us: "God, who has given animals the capacity of self-movement beyond the limits of our understanding, is doubtlessly capable of putting into bodies other principles of movement which are equally difficult for us to understand."[193] John Locke was convinced by him to believe that: "The gravitation of matter towards matter, which takes place in ways that are incomprehensible for me, not only demonstrates that God, when He regard it suitable, can attribute to bodies also powers and ways of action different from those that can be deduced from our idea of body, or explained by our idea of matter; but it also undeniably confirms that He really did so."[194] As a result, Koyré states: "Space for Newton (...) is the eternal kingdom of the divine presence and action – not only his *sensorium*, but also, so to speak, his *actorium*."[195]

[191] Leibniz 1960/65, v. 5, p. 313.

[192] Leibniz 1967, v. I, p. 310.

[193] Westfall 1989, p. 322.

[194] Koyré 1972, p. 172.

[195] Koyré 1972, p. 14.

We have already several times encountered the idea that the world, like a well-ordered kingdom, would be governed by laws. Marcus Tullius Cicero once said: "Omni in re consensio omnium gentium lex naturae putanda est." ["The agreement of all the peoples on every matter is to be considered the law of nature"]. In the same way, Roman jurists had identified the "jus naturale" ["natural law"] as the "jus gentium" ["law of the nations"].[196] We again refer here the conviction of Leibniz. "Quand nous voyons les corps suivre les loix mechaniques du choc dans quelque machine, et l'âme suivre les loix morales du bien et du mal apparent dans quelque deliberation: disons des autres cas que nous ne voyons pas ou que nous ne démêlons pas si bien, qu'il en est de même, et *que c'est tout comme icy*." ["When we see bodies follow the mechanical laws of a collision in some machine, and the soul follows the moral laws of good and evil apparent in some decision: we say of other cases, that we cannot see, or that we cannot discern so well, that the same happens, and that *it is all the same as here*."][197]

Also Newton invented laws. "Haec sunt motuum & virium leges & conditiones, quae ad philosophiam maxime spectant." ["These are the laws and conditions of movements and of the forces which mainly address philosophy."][198] Or, "... the said image of the Sun, if it became oblong, by means of rays refracted on the basis of no certain law, could not be delimited distinctly by straight lines in its length."[199] Descartes had written to Mersenne. "God provides nature with mathematical laws, as a king provides his kingdom with laws."[200] Noble has summarised for us innumerable books about Christian monastic communities, all governed by rules, canons and laws. "Benedictine monasticism, with its insistence on the law, the rule, discipline and obedience, seemed deliberately made for this ambitious reforming pope [Gregorius I Magnus, c. 540–605]."[201]

Leibniz had clearly and constantly sought necessary truths, to be expressed by means of laws which were guaranteed (how else?) by a transcendent Being. "In reality, we see that all the things in the world take place in accordance with the laws of eternal truths, not only geometrical but also metaphysical, that is to say, not only in accordance with material necessities, but also formal ones; ... it can be seen that the metaphysical laws are admirably valid in all the universe ... Thus, ..., we have the ultimate reason for the reality both of beings and of their existences in a single Being, which is necessarily greater than the world itself, superior to it, and antecedent to it." "Veritas est, inesse praedicatum subjecto." ["The truth is: the predicate is in the subject."][202] From the sentence we can deduce that also the "truth"

[196] Lovejoy 1966, pp. 312–313.
[197] Leibniz 1960, v. III, p. 346.
[198] Newton 1687/1972, p. 549.
[199] Newton 2006, p. 75.
[200] Noble 1994, p. 274.
[201] Noble 1994, p. 112.
[202] Lovejoy 1966, p. 186.

sought by our German metaphysicist-cum-natural philosopher depended on the verb "to be" as on the supreme Being.

Newton had written: "And if someone can make a conjecture about the truth of things only based on the possibility of hypotheses, I do not see how anything certain can be established in any science, seeing that it is always legitimate to formulate continually differing hypotheses, which will seem to procure new difficulties."[203] Like Mersenne, Nicolas Malebranche (1638–1715) even included the fateful word, 'truth' in his title: *La recherche de la vérité [The Search For Truth]*. "Les vérités sont immutable et nécessaires, aussi bien que les idées. Il a toujours été vrai que 2 fois 2 sont 4, et il est impossible que cela devienne faux." ["Truths are unchanging and necessary, like ideas; it has always been true that twice 2 is 4, and it is impossible for this to become false."][204] Funkenstein affirmed with certainty, "... in every historical period, what society has constructed (the gods, the laws, the institutions, [and I would add also the sciences]) has been its truth; for its members, what was *done* was *true*, with the same absoluteness because of which our science is true for us. There is no other truth than that constructed by man, except – obviously [sic!] – [divine] revelation".[205]

The dichotomy between truth and error, like that between God and the earthly world, would be for them a dualism governed by *tertium non datur* [there is no third option], capable of proliferating without ever stopping within innumerable other divisions into two sides. Everyone knows the famous dichotomy of Descartes. Leibniz followed it, too, in spite of his theological attempts to remain at a distance from the Zoroastrian Manicheans and from Pierre Bayle. "That comes from the appearances of good and of evil, which deceive us: while God is always led to the truth and to the greatest good, that is to say, to the true good in an absolute sense, which He is bound to know." His recourse to pre-established harmony underlined a strong link with dualism, rather than success in freeing himself from it. "Harmony and correspondence between the soul and the body is not a perpetual miracle, but the effect, or the consequence of an original miracle which goes back to the creation of things, as for all natural things." In logic and in mathematics, actually he could not give up "... that other great principle of contradiction: that is to say, that every intelligible enunciation must be either true or false."[206]

Newton did not refrain at all from first creating, and then solving, his dichotomies, with inexorable sharp cuts *à la* Francis Bacon. "Here, then, is ye wisdom of God, that he hath so framed ye Scriptures as to discern between ye good & ye bad, that they should be demonstration to ye one & foolishness to ye other."[207] Likewise, our English expert of gravity and of the optical prism wanted to decide truth by means of what he called "*Experimentum crucis*"

[203] Newton 2006, p. 96.

[204] Funkenstein 1996, p. 350.

[205] Funkenstein 1996, p. 340.

[206] Leibniz 1976, v. I, pp. 670, 363 and 372.

[207] Newton 1994, pp. 36–37.

["experiment of the cross"]. "The gradual removal of these suspitions at length led me to the *Experimentum Crucis*, . . .".[208] It is curious that none of the following commentators, careful to rediscover similar expressions in Hooke and Francis Bacon, made any reference to the experiment of the Christian Helen, the mother of the Emperor Constantine, performed with the aim of rediscovering the 'true' cross of Jesus Christ. Newton's dualism would also be found in his pages about alchemy. "A dead, heavy body is in itself an imperfect body. The Spirit which purges, illuminates and purifies the body is water. The soul which gives life to the imperfect body, when it does not possess it, or elevates it to a higher level, is leaven. The body is Venus, and is female; the spirit is Mercury, and is male; the soul is together Sun and Moon."[209] Dualism, of which we have found abundant traces in the European sciences, cropped up again, unawares, in the essay of Lovejoy. He wrote about the ultraworld and the world, about the single being and becoming multiple, about the sun of truth and the shadows of errors. According to him, this was justified by the "principle of fullness", that is to say, everything possible, whether good or evil (Q.E.D.), should exist.[210]

How would it have been easier to reach transcendent truths, than through dualist laws expressed in mathematical symbols, likewise outside historical spoken and written languages? If we accepted the terminology of a writer like Umberto Eco, we might wonder to what extent the language ultimately invented by the mathematicians of the seventeenth and eighteenth centuries was the "perfect language", desired for centuries by many in Europe, but never found. It will not be necessary to repeat again that the Italian philosopher's book would further reinforce the conviction of the limits shown by the West with its culture. In the outlines indicated by him (the dominant ones, clearly), it has allowed itself to be guided by "truth", "dualism", "laws" and its various "religious transcendences" as modelled in the language of "being". Sure enough giving heed, instead, to the polyphony of the other languages, ". . . the *aymara*, instead of the bivalent logic (true/false) on which Western thought is based, possesses a trivalent logic, and is thus capable of expressing modal subtleties which our languages capture only at the cost of heavy periphrases."[211]

But, even when writing about Leibniz, this scholar and successful novelist prudently avoided pronouncing himself about the possible universality and (divine?) eternity of the new symbolic-transcendent language used by natural philosophers starting from Descartes. And yet, in the world of Babel which gladdens our hearts so much, it possesses some suspicious and worrying characteristics. Unlike the many other "perfect" pretentious languages invented, which have remained without consequences, mathematical symbolism has been on the crest of the wave for

[208] Newton 1959, I, pp. 94 and 104. Newton 2006, p. 195. Francis Bacon had written: ". . . *examples of a cross*, metaphorically taken from the crosses placed at crossroads, to indicate the fork in the road." Bacon 1965, v. I, p. 422. Koyré 1972, p. 46.

[209] Westfall 1989, p. 313.

[210] Lovejoy 1966, pp. 57, 99, 104 and *passim*.

[211] Eco 1993, p. 373 and *passim*.

centuries, uniquely succeeding in occupying a position in all the schools of the world, even in Chinese schools (at least until Apophys or some other wandering asteroid will hit us). Everyone who has wanted to have access to scientific journals has been forced to express himself in this way. Today it is used willingly (or unwillingly, not being able to do without it, for better or worse) even by those literary-humanistic academic sectors who aspire to acquiring a certain scientific title. The closer we come to this mathematical language, the more we may claim to have reached the "truth", leaving behind all doubts, errors and subjective beliefs. Thus the preference is given to transcendent dualist laws (called deep structures in linguistics), far from deceptive appearances, (called surface structures). Where better can inspiration and roots be found for this than in Descartes?[212] Thanks to the arrival of computers, more recently still noble scholars have saved themselves the heavy task of learning it, without losing the sacred aura of justification, inventing new disciplines such as humanistic (and music) computer science.

While the dream of the "perfect" language periodically crops up, and regularly disappears, the unavowable wish of being able to invent it in the end thanks to the mathematical sciences still hovers too close. It would be better to look somewhere else, if need be. Because, in spite of some undoubted successes in certain limited fields (like physics) cultivated by the relative scholars, even mathematical symbolism has never possessed all the qualities required for a "perfect" language. In inventing the notation suitable to express movement, that is to say infinitesimal analysis, Leibniz would have come much closer to it than with his immobile combinatorial "universal characteristics". But even his symbolism, so effective for problems of mechanical movement, had to change in time. And the jurist from Hannover, already obscure enough in his own times for the non-initiated in the mystery, would not be understood centuries later. The scholar of analysis during the nineteenth and twentieth centuries were to prefer to introduce other notations into their books. Worse still, I have tried to show in the present work that even the mathematical sciences are neither neutral nor universal, nor eternal, and likewise depend for their efficacy on the historical and cultural contexts that invent them. Thus, although it may be presented as scientific, basing it on some current and jointly responsible mathematical theory, every research or construction of a "perfect" language will only be able to come more or less close to the many other attempts: to choose and re-found as such the Sanskrit of the Vedas, the Hebrew of the Bible, the Arabic of the Koran, Christian Latin, diplomatic French, or our contemporary (imperialistic?) English. To play the easy prophet, may we now expect some Chinese language made "perfect" by some alphabetic variant?

Following Leibniz, and thus the Jesuit sources known to us, together with others, Eco seems to misunderstand the symbols of the *Yijing (I Ching)*. It is not a question of doubtful differences only of interpretation, as the philosopher from Alessandria (in Italy, not Egypt) believes, misled perhaps by the secondary literature. Rather, there is a great distance, also on the *formal* level, between them and the numbers "1,

[212] Chomsky 1969.

0", which blocks the possibility of identifying them as the usual binary calculation. If, like the Yin and the Yang, 0 every now and then turned into 1 and vice versa, or, worse, if 0 and 1 were intertwined in such ways that even the sharp scalpel of Francis Bacon and Newton could not succeed in separating them,[213] what Chomsky-style linguist or illuminated philosopher in the Eco style could accept the consequences? Anyway, like the German of the pre-established harmony, after all the failures of the "perfect" languages, the renowned Italian scholar of languages and signs optimistically sought to distil some positive results for his semiotics students: the sophisticated logics of the nineteenth and twentieth centuries, and the binary model of the computer.[214]

In spite of all these efforts to discriminate and to simplify, the world continued, fortunately, to prove to be much more varied. Let us not deceive ourselves, either, that 3,000 years of history can be imprisoned in four little words like ours: law, truth, transcendence, dualism. We already know that even Leibniz, despite his faith in eternal, universal laws, recognized that he had to reconcile them, thanks to the Italian poet, Torquato Tasso, with the variety of phenomena. "…la sagesse doit varier. Multiplier uniquement la même chose, quelque noble qu'elle puisse être, ce seroit une superfluité, ce seroit une pauverté: avoire mille Virgilies bien reliés dans sa Bibliotheque, chanter tousjours les airs de l'opera de *Cadmus et Hermione*, …" ["…wisdom must vary. Multiplying only the same thing, however noble it may be, would be superfluous; it would be a kind of poverty. Having one thousand well-bound copies of Virgil in one's library; always singing the arias of the opera *Cadmus et Hermione*, …"].[215] *Cadmus et Hermione* was an opera that Giovan Battista Lulli had staged in 1673, amid the secular indifference of professors who were experts on Leibniz. Let us, too, return, then, to music, for an epoch which, in spite of everything, must have abounded in it.

The seventeenth century had opened with Giordano Bruno at the stake in Campo dei Fiori in Rome, being it at once illuminated in a sinister way. Galileo Galilei underwent several trials, and Johannes Kepler suffered religious persecution, which often forced him to change his residence. While he was finishing the *Harmonices mundi libri quinque*, Kepler had to defend his mother against the charge of witchcraft, luckily with success. In 1618, the 30 Years' War broke out, a conflict which was to overwhelm the population in bloodshed, reshuffling (to a relatively limited extent) the relations of the forces between the empires and kingdoms in the political and religious fields. For the famous General Wallestein, who fought on the side of the Catholic Habsburgs, ending up by being killed by them by treachery, Kepler had worked as an astronomer and a writer of horoscopes. Also René Descartes had gone around through Germany at war, with a sword dressed as a soldier, enjoying all the horrors of the events. As the further sign of His benevolence towards mankind, God also sent the plague here and there

[213] See Part I, Fig. 3.7.

[214] Eco 1993, pp. 305–308 and *passim*. We await with curiosity the so-called quantum computers.

[215] Leibniz 1960/65, v. 6, p. 179. Lovejoy 1966, p. 241.

several times. It threatened Descartes in 1635 and Newton in 1665. In England, Puritans led by Oliver Cromwell (1599–1658) organised a revolution and dethroned the philo-Catholic king, Charles I Stuart (1600–1649), beheading him. The Great Fire of London flared up in the year marked by the apocalyptic number 1666.

The screams of the wounded, the groans of the agonising, the crackling of the flames and the noise of firearms arrived sooner or later within the hearing range of everybody. The eyes of aggressive, carnivorous animals in search of prey to kill and devour were prevailing over the ears of gentle, herbivorous animals, ready to defend themselves in their flight. It might seem to be difficult to remain calm in meditation to perceive some notes of the heavenly harmony. Despite this, however, from a completely different point of view, for us faraway listeners, in the epoch which invented differential calculus and constructed the modern cosmos, the age of music truly began.

The Gabrielis, Andrea (1515?–1585) and Giovanni (1557–1612), spread their organ notes, choruses, symphonies, songs and sonatas from the Basilica of St. Mark's in Venice, all over Europe. William Byrd (1542–1623) played the virginal and sang sacred and profane songs at the English court. Claudio Monteverdi (1567–1643) filled up theatres, courts and churches with his newborn lyric operas, his profane madrigals and his masses. Heinrich Schütz (1585–1672) united polyphony and monody, Italian dramatic style and archaic religious rigour to elevate the Lutheran spirit toward the Heaven. Girolamo Frescobaldi (1583–1643) improvised on the organ, filling the Italian churches with iridescent musical flowers. Giacomo Carissimi (1605–1674) touched the soul with the human sufferings experienced by Biblical figures. The whole court of Louis XIV danced to the music of the Florentine Giovan Battista Lulli (1632–1687). The English enjoyed the theatrical compositions and the scene music irresistibly penned by the Purcell family, above all Henry (1659–1695). Dietrich Buxtehude (1637–1707) attracted crowds to the church of St. Mary at Lubeck, not only of believers, who wanted to hear him play the organ. Arcangelo Corelli (1653–1713) inaugurated, with his chamber sonatas, church sonatas and his famous grand concerts, the instrumental style based on strings. Antonio Vivaldi (1677–1741) travelled all around Europe with an enormous quantity of music, made of operas, oratorios, sonatas, and concertos. The Couperin family, above all with François II the elder (1668–1733), played the organ and the harpsichord in all the churches and palaces in France.

But more was yet to come, with the uncontainable production of Georg Philipp Telemann (1681–1767) as well as with the dozens and dozens of Italian-style works and oratorios by Georg Friedrich Händel (1685–1759), between the court of the Duke of Hannover and the English court in London, inherited by the Hannovers themselves with George I. It was very likely, even if no documentary proof exists, that Leibniz and Händel met at Hannover. Our German metaphysicist seemed to prefer church music, but did not disdain lyric opera. Jean Philippe Rameau (1683–1764) made his name in France with his lyric dance-operas and pieces for the harpsichord: thanks partly to his important theoretical writings, he aroused the interest of the most famous physicists and mathematicians of the period, as we shall soon see in the following chapter. Finally, the best arrived with Johann

Sebastian Bach (1685–1750), whose music seemed to be capable of absorbing and transfiguring into a superior harmony all the old and new styles, polyphonic and monodic, religious and profane, all the contrasts between Lutherans and Catholics, all the ambiguities between the body and the soul, between history and eternity.

As we are preferring to underline the differences exhibited in the subjects of our study, I would also like to investigate the characteristics of all these composers. But I am writing a book about the history of the mathematical sciences, which places music in the foreground, because of the function that it has had, not a history of music with hints to acoustic theories. We are therefore obliged to keep it short, and although the musicians mentioned above do not have too much in common, we must try to lead them back the context of their period. It was an era of great renewal, which we can sum up (and therefore also impoverish) in two words: the affirmation and the spread of Italian opera in Europe, together with the consolidation of tonal harmony, sometimes in conflict with polyphony. If we want to measure their success, it is sufficient to think that many of them are performed and listened to with pleasure still today by music-lovers.

The preceding vocal supremacy gave way in favour of music for instruments, like the organ, the harpsichord, without forgetting flutes, trumpets and other means which were to form richer and richer orchestras. The violin gradually took the place of the lute. Even in churches, with the luxury of timbre mixtures and chromatic richness, the sacred compositions betrayed a joy of living which opened up new spaces within religious purposes. The self-possessed heavenly harmony left its place to the harmonic technique of chord triads capable of arousing in the listener, whether sacred or profane, emotions of every kind: from melancholy to wrath, from love to heroism. Sometimes a sung literary text was underlined, at other times, the notes of an instrument were sufficient for the expression. Now the people who lived on the Earth manifested the sentiments they felt, and aroused them in the others with the new dramatic intensity permitted by the renewed tonal harmonic language.

The term *aria*, which indicates the way of singing lustily in lyric opera, where the music concentrates the dramatic expression between one recitative and another, represented appropriately the essence of freedom: do we not use the expression ‘as free as air’? The smooth flowing style of the *aria* became the *cantabile* of the violin, which, thanks to the ability displayed by makers of string instruments and by performers, was capable of constructing its world without the need of words. Suffice it here to recall Vivaldi *Il cimento dell’armonia e dell’invenzione [The Trial of Harmony and Invention]* with his naturalistic interpretations which have remained extremely popular. The whole gamut of notes that could be obtained on the strings of the violin, which, unlike the lute, does not have frets, well represented that *continuum* of sounds which the corresponding *continuum* of numbers, made completed by the irrationals, was now capable of calculating. Whereas in the *continuum*, the harpsichord had to operate its choices predisposed by the tuning. However, it had become free, abandoning traditional Pythagoreanism, and could follow the requirements of every composer who wanted to run his hands over the keyboard: from one note to another, from one chord to another, from one tonality to another.

Therefore, the harpsichord was no less capable of expression. On it, it had become easy to play three notes together placed at intervals of a third, $do-mi-sol$ the harmony, with the left hand, while the melody was played with the right hand. The theory of the chords was becoming harmony in the technical sense of the word. Thus tonal harmony was consolidated, forming the supporting framework of musical compositions. Note after note, they create the *tempo* in which they are carried out: *Allegro, Adagio, Presto, Lento, Finale, Vivace, . . . , Andante, Andantino*. The *tempi* of music mean movements, motions, emotions, commotions. In the music of the harpsichords played inside the houses of Huygens or the courts of Leibniz, the rhythm then was relative and varied, like their idea of the time for the movement of bodies in the vortices of ether. Filling up the stave, the composer transforms the *tempo* of his conscience in the space of the paper.

After the *cantus firmus* of the Gregorian chant, after the polyphonic masses composed of immobile vocal intarsias, music had now become *Vivace*, that is lively, moving. Which composers best represent the movement of Domenico Scarlatti (1685–1757), with his 550 *Sonatas* incapable of boredom, well-marked by rhythm of the harpsichord, with an inexhaustible imagination? Or which music is better than the *Pièces de clavecin* written by François Couperin to describe kings and courtiers, and sometimes make them dance? Nor should we overlook the instrumental works of Telemann, requested for religious functions as for banquets, so popular at his times as to be better known than Bach. The North German appears to be a perfect example of musical orthodoxy during this period. "The *tempi* evolve in their brief parabola with that simple, concise logic which was the weapon of the century. The timid thematic evolutions are regular and mechanical, like the exercises of Frederick II's grenadiers on the parade ground."[216]

For reasons of time, space and argument, we have had to limit ourselves here to a few synthetic considerations, based above all on analogy (which Kepler appreciated). But we shall immediately find in the following chapters more solid, documented moments of dialogue between musicians and those natural scientists who were becoming the physicists and mathematicians of today.

The impressive torrent of music produced in Europe between the seventeenth and eighteenth centuries formed vortices and currents very equal to those of the planets, and capable of overcoming every obstacle. Above all, no old theory of harmony or of the scale could have resisted. Practice, taste, consumption, inspiration, imagination conquered new realms of freedom, and then fixed new rules. Polyphonic compositions increasingly left space for accompanied music. The dualism was imposed between major and minor, founded on the interval of a third with the 24 tonalities. Keyboard instruments (organ, harpsichord, piano) chose the fixed tuning compatible with the interplay of modulations and tonalities, that is to say, the equable temperament which divided the musical scale into 12 equal semitones.

[216] Mila 1963, pp. 105–138, 163–178, 177, *passim*.

Thus the ratio $\sqrt[12]{2}$, on which the equable temperament is based, and which had never been tolerated by the theoreticians of the Pythagorean-Platonic tradition, because it was irrational and could not be expressed as a ratio of whole numbers, triumphed in practice, winning the attention of theoreticians. Having arrived at this point, have I succeeded in reconstructing some links, some form of consistency, or at least in narrating what was a singular coincidence to a certain extent? We have seen that one existed between rationalising the irrationals, together with transcendence in the infinite through differential calculus or infinite series, and rationalising the musical scale, through $\sqrt{2}$. Even New Tone/Newton had actually occupied himself, in his notes dedicated to music in 1665, with dividing the scale into 12 equal semitones whose value was $\sqrt[12]{2}$, also using logarithms. Above all, we have written that Leibniz must have preferred the equable temperament, in spite of everything. A more or less equable temperament, with a good approximation, was codified in 1697 by Andreas Werckmeister, and was to be brought to perfection and to success in the *Well-tempered harpsichord* (1722–1744) of Johann Sebastian Bach. But this, too, is another story.

The struggle between death and music seemed to mould the character of the period in which the modern physical and mathematical sciences were born. In the face of so much evil spread all over the world, Leibniz succeeded all the same in justifying his God of pre-established harmony. He permitted sins, he would say, as the musician used dissonances, not wanting them in themselves, but in order to reach a better melody: "the shadows, like dissonances, put the rest into relief", as we already have read above. However, neither the dream of Leibniz, nor that of Descartes, of succeeding by means of geometry and mathematics in cancelling conflicts and controversies, came true. Many different harmonies continued to present themselves and to be confront each other. But this did not seem to be a terrible thing, because thus the history proceeded cheerfully in its variety, rather than ending up in boredom. Everyone may thus continue to choose the music, or the death and war, that he prefers.

We are narrating a conflict between the community of composers or players, who wanted to choose the music of their liking, without any limitations, and the community of natural philosophers and mathematicians, who, on the contrary, would claim to discipline it, by inventing rules and numerical laws. Rare to the point of being unique during the history of earthly cultures, the former prevailed. Now, conquered by variety and by the pleasure of listening to music, if for no other reason, even some physicists and mathematicians were becoming convinced to abandon their ancient games with the ratios of whole numbers. Anyway, music was not to leave the stage of the mathematical sciences so easily and the patient reader will immediately encounter it again in the following chapters.

Dulce bellum inexpertis
[To those who do not know it, war seems sweet]
Erasmus from Rotterdam

Oportet et haereses esse
[It is also necessary to be heretics]
Saul from Tarsus

Nature and nature's laws lay hid in night;
God said 'Let Newton be!' and all was light.

He hangs between; in doubt to act or rest;
In doubt to deem himself a god or beast;
In doubt his Mind or Body to prefer;
Born but to die, and reas'ning but to err; ...
Chaos of thought and Passion all confus'd
Still by himself abus'd, or disabus'd;
Created half to rise, and half to fall,
Great lord of all things, yet a prey to all;
Sole judge of Truth, in endless error hurl'd;
The glory, jest and riddle of the world.

Superior Beings when of late they saw
A mortal man unfold all Nature's law,
Admir'd such wisdom in an earthly shape,
And shew'd a Newton as we shew an Ape.

Alexander Pope

Chapter 11
Between Latin and French

Nous avions pourtant une pure tradition française dans l'oeuvre de Rameau, faite de tendresse délicate et charmante, d'accents justes, de déclamation rigoureuse dans le récit, sans cette affectaction à la profondeur allemande, ni au besoin de souligner à coups de poing, d'expliquer à perdre haleine, qui semble dire: 'Vous êtes une collection d'idiots particuliers, qui ne comprenez rien, si on ne vous force pas d'avance à prendre des vessies pour des lanternes'.
[Therefore we have a pure French tradition in the work of Rameau, made of delicate and charming tenderness, correct accents, rigorous declamation in the recitative, without that German affectation towards depth, or the need to underline with blows of the fist, or to explain at the top of one's voice, as if to say: 'You are a mass of real idiots who do not understand anything if you are not forced in advance to get hold of the wrong end of the stick'.]

Claude Debussy

11.1 Jean-Philippe Rameau, the Bernoullis and Leonhard Euler

"La Musique est une science qui doit avoir des regles certaines; ces regles doivent être tirées d'un principe évident, & ce principe ne peut gueres nous être connu sans le secours des Mathematiques." ["Music is a science which must have certain rules; these rules must be derived from a clear principle, and we cannot have a knowledge of this principle without the help of mathematics."] This was written in the Preface to his *Traité de l'harmonie réduite à ses principes naturels [Treaty of Harmony Reduced To Its Natural Principles]*, not by one of our usual natural philosophers, but by one of the most famous musicians of the eighteenth century: Jean-Philippe

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Rameau (1683–1764).[1] After travelling around various cities in France as an organist, he arrived in Paris, where he became famous as a composer of pieces for the harpsichord, and above all, of operas and ballets. These won him the favour of the Royal Court, but they also aroused endless criticism and controversies, in spite of himself.[2]

Rameau's books, to which he dedicated himself with equal passion, succeeded in justifying, by means of his theory, those new ways of writing music which were in practice already widespread in Italy and Germany. As we are about to relate, two of the most important physicist-mathematicians of the century were to tackle this question. The French composer started from the point where Zarlino had arrived, as many others had done, as we know, but he did not stop there. Also Descartes had made reference to Zarlino, and above all Rameau read the work of the former: the *Abregé de la Musique [Compendium of music]*, which had been republished in French in 1668. He copied some passages from it in his language: "…dans chaque son, tous les aigues sont contenus dans le grave, mais non pas reciproquement …" ["…in each sound, all the acutes are contained in the deep, but not reciprocally …"].

In this way, he arrived at the execution of trials on "…une *viole* dont les cordes sont assez longues pour pouvoir en distinguer les battemens, l'on y remarquera qu'en faisant resonner une corde avec un peu de violence, celles qui seront plus basses ou plus élevées d'une Octave, trembleront d'elles-mêmes, au lieu qu'il n'y a que le Son aigu de la Quinte qui tremble, & non pas le grave; ce qui prouve que le principe de l'Octave est confondu dans les deux Sons qui la forment, & que celuy de la Quinte, & par consequent de tous les autres intervales, reside uniquement dans le Son grave & fondamental. Descartes s'étant trompé icy par la fausse preuve qu'il tire d'un Luth, à l'égard de l'Octave. […] …un Son quelconque est toujours sous-entendu dans son Octave; Descartes en convient en partie, (lorqu'il dit, *qu'on n'entend jamais aucun Son, que son Octave en dessus ne semble frapper les oreilles en quelque façon*; & il y auroit peut-être ajoûté *l'octave au dessous*, s'il ne se fut pas trompé dans la preuve qu'il en a tirée d'un Luth …" ["…a viola [*da gamba*?] the strings of which are very long, so as to distinguish their vibrations; it will be noted that, if we pluck a string with a certain violence, the strings one octave lower or higher will likewise vibrate by themselves as well, whereas as regards the fifth, only the acute sound vibrates, and not the deep one. This demonstrates: the principle of the octave is founded in the two sounds that compose it, and that of the fifth, like those of all the other intervals, as a result, resides only in the deep, fundamental sound. Descartes had made a mistake here, as a result of the false test obtained from a lute, regarding the octave. […] …any sound is always implicit in his octave. Descartes partly agrees (when he says 'that no sound is ever heard, such that its higher octave does not seem to strike the ears in some way' [)] and he

[1] Rameau 1722/1992, preface [c].

[2] Chailley 1986.

would have added, perhaps, 'the lower octave', if he had not made a mistake in the experience deriving from a lute . . . "].[3]

In reality and in this case, the eyes if not the ears of Descartes would seem to be better than those of Rameau, because nobody would ever have succeeded in seeing or hearing the lower octave of a sound vibrate by itself. Today, we would say that the lower harmonics, or sub-multiples of the fundamental frequency, do not exist. Actually, only a phantom string twice as long as the real one could emit them, and a string twice as long could not be seen to vibrate by sympathy and resonance by itself, if we plucked one half as long. On the contrary, the opposite case can easily be observed.

But why would the renowned eighteenth-century composer have needed to see or hear similar phenomena, even if partly too imaginary? His book was founding a new conception of music based on the intervals of third, superimposed in chords of three of four notes. Rameau was thus trying to justify the reason why he, together with many others by now, including Bach, were composing that kind of music called "tonal". It was different from the Greek-ecclesiastic modes, such as, for example, those considered by Francesco Maurolico. In the case of Rameau, a tonality was characterised by its chord of three sounds a "major" third (*do* − *mi*) and a fifth (*do* − *sol*) distant. Thus in the end he obtained the major "accord parfait" ["perfect chord"] (*do* − *mi* − *sol*), if the fundamental note was *do*, that is to say, the perfect chord of *do* "major". Other similar "major" chords could be constructed on all the remaining 11 notes of the scale. But he would have liked to give an equal justification similarly to the 12 "minor" tonalities, which can be obtained by superimposing the "minor" third (*la* − *do*) on the fifth (*la* − *mi*). This latter fifth of the "minor" tonality now contains first the "minor" third and then the "major" third (*do* − *mi*). The opposite happened with the "major" tonality, because the "major" came first, followed by the "minor" (*mi* − *sol*).

To cut a long story short, the harmonic theory proposed by Rameau placed everything within the octave, the ratio of which was 1:2, and shifted attention from the consonances, fifths and fourths of the previous Greek tradition to thirds, which were divided in a dualist (Cartesian?) style between "major" thirds, 4:5 (formed by two whole tones), and "minor" thirds, 5:6 (formed by one tone and one semitone). He linked intervals together by means of appropriate operations of "renversement" ["inversion"]; for example, by inverting the fifth *do* − *sol* in the octave, we obtain the fourth *sol* − *do*. Likewise, he reasoned on the natural increasing "arithmetic" succession of the numbers $1, 2, 3, 4, 5, \ldots$ which could be inverted into the decreasing "harmonic" succession of reciprocals, which he did not write, however, because he wanted to avoid fractions. He criticised Zarlino because, though invoking it, he had not always followed it.[4] At that moment, therefore, the numerical ratios had still remained those of Tolomeo-Zarlino, but the musical conception, as a result of the way of combining them together, was deeply modified.

[3] Descartes 1979, pp. 120, 81, 83 and 87. Rameau 1722/1992, pp. 3, 6–9 and passim.

[4] Rameau 1722/1992, passim.

From then on, for musicians, "harmony" was to mean the art of constructing chords of three notes superimposed by thirds, with the aim of giving the piece the "major" or "minor" sense of a chosen tonality in order to express the feeling desired.

For example, Antonio Vivaldi composed a concerto for flautino and orchestra in *do* "major"; Wolfgang Amadeus Mozart (1756–1791) chose the same tonality for his last symphony. With him, it has been noted that *do* major expressed "…an aspiration to monumentality, to imposing grandeur of architectonic values, which has gained for it the mythological nickname of 'Jupiter-Symphony'….solar grandeur …vital energy …fundamentally optimistic creative impetus …". "…it is Mozart's grandest and most dazzling symphony and it breathes a proud sense of strength like no other. …it celebrates life both in its vigorous masculine aspect and in its serene feminine aspect."[5] In the ancient tradition of cosmic harmony, the "lepidus, jocosus" ["pleasant, playful"] character was compared to the Sun, and the "hilaris, petulans" ["cheerful, jovial"] one to Jupiter; the former was evoked by the "Dorian" mode, and the latter by the "Lydian" mode.[6] Something of all this had still remained with the new tonalities, but the main difference was achieved by the freedom and richness in the modulations between one tonality and another, and by the transpositions made possible by the equable temperament.

For his new "harmony", in which we shall no longer find the relationships between the planets, the French composer tried to procure "natural principles", so as to give it sure foundations. For this reason, he referred back to Descartes and entered into our history. But the philosopher of Method still represented the old style, having made reference to the lute, where two tones had not formed the "major" third, but like every ancient Pythagorean, the ditone, even if he had considered it to be a consonance. For that Descartes, as deaf as a post, the notes had followed each other in a more or less simple melodic line, possible in parallel with others. Now with Rameau, instead, the notes came to form vertical bunches three by three, and the consonances had been enriched by two thirds, both the "major" and the "minor" one. "…la Melodie n'est qu'une suite de l'Harmonie." ["…melody is only a consequence of harmony."] "…l'Harmonie est donc engendrée la primière: …" ["…thus harmony is generated first: …"].[7] For this reason, he did not spare criticism for the famous philosopher, even though in the end he concluded "…, ces défauts sont pardonnables à un Auteur qui n'a fait qu'éfleurer la matière, & qui nous fait assez connoître d'ailleurs qu'il l'auroit poussée plus loin qu'un autre, s'il s'y fût attaché." ["…, these defects are forgiveable in the case of an author who has only touched upon the subject, and who shows us that otherwise he would have advanced it further than others, if he had dedicated himself to it."][8]

[5]Mila 1967, pp. 134–135 and 145. Abert 1985, v. II, p. 511. Alfred Einstein 1951, pp. 170–171. Paumgartner 1956, pp. 370–371.

[6]Maurolico 2000, e 201?. Tonietti 2006b, p. 151. See Appendix C.

[7]Rameau 1722/1992, pp. 23 and 139.

[8]Rameau 1722/1992, p. 14.

It is true that Descartes had justified consonances partly by the phenomenon of resonance by sympathy,[9] but in the end he continued to exploit the theory of coinciding strokes in order to classify them (into perfect and imperfect): it was, after all, an attempt to confirm the Pythagorean style. Rameau, on the contrary, was looking for something else, and was to find it, even if, for the moment like Zarlino and others, he was lingering on games with whole numbers. He was still calculating ratios as proportions and not as fractions. Thus, instead of division, to "subtract" intervals, he put in his book the cross typical of the mathematics used at the time of Luca Pacioli and Francesco Maurolico. And yet he did not do the same in order to "add them" because he could write multiplications more easily.[10]

For two centuries or more up to today, the terms and reasonings of our French theoretician and composer on harmony were to enter into the use of musicians. By a singular coincidence, in the same year, 1722, Johann Sebastian Bach, instead of writing a treatise on harmony, started to compose the First Book of *Das wohltemperierte Clavier [The Well-tempered Harpsichord]*. Here the preludes and fugues followed one another in the order of the major and minor tonalities on the levels of the scale as justified by Rameau: *do* major, *do* minor, *do*♯ major, *do*♯ minor, *re* major, *re* minor, ... We may debate which of the two attitudes had a greater influence of the following evolution of music.[11]

Anyway, our French musician and theoretician, often criticising Zarlino and others, gave those rules for the new concept of harmony which were to pass into the following manuals on the subject. But it was to prove to be a completely different matter to succeed in justifying them on some alleged 'naturalistic' basis. Besides having to convince the other composers, the problem remained of having to play them on keyboard instruments. At that time, Rameau seemed to ignore, or underestimate, this non-indifferent obstacle, above all because the beautiful and the good of the new music consisted in the great variety made possible by the modulations between the 24 different tonalities. Here, in the first treatise, in accordance with the numbers of Zarlino, he took tones and semitones of "... differentes especies ..." ["... different kinds ..."],[12] which were therefore not equal, not tempered, and not easily transposable.

All the numerous rules were illustrated by means of a wealth of examples and fixed in the treatise. They aspired, as we can imagine, to the status of laws, as they were later to become, and were to serve to guide the first steps of uncertain beginners as regards "... bon goût ..." ["... good taste ..."]. Also the most gifted in this field could benefit from them, however. "... mais malgré ce don naturel, il est difficile de ne pas s'écarter de la verité, quand il n'est pas soûtenu par la connoissance; & la connoissance ne suffit pas pour la perfection, se le bon goût ne vient à son secours; ..." ["... but despite this natural gift, it is difficult not to move away from the truth

[9] Descartes 1979, pp. 93–94.

[10] Rameau 1722/1992, pp. 23–24 and passim.

[11] Basso 1979/1983, v. I, pp. 650–663.

[12] Rameau 1722/1992, p. 364.

if it is not sustained by knowledge; and knowledge is not sufficient for perfection, if good taste does not come to its aid; …"]. Having repeated for the umpteenth time the perfect chord on the tonic of *do* major, he let slip expressions that betrayed his experience and practice on the harpsichord: "… les doigts préviennent l'esprit …" ["… the fingers anticipate the spirit …"].[13]

Having learnt of the phenomenon of resonating harmonics from Joseph Sauveur years later, he made this a new basis in order to justify his own idea of harmony: *Génération harmonique [Harmonic Generation]*. Effectively, the first harmonic sounds that he heard (1, 2, 3, 4, 5, 6) appeared to him to contain exactly the perfect major chord: *do*, *do*, *sol*, *do*, *mi*, *sol*. "Prenez une viole, ou un Violoncello dont vous accorderez deux cordes à la Douzième l'une de l'autre; raclez la grave, vous verrez frémir l'aigüe, vous l'entendrez peut-être même résonner, …; raclez ensuite l'aigüe, vous verrez non seulement la grave frémir dans sa totalité, vous la verrez encore se diviser en trois parties égales, formant trois ventres de vibrations entre deux noeuds, ou points fixes." ["Take a viola [*da gamba*?] or a violoncello, of which you will tune two strings one to the twelfth of the other; scratch the deeper one, and you will see the higher one vibrate, and maybe you will even hear it resound, …; then scratch the higher one, and you will not only see the deeper one vibrate in all its length, you will see it divide up into three equal parts, forming three loops of vibration between two nodes, or fixed points."][14]

But, unfortunately for him, he was hardly to enjoy any success with the minor chord, which, in spite of various efforts of calculation, could not enjoy a similar simplicity in its justification. In the end, not the least step of all, also Rameau was to end up by accepting and proposing (reluctantly?) his own approximate equable temperament, to make it possible for keyboards to play the whole variety of tonal combinations admitted by his theory. "Origine du Tempérament, sa theorie & sa pratique. … L'oreille ne suit pas servilement le Tempérament des Instrumens elle a le sien particulier; ces Instrumens servent seulement à la mettre sur les voies des sons fondamentaux, par eux-même, ou par quelques-uns de leurs sons Harmoniques, d'où prenant l'essor, elle tempére, sans réflexion, par sa Justesse, tout ce qui peut s'opposer aux justes rapports de ces sons fondamentaux; …". ["Origin of the temperament, its theory and practice. … The ear does not follow the temperament of instruments in a servile manner, but has its own; these instruments serve only to put it on the right road of fundamental sounds, by means of the sounds themselves or the harmonic sounds, from which, springing forward, it tempers everything that can be in opposition to the just ratios of these fundamental sounds, without minding its justness; …"].

Rameau admitted "… que le Tempérament est naturel, ou du moins necessaire, … L'altération des Quintes est si petite dans ce Tempérament, qu'à peine l'oreille s'en apperçoit, …". ["… that the temperament is natural, or at least necessary, … The alteration of the fifths is so small in this temperament that the ear hardly

[13] Rameau 1722/1992, pp. 323 and 397.

[14] Rameau 1737, pp. 7–9. Chailley 1986, pp. 219–220.

notices it, …"]. He calculated the *comma* of Pythagoras by means of the ancient characteristic cross : "…524288 531441 …Si l'Harmonie en souffre, l'Oreille ne sçait-elle pas en sous-entendre la perfection dans chacun des Sons fondamentaux?" ["…If the harmony suffers, can the ear not imply the perfection in each fundamental sound?"] He did not extract any roots to calculate the exact temperament. Rather, he considered the "Pratique du Tempérament sur le Clavecin ou sur l'Orgue." ["Practice of the temperament on the harpsichord and on the organ."][15]

Not satisfied with the celebrity that his *opéra-ballets* and their relative controversies aroused in Paris, in 1750 our musician presented a memorandum to the Académie des Sciences: "Démonstration du principe de l'harmonie, servant de base à tout l'art musical théorique et pratique" ["Demonstration of the principle of harmony, which serves as the basis for all theoretical and practical musical art"]. He had been "Eclairé par la *Méthode* de Descartes que j'avais heureusement lue, et dont j'avais été frappé, …" ["Illuminated by the *Method* of Descartes, which I had fortunately read, and by which I had been impressed, …"]. He adopted the symbolic language of mathematicians, "…pour qu'ils ne puissent rien révoquer en doute." ["…so that they may not put anything in doubt."] and explained ♯ and ♭ to them, also in the sense of "major" and "minor".

He obtained from the resonating "corps sonore" ["sounding body"] *ut* [*do*] (1) − *sol* ($\frac{1}{3}$) − *mi* ($\frac{1}{5}$) and at the higher octaves *ut* [*do*] ($\frac{1}{4}$) − *mi* ($\frac{1}{5}$) − *sol* ($\frac{1}{6}$), thus the ratios of the major and minor thirds. The original sounding body "…force les derniers à se diviser dans toutes les parties qui en font l'unisson; de sorte qu'en ce cas, il a sur ses multiples même puissance que sur ses sous-multiples. Ces expériences sont également sensibles à l'oreille, à l'oeil, et au tact." ["…forces the latter [other sounding bodies in those particular ratios for resonance] to divide themselves into all those parts which make up unison. With the result that, in this case, it exerts over its multiples that same power that it has over its sub-multiples. These experiences can likewise be perceived by the hearing, the sight and the touch."] And yet, although he succeeded brilliantly in providing a new physical basis for harmony, Rameau did not abandon at all those well-known numerical ratios distributed in harmonic, arithmetic and geometrical successions. And thus, for the moment, also the notorious *commas* of the Greeks continued to appear here.[16]

Shortly before in 1748, he continued, he had put his harmonic laws into practice in the one-act ballet, *Pygmalion*. Every now and then, he invoked nature to justify his own rules. "C'est donc une lois dictée par la nature même qu'on ne peut monter diatoniquement au générateur d'un mode qu'à la faveur de sa note sensible; …" ["It is therefore a law dictated by nature itself that it is not possible to ascend diatonically to the note that generates a mode [tonality] except with the favour of its sensible note;" that is to say, passing from B to C, if we were in C major.] Our French musician noted "…que l'oreille penche toujours du côté des sous-multiples, dont la résonance, causée par celle du corps sonore, l'emporte sur le simple frémissement

[15]Rameau 1737, pp. 14 and 75–104.

[16]Rameau 1980, pp. 66, 70–72 and passim.

des multiples." ["...that the ear always tends towards the side of the sub-multiples [contained in the length of the vibrating string]; thus the resonance, caused by that of the sounding body, exceeds the simple quiver of the multiples."][17] Like the phantom limb which still hurts, even if it has already been amputated, Rameau's brain, with his senses involved here, thought that he could perceive the vibrations of the imaginary multiples of the vibrating string (that is to say, the sub-multiples of the basic frequency).

He reserved a separate section for the minor mode. Bur in this case, compared with the sure resonances of the major mode, Rameau could only indicate a "...simple frémissement ..." ["...simple trembling ..."]. And, luckily for us, on this weak [inexistent?] thrill, he started to construct the art which could never be forgotten by him. In this case, nature only gave few [no?] indications and the only way to proceed would be "...qu'en s'écartant des premières lois de la nature." ["...by moving away from the first laws of nature."] The sub-multiples of the basic frequency, acccording to him, would generate from C the fifth below, in other words F, together with the major third below, that is to say A flat. Thus the notes F, A flat and C would be obtained, which divide the fifth into a minor third and a major third, thus forming the minor mode.

Art desired variety, without worrying about harmonics, and thus our musician, starting from A constructed the scale of A minor. "Le mode majeur, ce premier jet de la nature, a une force, un brillant, si j'osais dire, une virilité, qui l'emportent sur le mineur, ...Le mineur, ..., existant moins par la seul et simple nature, reçoit de l'arte dont il est en partie formé, une fablesse qui caractérise son émanation et sa subordination, ..." ["The major mode, this first bud of nature, possesses a force, a splendour, if I may say so, a virility which make it prevail over the minor, ...The minor, ..., existing less thanks simply only to nature, receives from art, by which it is partly formed, a weakness which characterises its emanation and its subordination, ..."]. But here, Rameau did not dare to write that the minor mode was feminine, even though the Cartesian dualism continues to make itself clearly felt, undoubtedly better than the sub-multiple harmonics.

For the enharmonic genre with quarters of a tone, he referred to his lyric tragedy of 1739, *Dardanus*, and to that of 1733, *Hippolyte et Aricie*. He had tried, but without success, also in his most famous *opéra-ballet* of 1735, *Les Indes galantes [The Gallant Indies]*. In *Castor et Pollux*, a lyric tragedy of 1737, the chromatic effects of changing tonality at every moment could be heard. Then, he rejected the enharmonic effects because they would led to excesses of passions, they were frightening and they brought disorder, distracting the ear.[18]

Finally, "De la seule résonance du corps sonore, vous venez de voir naître l'harmonie, la basse fondamentale, le mode, ..., même la nécessité d'un tempérament, ...il importe peu à l'oreille que ces tons, quarts de tons, et demi-tons soient dans leur juste proportions. On s'en est toujours douté, mais la raison en serait

[17]Rameau 1980, pp. 80, 82 and passim.

[18]Rameau 1980, pp. 89, 95 and passim.

encore inconnue, …" ["From the resonance alone of the sounding body, you have seen the birth of harmony, the fundamental bass, the modes, …, the very necessity of a temperament, …it matters little to the ear if these tones, quarter-tones and semitones are in their just ratio. We have always had doubts about it, but the reason is still unknown, …"]. The important thing for him remained to show respect in all cases for the order of harmony. Keyboard instruments were constructed with the semitones equal, or almost. Thanks to nature, our ears would be insensitive to the differences between the 12 semitones of the Zarlino-Rameau scales, which thus remained governed only by the fundamental sounds.[19]

Rameau caused the proportions of his numbers to be created with harmony "…ces premiers principes mathématiques trouvent eux-mêmes ici leur principe physique dans la nature. …l'organe, sans le secours de l'esprit, éprouve ici ce que l'esprit avait découvert sans l'entremise de l'organe." ["…these first mathematical principles find their physical principle here themselves in the nature. …the [auditory] organ, without the help of the spirit, experiences here what the spirit had discovered without the intrusion of the organ."] He offered as suitable for scholars guided by "…lumiéres métaphysique, qu'un phénomène où la nature justifie et fonde pleinement des principes abstraits." ["…metaphysical beacons, only one phenomenon where nature justfies and fully founds abstract principles."][20]

In the Académie des Sciences, Rameau's main interlocutor became for the moment Jean-Baptist le Rond d'Alembert (1717–1783), who was to mitigate his initial enthusiasm for our composer subsequently with various kinds of criticism, which we shall see in the following section. In the meantime, also the initial entries of the renowned *Encyclopédie*, edited by himself and Denis Diderot (1713–1784), were published. Among these, those dedicated to music were not in line with the ideas of Rameau, leading to the development of a controversy, which we shall also see below. We only anticipate here that his main adversary appeared Jean-Jacques Rousseau (1712–1778), the most musically inclined of the various collaborators in the project. Heated disputes were to gather around the comic lyrical opera *La serva padrona [The Bossy Maid]* by Giovanni Battista Pergolesi (1710–1736), which was presented in Paris by a company of Italian singers without wigs in 1752. The well-known related "Querelle des Bouffons" ["Jesters' Dispute"] saw the confrontation between the party of the French *Opéra* and that of the Italian lyrical opera, the party of harmony against that of melody, the party of theoretical reason compared with that of the spontaneous heart, the King of France even drifted away from his Queen. Over all this hovered, *cela va sans dire*, the spectre of Cartesian dualism.

But even amid these polemics, Rameau cannot have considered it sufficient to have successfully reached the highest French scientific assembly, and he even started corresponding with various foreign scholars. These also included an Italian musician and theoretician, like himself: the Franciscan father from Bologna, Giovanni Battista Martini (1706–1784), who had become an authority recognized all over Europe on

[19]Rameau 1980, pp. 87, 103–105 and passim.

[20]Rameau 1980, p. 88.

questions of music. Father Martini was to have had Italian translations made, both of Rameau's reply to Euler, as "Risposta di Mons. Rameau a M. Eulero sopra l'identità delle Ottave" [Reply of Mr. Rameau to Mr. Euler on the identity of Octaves], and of the exchange of open letters between the French composer and d'Alembert.[21]

Rameau wrote to the Father from Bologna in the following terms: "De quelques raisons que le géomètre se pare pour autoriser ses découvertes, elles ne paraîtront jamais que l'ouvrage d'un instinct dont la nature nous présente le germe dans le corps sonore: elle ne pouvait s'en expliquer qu'à l'oreille …". ["Whatever the reasons may be that the geometrician shows to authorise his discoveries, they will never be anything other than the work of an instinct, the germ of which nature offers us in the sounding body: it could only reveal itself to the ear …"]. For him, only music would be capable of disclosing true harmony: "… harmonie universelle, harmonie céleste, harmonie du corps humain, harmonie en peinture, en architecture, harmonie de gouvernement, … parvenir cependant à la justesse exacte rigoureuse et sensible qu'on trouve dans la musique, laquelle semble nous être donnée par la nature comme le type sensible de ce qui doit être en proportions, c'est-à-dire de toute perfection." ["… universal harmony, celestial harmony, harmony of the human body, harmony in painting, in architecture, harmony in government, … [through the ear it would have been possible] to arrive, however, at the exact precision, rigorous and sensible, that we find in music, which seems to have been given to us by nature as the sensible model of what should be present in the proportions, that is to say, every perfection."] With another interlocutor, later, he was to sustain that the principle of harmony was: "… inépuisable, et tient à la théologie comme à la géométrie et à la physique." ["… inexhaustible, and it refers to theology, as well as to geometry and to physics."][22]

If we did not want to hear echoes of an ancient Pythagoreanism again in these explicit sentences, albeit justified now by the physics of harmonic resonances, how can we avoid exercising our memory and considering it a continuation of the equally universal general programme of Marin Mersenne? But the eighteenth century had changed its look, and even if it had been carried forward successfully, that project would no longer have had suitable listeners. Anyway, Rameau had remained so convinced that he had revealed hidden laws of nature that he preferred to write to scholars of physics and mathematics like Leonhard Euler (1707–1783) or Johann II Bernoulli (1710–1790). Not one of the most famous members of the Bernoulli family, may the French musician have chosen this young son of Johann I Bernoulli (1667–1748) because he seemed curious about many different phenomena, like light, magnetism and navigation?[23]

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[21] Rameau 1969, v. V, pp. xxxiv and xliii–xliv.

[22] Rameau 1980, pp. 30–32.

[23] At the time, Johann II enjoyed success, and won four prizes at the Académie des Sciences. Fleckenstein 1981, p. 56. It will be seen below that his best interlocutor in the family would have been Daniel Bernoulli, a scholar of harmonics.

But in spite of three letters, our Frenchman does not seem to have found any willingness on the part of the Swiss scholar. Although they had taken a stand with Leibniz, was the powerful and numerous Bernoulli family little interested in music? By means of his theory, our French composer claimed that he would make Johann Bernoulli junior "... autant musicien que je suis géomètre pour peu que vous vous livriez à la matière. ... Ce principe est en même temps le principe physique des proportions, ce qui prouve bien que rien ne plaît à nos sens qu'autant qu'il s'y trouve des proportions, ..." ["... as truly a musician as I am a geometrician, however little you dedicate yourself to the subject. ... This principle is at the same time the physical principle of proportions, which clearly demonstrates that nothing is pleasant to our senses until its proportions are discovered, ...".][24]

However, beyond the curious comparison between Rameau, the geometrician, and Bernoulli, the musician, which might have made the Swiss scholar smile (or get angry?), the European scientific community had already considerably enlarged, and superseded the mathematical model of proportions. In order to study natural phenomena, that more powerful instrument called infinitesimal calculus had been invented. Therefore, the approval sought by Rameau half-way around Europe did not arrive. Actually those ancient proportions of his no longer aroused much interest, at least among the new physicists and mathematicians who were followers of Leibniz and Newton, with the next exception.

Sure enough, things went a bit better with Euler for the composer, because the latter, in his manifold and plentiful scientific production, had truly already dealt with music, since the beginning of his studies. In this sense, we can see immediately, he had written to Johann I Bernoulli. Thus, when Rameau were to present his harmonic theory to him, as well, Euler would have reacted by creating a confrontation between the two. Let us read, then, what Euler had written in his letter sent from Petersburg to Johann Bernoulli senior on May 25th, (the Catholic June 5th), 1731.

> I have delayed writing to Your Excellency so long and assuring you of my due respect, because I have not worked for some time on anything that I have considered worthy of being transmitted to you. For almost all this time, I have dedicated myself to constructing a *Systematis Musici*, for which I have now almost arrived at the end. Thus I take the liberty of sending to Your Excellency what I think I have achieved; for I imagine that You yourself may be curious about it, having recently heard from Your son [Daniel (1700–1782)] that you have heard something about it from Mr. Hermann [Jakob (1678–1733)]. My aim in this work was to try to develop music as a part of mathematics ["die Musick als einen Theil der Mathematik auszuführen"] and to derive with order, from the correct foundations, ["richtigen Gründen"] everything that a combination and a mixture of ["Zusammenfügung und Vermischung"] of notes can render pleasing. For the whole discussion, I have needed a metaphysical foundation ["metaphysischen Grundsatz"], in which to contain the cause ["Ursach"] why we can take so much delight ["Wohlgefallen"] from one piece of music, and to set out the reason ["Grund"] why one thing proves to be pleasant ["angenehm"] to us, but another is unpleasant. By means of the notes ["Thönen"], I have demonstrated that the following situation is created: we like many notes combined if we recognize their ratio of height and depth ["die Verhältnuß der Höhe und Tiefe"], to be precise: the *rationem*

[24] Rameau 1980, pp. 113–115. Rameau 1752a. Cohen 2001. Lester 2004.

intervallorum pulsuum [the ratio of the strokes of the intervals] that the strings ["Saiten"] produce. And I have thus also derived some rules ["Regeln"], about how the notes should be put together so that an intelligent ear ["verständiges Ohr"] can draw pleasure from them ["Gelustigen"].

But there is yet another reason ["Grund"] why we feel pleasure ["ein Lust"] with one piece of music, which consists of the duration of the notes. Thus one piece of music is pleasant to us, partly, simply because we apprehend the relationship in the duration of the notes ["die Verhältnuß der Daur der Thönen"]. According to the first *Principio*[25] [principle], it is thus the height and the different depth of the notes ["die verschiedene Höhe und Tiefe der Thönen"] that delights us ["belustigen"], but according to the second one, it is their different duration. For a perfect piece of music, ["vollkommenen Musick"], both things should be found together; thus we appreciate it both *ratione acuminis et gravitatis* and *ratione durationе sonorum* [for the ratio between the acute and the deep, and for the ratio in the duration of the sounds]. The Choral ["Choral"] is a kind of music which we appreciate only in the first way, in which there is only the harmony of the voices [" Zusammenstimmung der Stimmen"] and no attention is paid to the duration. But an *Exempel* [sample] of music which can arouse pleasure only in the other way ["nach dem andren Grundsatz ein Gefallen"] is offered by the sound of drums ["Trommelspiel"] in which we look only at the duration and not the height of the notes. First I have described music in accordance with the first *Principio*, and only afterwards in accordance with the second one, and at the end, I shall take the two together.

In the first part, which is without doubt the more important [? "fürnehmste"], the following pieces were taken into consideration. Firstly, the *Accorten* [chords], where I demonstrated how two or more notes have to be employed in such a way as to create a pleasing *Harmonie* when they play together. Secondly, I examined how two chords should be employed, in such a way that, played one after the other, they sound comfortable ["annehmlich klingen"]. Thirdly, I considered an entire succession of chords, and it is demonstrated what is required for their pleasantness ["Annehmlichkeit"]. The limitations in which such a succession is found constitute the *Modos* which I have described subsequently. I have also demonstrated which notes are needed on instruments for every *Modum*, and consequently, I have found very beautiful the *genus chromaticum*, with 12 notes in an octave ["Octav"]. In the following treatment, I considered everything in accordance with the *Systemate chromatico*. After this, I carried out something more particular than the above, and I no longer applied myself to *Praxin* [practice]. Seeing that I have thus performed a *perfectam enumerationem modorum* [perfect numbering of the modes], it can thus be seen that we could go much further on in music, considering that out of all these many *Modes*, only two are now in fashion ["nur 2 im Schwange sind"]. These are included in the music that is based on the *Genus chromaticum*. Furthermore, if we wished to use others, over and above the 12 notes, as we find in an octave ["Octav"] or in their place, then we could obtain an infinity of different types of music ["unendlich viel verschiedene Arten Music"].

Thus I found all this from the first *Principio*. Let there be many different notes, whose *numeri pulsuum* [numbers of strokes] concluded at the same time stand to each other as whole numbers ["ganze Zahlen"] a, b, c, d etc., through which also the same notes are usually expressed. Let A be the *minimus communis dividuus* [lowest common multiple]. I call this number the *Exponentem* [exponent] of the same notes, because it is from this that we recognize the gracefulness ["Anmuth"] that is produced ["verursachet"] when these notes are played, either simultaneously, or one after another. I have also established a *gradus suavitatis* [degree of sweetness] the first of which includes the most perfect chord, that is to say, when all the notes are relatively equal. The following include the least perfect according to the order among them. From the *Exponent* the *gradus suavitatis* can be recognized in the

[25] In Italian in the text.

following way. I resolve it into its *factores simplicissimos* [most simple factors], I add these in a sum, and from this I subtract $n - 1$ (n indicates the *numerum factorum* [number of factors]): the number thus obtained gives the *gradum* [degree]. For example, let the notes be 1, 2, 3, 4, 6, then their *Exponent* will be 12. The *factores simplicissimi* of this are the following three: 2.2.3, the sum of which minus 2 is 5, from which it is clear that the harmony of these notes will be pleasant to the 5th level. In this way, we can find the *Exponent* of an entire piece of music, when all the notes are expressed by means of whole numbers, and we take the *minimum commune dividuum*. But this *Exponent* is nothing other than the *modus musicus* of the same piece. As all the notes are present on the keyboard ["Clavier"], in any piece, the *Exponent* is $2^n.3^3.5^2$, where n depends on the number of octaves. According to this, in one octave there must be the following 12 notes:

$$480, 512, 540, 576, 600, 620, 675, 720, 768, 800, 864, 900$$

the thirteenth note is one octave above the first one, that is to say, 960.
This is what I desired to communicate to Your Excellency regarding my *Musica Theoretica*.
...

On August 11th, (the non-Catholic July 31st?), Johann I Bernoulli wrote his reply to the letter that had been transmitted to him by the father of Euler, who was a Protestant pastor in Basle.

I was very glad to learn that you are working on the completion of a *Systematis Musici* (which must have been almost finished now). I do not doubt that will be born a fine work, which will adequately reveal the excellent *ingenium* [brain] of its *Author*. I can well imagine that it will be unlikely a similar *opus* will be produced, in which everything is drawn from the mathematical foundations, seeing that few *Scriptores Musici* [writers of music] exist, not to say none, who are gifted with such great, and unlimited mathematical science as yourself the professor. I therefore greatly look forward to seeing also your work. It would certainly not be easy to imagine what that foundation ["Grundsatz"] consists of, which must be so much metaphysical ["metaphysisch"], as You say, by means of which you succeed in giving the cause ["Ursach"] of the reason why a piece of music can bring pleasure ["Wohlgefallen"], and one thing proves to be pleasant ["angenehm"] to us, but is unpleasant to another. Of course, we have a general idea of harmony, which is charming ["lieblich"] when it is well founded, and consonances are well handled. Then, as it is known, in music also dissonances have their use, with the result that the gracefulness ["Lieblichkeit"] of the consonances that follow them is all the more enhanced, in accordance with the common proverb *opposita juxta se posita magis elucescunt* [opposites set next to each other stand out even more]. Thus it is also connected with shading in the art of painting, which is supposed to make the light stand out. This depends, I believe, in *musica practica* [practical music] above all on the type and the modification ["modification", modulation?] to which we are accustomed. This type depends largely on the nature ["Naturel"] and the temperament ["Temperament"] of people; as a result, some consider one thing to be sweet and pleasing ["süeß und angenhem"] and others feel sweet this different thing. Thus, the kind of Italian music is discordant ["discrepant"] with French music, and this with English music, etc. In a word, *de gustibus non est disputandum* [there is no arguing about tastes].
If we want to base the pleasantness ["Lieblichkeit"] of a piece of music on nature ["Natur"] itself, then we must define clearly what we mean by pleasantness ["Lieblichkeit"], and not say that this or that is pleasant ["lieblich"] because I really like it ["wohl gefället"], seeing that another person may not like ["mißfallen"] this same thing. For example, Midas preferred the making din in music ["schnader Music"] of Pan to the lyre-playing of Apollo. You, the professor, may say: we might judge the pleasure ["wohlgefallen"] or the unpleasantness ["mißgefallen"] of four notes played together, if we examined the ratio of the notes with height and depth, or to be precise, the *rationem intervallorum pulsuum*

> [ratio of the strokes of the intervals] given by the strings ["Säiten"]. From this, you have derived rules ["Regeln"], in accordance with which the notes should be combined, so that an intelligent ear ["verständiges Ohr"] can enjoy it ["belustigen"]. In my opinion, this is valid for a master ["Meister"] who considers more the accuracy ["Accuratesse"] of a piece of music than its effect, which is all right for the listener. A similar person will be delighted and will take pleasure ["ergötzen und belustigen"] without any doubt, when you have only written it down on paper, and he examines it, finding that it is composed well, in line with the fundamental rules ["nach den Grundreglen wohl componirte ist"]. But as a piece of music is, for the most part, played in front of ears that do not understand, which cannot recognize the *rationem intervallorum pulsuum* of the strings, and much less count, then, I believe, these ears will like or reject the same piece of music in accordance with whether they are accustomed to this or that genre of music. For the rest, I like your design ["Dessein"] very much, because at least, with this, the *Theoria Musices* is improved, and it is demonstrated that a *Mathematicus* is capable of discussing almost all the sciences ["Wüßenschafften"]. On the contrary, other masters ["Meister"] who are only *Practici* [practicians] write about their own art in the same way as someone who is blind about colours ["wie ein Blinder von der Farb"]. When this *Tractatus Musices* will be finished, you will undoubtedly seriously take up your planned *Mechanicam* (about which somebody wrote to me), from which I expect something special, ...

Euler's *Tractatus Musices* was to change its title to *Tentamen novae theoriae musicae*, and was only to appear in 1739, after the *Mechanica*, first published in 1736.[26]

In this reply to the young professor of mathematics who had moved to Russia, we notice that the topic offered in return by Johann father was in the style used by Leibniz to justify dissonances. But the "pleasantness" of listening to music was considered to be subjective and changeable with the "nature and temperament of people", just as Italian music could be distinguished from French or English music, whereas German music was not mentioned by name. How, then, would it have been possible to create a theory of it by means of the mathematical sciences of nature, which were clearly considered universal and stable? And yet Euler, who was destined to become the mathematician and physicist who was to set the tone A for the following centuries in his disciplines, was searching for the *Principio* of music in the physical and mathematical nature of sound. He used this term taken from the Italian language, which was much more popular among musicians than among his habitual interlocutors.

Unlike Rameau, our Swiss mathematician who had emigrated to St. Petersburg searched for his rules, by pursuing the ancient theory of strokes coinciding in the ear, with the *rationem intervallorum pulsuum*. Instead, he ignored the recent novelties brought to the attention of scholars by the higher harmonics of sounds. It is particularly curious that in him, novelties for physical and mathematical sciences co-existed side by side with a similar archaic neo-Pythagorean concept of consonances. Besides, the results of his research were still presented in Latin, under the complete title *Tentamen novae theoriae musicae ex certissimis harmoniae principiis dilucide*

[26] Euler 1731, pp. 146–150.

expositae [Essay On a New Musical Theory Expounded Clearly In Accordance With Certain Principles of Harmony.][27]

Some preliminary drafts of the long text are extant, which go back to 1726. We can see from these how often Euler dealt with music. He even considered actual compositions, such as dances of his period, called "Sarraband", "Menuete" and "Allemands", or fugues. However, he did not indicate notes on the stave, but by means of whole numbers, counting from 1 to 7, both on the scale of C major "Dur" and on that of C minor "Mol", and consequently with E flat, A flat, and B flat. From the top downwards, the four parts corresponded to the soprano, contralto, tenor and bass. The octave was not specified (Fig. 11.1).[28]

> Cum musicam nobis propositum sit ad modum philosophicarum disciplinarum pertractare, in quibus nihil, nisi cuius cognitio et veritas ex praecedentibus explicari possit, proferre licet, ante omnia est exponenda doctrina de sonis et auditu, ... quarum rerum sufficiens cognitio ex Physica et Mathesi est petenda. [As our project is to deal with music as we do with philosophical disciplines, where you are not allowed to propose anything the knowledge and the truth of which cannot be demonstrated on the basis of what goes before, we must first of all expound the doctrine of sound and of hearing, ... for a sufficient knowledge of which we must search in the fields of physics and mathematics.][29]

While Pythagoras had already long before identified ratios as the causes of the effect exerted by sounds on the ears, according to Euler, he showed too much confidence in it. Thus Aristoxenus had some reasons to criticise him. But the real causes could not lie only in the ear. Besides, they must exist, because in his opinion the ratios of the octave, the fifth, the fourth and the third are found to be pleasing in all nations. In his *Tentamen*, he would have identified some of them "... quodsi ergo theoria cum experientia consentiat, ..." ["... if theory then agrees with experience ..."]. It was the task of physics to provide an exact knowledge of sounds, but to judge the pleasure or displeasure, metaphysics needed. The species had to be of this kind, in the case of the criterion "... secundum simplicitatem ..." ["... in accordance with simplicity ..."], to classify the "... gradus suavitatis ..." ["... degrees of sweetness ..."], these, too, "... maxime ..." ["... particularly ..."] in agreement with experience.

In the wake of Ptolemy, Euler thus would have reconstructed, among other things, also the "... diatonico-chromaticum ..." ["... diatonic-chromatic ..."] genre in fashion during his period, putting "... in una octava duodecim sonos aequalibus fere intervallis a se invicem distantes, hemitoniis scilicet et limmatis sive maioribus sive minoribus." ["... in a single octave twelve sounds separated from one another by intervals that were almost equal, which are semitones or major or minor *limmas*."][30]

Notwithstanding, he considered sound to be the vibration of a body transmitted into the air, and from there to the ears: "Est igitur sonus nihil aliud nisi perceptio

[27] Euler 1739.

[28] Euler 1926, pp. vii–xx.

[29] Euler 1739, p. 207; cf. 1865, p. 1.

[30] Euler 1739, pp. 200–203; cf. 1865, pp. i–vi.

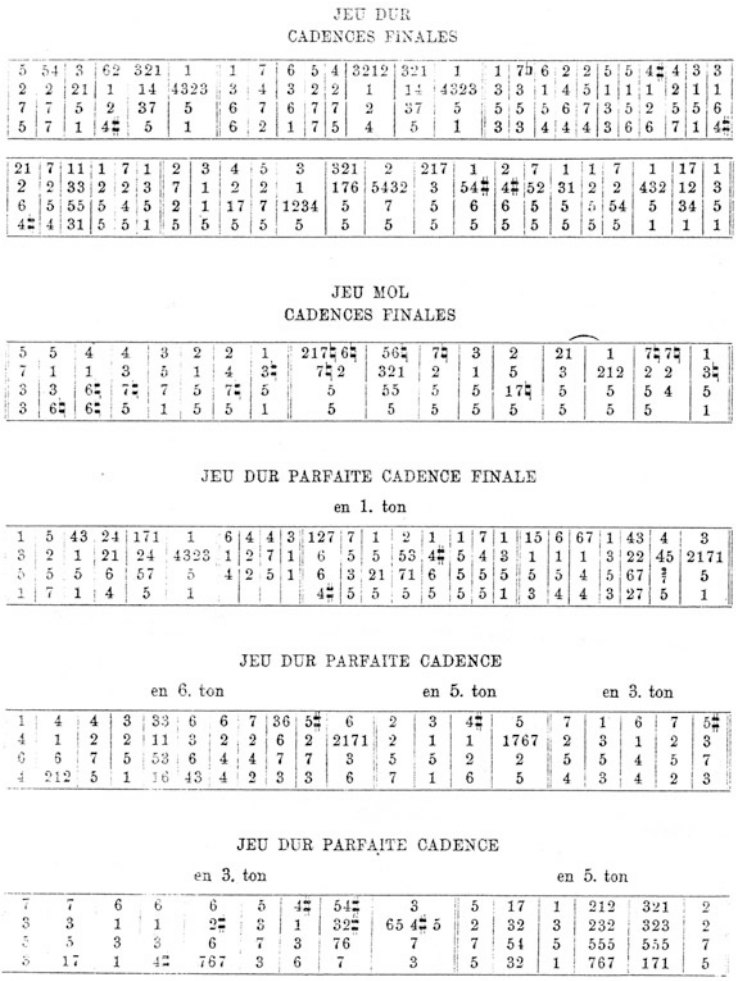

Fig. 11.1 How Euler wrote music, using whole numbers (Euler, "Cadences", 1926, p. xii)

ictuum successivorum, ..." ["Thus sound is nothing other than the perception of successive strokes, ..."][31] The term 'wave' was not used in the treatise. From the appropriate already-known formula, Euler obtained that, for a certain length and a given weight to stretch it, a string vibrated 392 times a second: "...in instrumento clavem signatam *a*." ["...on the instrument, the note indicated *a* [the note A]"]. It was therefore a much lower A than the present-day one, which is fixed at 440 Hz. For wind instruments, our Swiss physicist and mathematician supposed that their

[31] Euler 1739, pp. 208–209; cf. 1865, p. 3.

sound depended only on the length of the pipe. He knew about the phenomenon of higher harmonics, which he quoted through the studies published by Sauveur in 1701.[32]

Euler made the pleasure of consonances and of music depend on our capacity to understand their order. The more simple these were, therefore, the better they would be understood and appreciated. From this presupposition, he derived his laws of harmony. For him, disorder could only prove to be incomprehensible and repugnant. He represented the effect of sounds in the ear by means of lines of dots at different intervals, which could then be compared together (Fig. 11.2).

Hence from that, he obtained a visual classification of consonances which should explain the scale of liking. If strokes (which he now called vibrations) always would coincide in unison 1:1, and would coincide alternately in the octave 2:1 one stroke every two, with the other ratios they could still coincide, but increasingly rarely, one every three, one every four strokes and so on. For his levels of liking, he excluded fractional values, "...quia in hoc casu ratio fit irrationalis et prorsus non perceptibilis. [...] Sed numeri hi rationem propositam constituentes debent esse rationales, integri et minimi. Horum quidem primum facile intelligitur, cum in irrationalibus nullus huiusmodi insit ordo." ["...because in this case the ratio becomes irrational and is consequently not recognizable. [...] But these numbers which constitute the ratios proposed must be rational, whole and as small as possible. It is easy to understand the first of these conditions, because no order of the kind considered here exists in irrational numbers."][33] Having excluded irrational ratios from the *continuum* of sounds, the others were broken down into prime factors and their lowest common multiple, also known as their exponent, was calculated, in order to establish how many strokes coincided, and thus, finally, what their index of liking were. For example, 3:2 gave 6 and thus it was included in the IV degree; instead, 5:7 gave 35, and descended to the XI level.

Having defined music, "...eam esse scientiam dicam varios sonos ita coniungendi, ut auditui gratam exhibeant harmoniam." ["...I will say that it is the science of combining various sounds so as to obtain a harmony that is pleasing to the hearing."], Euler classified diligently and completely the chords, also reduced to whole numbers. The exponent of a chord was calculated as the lowest common multiple of the simple sounds that composed it. His theory should have led "...ad musicam ipsa praxi tractandam." ["...to the music to be treated as in the same practice."]. Below level VIII, however, there were both the dissonance of the tone, 9:8, and the consonances, 5:6 and 5:8, of the minor third and the minor sixth. The fourth, 4:3, remained a consonance, but the mathematician believed that also the ancient musicians had considered it to be such, without problems. Curiously, he sustained that the school of Aristoxenus had not departed very far away from

[32]Euler 1739, pp. 210–211 and 221; cf. 1865, pp. 5–6, 14–16 and 19.

[33]Euler 1759, pp. 217–219, 230 and 235; cf. 1865, pp. 30 and 34.

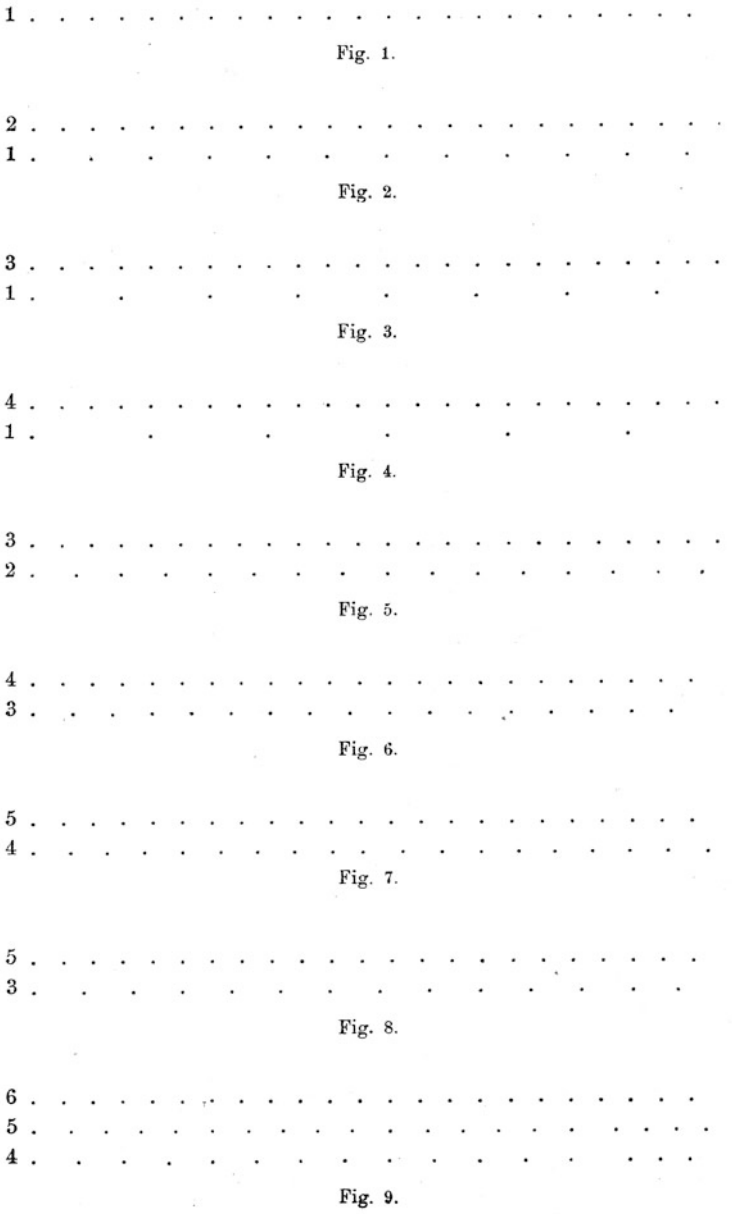

Fig. 11.2 How Euler represented the different coincidence of strokes in the ear, due to numeric ratios (Euler, 1926, p. 231)

the opinion of the supporters of Pythagoras, although this school had preferred to trust its senses rather than numbers.[34]

Euler measured the interval between two notes by means of the logarithm of the relative fraction. For equal sounds, he obtained $log\frac{a}{a} = loga - loga = 0$; for the octave $log\frac{2}{1} = log2 - log1 = log2 - 0 = 0,3010300$. Using logarithms, he could easily have divided every interval into equal parts, but he ignored (of course?) the equable temperament.[35] Gradually, all musical composition was reduced to its numerical "exponents". "Qui ergo haec omnia perspicit, is demum opus musicum perfecte cognoscit de eoque recte iudicare potest." ["He who considers all these things in depth will only then know a work of music perfectly, and can judge it correctly."]

But despite his initial appeal to simplicity, the young Swiss mathematician understood very well that his theory was bound to sound too complicated. It would be "... summopere difficilis, imo etiam vires humani intellectus longe superans videatur propter exponentem totius operis musici tam compositum numerum, ut animo comprehendi omnino nequeat." ["... extremely difficult, indeed it seems to exceed the forces of the human spirit by far, because the exponent of a whole musical work is such a compound number that it cannot be completely understood by the mind."]. Consequently, he suggested appreciating it by degrees so that "... sublevatur intellectus ..." ["... the intellect is to be relieved ..."].[36]

Starting from the Pythagorean sects, it was destiny that every initial numerical simplicity of this tradition should founder in unbearable and fundamentally useless complications. It would appear that even the brilliant Euler fell into this trap. However, as a partial excuse for him, he was undoubtedly more interested in calculating and playing with prime numbers, and with their lowest common multiples, than in enjoying the charms of music. He was amazed, however, that practical musicians should dedicate themselves to composing new works "... modorum autem numerum, qui satis est parvus et a longo abhinc tempore iam receptus, augere omnino non curent." ["... and do not care at all, instead, about increasing the number of modes, which is quite limited, and was already obtained a long time ago."][37] The criticism sounds curious, because it was advanced in a period when the numerous Greek-ecclesiastic modes had just been simplified into the two major and minor modes, to the reciprocal satisfaction both of composers and of listeners.

To cut a long story short, with the exponent $2^m \times 3^3 \times 5^2$, Euler obtained the classic, widely-used diatonic-chromatic genre, with its division of the octave into 12 intervals, as he had written at the bottom of the letter to Johann I Bernoulli, translated above. Using these, our mathematician who had now emigrated to Russia was to indulge himself in 'composing' music, combining 2, 3, 5 variously in numerous tables re-translated into alphabetic symbols for the notes.

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[34]Euler 1759, pp. 237 and 249–252; cf. 1865, pp. 36 and 48–51.

[35]Euler 1759, pp. 258–260 and 281–288; cf. 1865, pp. 58–59 and 78–85.

[36]Euler 1759, p. 276; cf. 1865, p. 73.

[37]Euler 1759, p. 280; cf. 1865, p. 77.

Our Swiss mathematician knew that theoreticians divided the octave in different ways. He referred to the one proposed by Johann Mattheson (1681–1764, famous a duel he fought with Händel), which had been reprinted shortly before in 1731 as *Generalbaßschule [School of the Continuous Bass]*. Musicians had for some years been using a division of theirs; "... dubitandum non est, quin experientia deprehenderint hunc modum ad harmoniam producendam magis esse idoneum." ["... it is doubtless that by experience they have found this mode is most suitable for creating harmony."][38]

For his part, he continued to calculate the powers of 2, 3 and 5 to obtain other scales, which were proposed in numerous tables and made far more complicated by different intervals, which were not easy to transpose on the notes of the scale. "... plures Musici putaverint veram musicam potius in aequalitate intervallorum consistere quam in eorum simplicitate. Hi igitur, ut sibi magis quam harmoniae satisfacerent, non dubitaverunt intervallum diapason in duodecim partes aequales dissecare atque secundum hanc divisionem sonos 12 consuetos constituere." ["... many musicians would have thought that true music consisted more in the equality of intervals than in their simplicity. In order to satisfy more themselves than harmony, therefore these have not hesitated to divide the octave interval into twelve equal parts and to create the twelve habitual sounds in accordance with this division."] But Euler calculated the numerical differences between the equable temperament and his own scale, concluding that "... harmonia non parum turbatur. ... Denique ob nullam sonorum rationem rationalem praeter octavas hoc genus harmoniae maxime contrarium est censendum, etiamsi hebetiores aures discrepantiam vix percipiant." ["... the harmony is not a little disturbed. ... In the end, as it does not present any rational ratio between the sounds, except in the case of the octaves, this genre must be considered as supremely contrary to harmony, although the most obtuse ears can hardly perceive the differences."] Besides, according to him, musicians would have followed their imagination and found solutions for their problems by chance. Our Swiss mathematician also rejected the approximation of dividing the tone into two almost equal parts with the ratios 17:16 and 17:18, which was attributed to England; I wonder why.[39] Clearly, 17 cannot be divided either by 2, or by 3, or by 5.

Using the fifth power of 5, on the contrary, he divided the octave into 24 fairly equal intervals, for which he hypothesised that every key of the instrument corresponded to two sounds. At that time, he quoted the master of his masters Bernoulli: "...; adeo ut Leibnitii effatum omnino locum abeat, cum diceret in musica etiamnum ultra quinarium numerari non solere." ["...; to the point that Leibniz was quite right when he said: the custom in music was still not to count beyond 5."] Thus he believed that the introduction of the number 7 was contrary to harmony. However, he calculated also with the exponent $2^m \times 3^3 \times 5^2 \times 7$ another division of the octave into 24 intervals (Fig. 11.3). But on this point, we shall see

[38] Euler 1739, p. 307; cf. 1865, p. 101.

[39] Euler 1739, pp. 320–321; cf. 1865, pp. 112–114.

Generis exponens $2^m \cdot 3^3 \cdot 5^2 \cdot 7$.

Signa Sonorum	Soni	Logarithmi Sonorum	Intervalla	
F	2^{12}	12,00000	0,03617	512 : 525
Fs^*	$2^3 \cdot 3 \cdot 5^2 \cdot 7$	12,03617	0,04064	35 : 36
Fs	$2^5 \cdot 3^3 \cdot 5$	12,07681	0,05247	27 : 28
G^*	$2^7 \cdot 5 \cdot 7$	12,12928	0,04064	35 : 36
G	$2^9 \cdot 3^2$	12,16992	0,03618	512 : 525
Gs^*	$3^3 \cdot 5^2 \cdot 7$	12,20610	0,02272	63 : 64
Gs	$2^6 \cdot 3 \cdot 5^2$	12,22882	0,07039	20 : 21
A^*	$2^4 \cdot 3^2 \cdot 5 \cdot 7$	12,29921	0,02272	63 : 64
A	$2^{10} \cdot 5$	12,32193	0,07039	20 : 21
B^*	$2^8 \cdot 3 \cdot 7$	12,39232	0,00642	224 : 225
B	$2^3 \cdot 3^3 \cdot 5^2$	12,39874	0,05247	27 : 28
H^*	$2^5 \cdot 5^2 \cdot 7$	12,45121	0,04064	35 : 36
H	$2^7 \cdot 3^2 \cdot 5$	12,49185	0,07039	20 : 21
c^*	$2^5 \cdot 3^3 \cdot 7$	12,56224	0,02272	63 : 64
c	$2^{11} \cdot 3$	12,58496	0,03618	512 : 525
cs^*	$2^2 \cdot 3^2 \cdot 5^2 \cdot 7$	12,62114	0,02272	63 : 64
cs	$2^8 \cdot 5^2$	12,64386	0,07039	20 : 21
d^*	$2^6 \cdot 3 \cdot 5 \cdot 7$	12,71425	0,04064	35 : 36
d	$2^6 \cdot 3^3$	12,75489	0,05247	27 : 28
ds^*	$2^{10} \cdot 7$	12,80736	0,00642	224 : 225
ds	$2^5 \cdot 3^2 \cdot 5^2$	12,81378	0,07039	20 : 21
e^*	$2^9 \cdot 3^3 \cdot 5 \cdot 7$	12,88417	0,02272	63 : 64
e	$2^9 \cdot 3 \cdot 5$	12,90689	0,07039	20 : 21
f^*	$2^7 \cdot 3^2 \cdot 7$	12,97728	0,02272	63 : 64
f	2^{13}	13,00000		

Fig. 11.3 Euler divided the octave also into 24 parts (Euler 1739, p. 333; cf. 1865, p. 124; 1926, p. 333)

that 30 years later, he was to change his mind. Perhaps, in the meantime, someone like Christian Goldbach, to whom we have already seen that the letter of the German mathematician was addressed, might have explained to him that Leibniz was only joking when he spoke about the limited ability of musicians to count.[40]

"Quemadmodum igitur per compositionem ...exponens systematis auditui repraesentatur, ita, cum is iam satis fuerit perspectus, ad aliud systema transitus fieri debebit." ["Therefore, as the exponent of a system is presented to the ear by the

[40]Euler 1739, p. 332; cf. 1865, p. 123.

composition . . . then, as soon as he has recognized it sufficiently, it will be necessary to pass to another system."] In this way, music gained in variety.[41]

Euler appeared to be continuing the idea of Leibniz, in which the ear calculated by 2, by 3, or by 5, without realising it. If the exponents were all the right ones, then we would receive pleasure from this unconscious mathematics carried out with prime numbers. As in the case of the German metaphysician, the variety stemmed here from the infinite combinations of numbers, letters, or notes. And yet, from his numbers, Euler derived only 10 major tonalities and 6 minor ones, instead of the usual 24. The major modes were contained in the exponent $2^n \times 3^3 \times 5$, and the minor ones in $2^n \times 3^2 \times 5^2$. Nevertheless, the Swiss scholar who had emigrated to Russia, remained convinced that his theory was in line with compositing music in his period, and ". . . occasionem sit praebitura hanc scientiam ope verae theoriae etiamnum ignoratae ad maiorem perfectionis gradum evehendi." [". . . was going to offer the chance to elevate this science to a higher level of perfection, by the agency of the true theory, so far ignored."][42]

It was to such a mathematician and theoretician of music that the letter of the musician Rameau was delivered in 1752. In those years, Euler had moved from St. Petersburg to Berlin, where Frederick the Great of Prussia was revitalising the academy founded by Leibniz. This became the Académie Royale des Sciences et des Belles Lettres de Berlin, seeing that French was a language preferred by the king both to Latin and to German. The indefatigable Swiss scholar was to dedicate himself here to the most varied subjects: from State lotteries to demography, from cannons to navigation, from the movements of the Moon to hydrodynamics, from the theory of numbers to integral calculus and calculus of variations.[43]

From a similar character, our French composer solicited an answer to his ideas based on the ". . . principe Physico-mathématique . . . commune à tous les arts de goût, . . . , qui ont nos sens pour object et pour règle les proportions." [". . . physical-mathematical principle . . . common to all the arts of [good] taste, . . . , which have our senses as their object and proportions as their rule."] He had previously sent also to him the note on the demonstration for the principle of harmony submitted to the luminaries of Paris. Not having received any reaction, now he added some *Nouvelles réflexions sur le principe sonore [New Reflections On the Principle of Sound]*.

Here, Rameau had raised a series of questions. "Pourquoi n'est-ce que dans l'acoustique qu'on peut prendre une connaissance certaine des rapports? Les sciences n'en tireraient-elles pas leur origine, puisque le principe d'où naissent les moyens d'opérer, de découvrir et de démontrer, s'y trouve renfermé? [. . .] D'où . . . les sauvages ont-ils appris à chanter, eux qui n'ont aucune méthode, et qui chantent aussi juste que nous? [. . .] nous sommes passivement harmoniques; notre voix est un corps sonore, . . . " ["Why is it only in acoustics that a certain knowledge of ratios can be obtained? Don't the sciences find their origin there, seeing that it

[41] Euler 1739, p. 419; cf. 1865, p. 205.

[42] Euler 1739, pp. 421 and 427; cf. 1865, pp. 208 and 215.

[43] Youschkevitch 1970, pp. 470–471.

is there that we find the principle from which the means to operate, to discover and to demonstrate are derived? [...] Where did the savages learn how to sing, they who do not possess any method, but who sing as correctly as we do? [...] we are passively harmonic; our voice is a sounding body, ..."].[44]

This time, the busy Euler answered that he, too, had taken the principle of harmony as "...unique base de toute la Musique, et comme ce principe se reduit à la simplicité des rapports, je prétend que mon *Essai d'une Theorie de la Musique* est uniquement fondé sur ce même principe, et que les exposants auxquels j'ai reduit mes recherches, en sont une suite tres naturelle. ...je tombe aussi d'accord que la diversité d'octaves ne change pas la nature d'une consonance, quoiqu'elle en change l'agrément, ...Or c'est aussi ce même principe, que Pythagore a decouvert, et il me semble que les Aristoxeniens n'ont jamais nié ce principe, même, leur sentiment etoit seulement, qu'on ne devoit pas chercher la cause des consonances dans les nombres, et qu'il faloit se rapporter uniquement au jugement des oreilles. Cependant les Pythagoriciens se sont bien tôt égarés dans leur nombres, ...à cet egard les Aristoxeniens ont bien eu raison de se moquer de leur Theorie pretendue. ...je me flatte avoir eviter [evité?] ce defaut dans mon ouvrage, ...", he had taken that principle of harmony as the ["...only basis of all music, and as this principle is reduced to the simplicity of ratios, I pretend that my *Essay On a Theory For Music* should be founded only on this same principle, and that the exponents to which I have reduced my research should be a thoroughly natural consequence of it. [...] I also agree that the diversity of the octave does not change the nature of a consonance, though it may change the [degree of] pleasantness ...Now, it is this same principle that Pythagoras discovered, and it seems to me that the Aristoxenians have never denied this; although their opinion was not to look for the cause of consonances in numbers, and to make reference only to the judgement of the ears. However, the Pythagoreans soon got lost in their numbers. [...] on this point, the Aristoxenians were quite right to make fun of their presumed theory. ...I flatter myself that I have avoided this defect in my work, ..."].[45]

Rameau's answer was not long in arriving: *Extrait d'une réponse de M. Rameau à M. Euler sur l'identité des octaves [Extract of an Answer From Mr. Rameau To Mr. Euler On the Identity of Octaves].* The French musician used the harmonics of the sounding body, the ear and the human voice to maintain that all the octaves were identical. Actually the ear could hear the second harmonic 1:3, the twelfth, as if it were the fifth of the fundamental sound 2:3; that is to say, from the second octave 2:4, it automatically took it back to the first one 1:2. It is interesting that he wrote in a note: "Un son se multiplie dans le corps sonore, en autant d'Octaves qu'on veut au-dessus de lui, mais jamais au-dessous." ["Within the sounding body, a sound is multiplied in any octave above it, but never below it."] Was he now retracting his previous idea of the existence also of lower harmonics? With his calculations for the octaves, Euler was also considering the different multiples of 2, and for

[44]Rameau 1752b; Rameau 1980, pp. 116–120.

[45]Euler 1752.

them, he arrived at different degrees of liking. For Rameau, in transpositions and in inversions, the combinations of notes for chords were based on identical octaves. Only in particular musical contexts would it be possible to appreciate differences of octaves. In this connection, he quoted his own chorus "L'amour triomphe" ["Love triumphs"] from the ballet *Pygmalion* of 1748.

"Tel est l'empire de l'oreille qui, ...a toujours tenu dans l'esclavage les plus grands Philosophes, les plus grands Géomètres, & les plus grands Musiciens de pratique: ..." ["The dominion of the ear is such that it has always held in bondage the greatest philosophers, the leading geometricians and the greatest practical musicians: ..."]. Instead, "Depuis Pythagore jusqu'à Zarlino, ...tous ont également souscrit à la même Hypothèse, fondée sur des produits de produits: ...Aussi sourd que les anciens à la voix de la nature, quoique, d'accord avec Aristoxène, il eut bien recommandé de l'écouter, cet Auteur s'imagine, comme eux, pourvoir combiner à son gré ce même ordre qu'elle vient de lui dicter, & va jusqu'à en former douze *modes* sans ordre déterminé & sans bornes, ...À la fin l'oreille a pris le dessus, & le Musicien rebuté des préceptes que contrarioient à chaque instant les effets qu'il éprouvoit, ne s'en est presque plus rapporté qu'à son expérience: ...qu'il ne pouvoit y avoir que deux *modes*, le *majeur* & le *mineur*, ..." ["From Pythagoras to Zarlino, ...everybody endorsed the same hypothesis founded on some products of products [among numbers]: [...] As deaf as the ancients to the voice of nature, though in agreement with Aristoxenus they had undoubtedly been advised to listen, like them this author [Euler] imagines that he can combine at will the same order that she comes to dictate to him, and arrives at the formation of twelve modes without any determined order and without limits, ...[...] In the end, having refused precepts which contradicted at every instant the effects that it experienced, the ear prevailed, and the musician practically only trusted his experience ...that he could have only two modes, the *major* and the *minor*, ..."].[46]

For Rameau, probably, Vincenzio Galilei had already fallen into oblivion. If he had been aware of his writings, would he perhaps have appreciated them? Euler had not obtained from a musician like Rameau the approval that he expected for his *Essay*. Nor, vice versa, did the Frenchman appreciate the veiled reservations contained in the letter of the Swiss mathematician, who had remained faithful to the Greek ecclesiastic modes, and was only prepared to multiply them. Now, the Swiss scholar was enlisted in the Prussian Berlin, while the usual umpteenth war (called the Seven Years' War) was being fought between his old and new employers, with the Russians who had arrived as far as Berlin. Euler had undertaken the task of educating Princess Friederike Charlotte Leopoldine, the daughter of the margrave of Brandenburg: *Lettres à une princesse d'Allemagne sur divers sujets de Physique & de Philosophie [Letters To a German Princess About Different Subjects of Physics And Philosophy]*. Among these, lessons on music could not fail to be present.[47]

[46] Rameau 1753, pp. 10, 23, 28–30.

[47] Youschkevitch 1970, p. 471. Euler 1958.

The *Letters* enjoyed such wide-ranging success that they were published and translated in numerous editions all over Europe, while Euler, at variance with Frederick II over the running of the Académie in Berlin, went back to St. Petersburg, to the Czarina Katerina II. We can thus see that music had not been a youthful confusion to be forgotten for Euler, but a constant presence in his work, to be compared with the other numerous disciplines to which he had dedicated himself.

The wartime context emerged here and there from the prose of our scholar. The Omnipotent who governs the universe "... brings success to our [Prussian] army". He investigated the speed, not only of the wind and of sound (1,000 ft/s, about 313.85 m/s), but also of the shells fired by cannons (2,000 ft/s). The cannon shot became the phenomenon of sound. It was thought to be generated by the "shudders and vibrations" sustained by the parts of a body. Through the air, "... our ears receive the same number of strokes as the vibrations that the string has emitted ...". Music would be created when the strokes were regular. If a low C emitted 100 vibrations per second, an acute C four octaves higher will arrive at 1,600. The Swiss physicist and mathematician, for the moment still in Prussia, explained to his illustrious pupil the difference between consonances and dissonances, as in the *Tentamen*, attaching to the letter drawings of dots ranged one below the other. If the ear could easily evaluate the ratio between two sounds, as by counting the coincidences between the strokes-dots, there would be a consonance, otherwise in difficult or impossible cases (this happened with irrational ratios), dissonances would be obtained.

Almost 30 years had passed since the letter written to Johann I Bernoulli, but his idea of music remained the same: "... the principles of harmony are ultimately reduced to numbers, ...". These were, as we know, 2, 3 and 5. The equable temperament remained in contrast with the principles of harmony, although it was used by musicians in order to transpose melodies from one tonality to another. By also admitting the number 7, music would be brought "... to a higher level of perfection. But it is here that mathematics leaves harmony to music."

The only novelty compared with the past would seem to be the following one. "But the simple knowledge of all the ratios ... is not sufficient by itself to arouse the sense of pleasure; something else is needed, which nobody has succeeded in determining yet." Euler admitted that it was also necessary to capture "... the intentions and the sentiments of the composer."[48] As these were the first eight letters to the German princess, our physicist and mathematician thought that by means of music, he would initiate her better to the mysteries of these disciplines of his. The phenomenon of harmonics, so dear to Rameau, was ignored. The diversity of a flute from a horn (in the timbre) was observed, but it would be "... almost impossible to say what this quality consists of", as the different ways of pronouncing or singing with the voice the vowels a, e, i, o, u remained "hidden" to philosophers.[49]

To study how sound propagated in the air, Euler considered "... le vrai mouvement dont les particules de l'air sont ébranlées successivement ..." ["... the true

[48]Euler 1958, pp. 5, 10, 22–28.

[49]Cf. Euler 1862.

movement by which the particles of air are moved successively . . . "]. He criticised the model of Newton as insufficient; that of Joseph Louis de Lagrange (1736–1813) seemed indecipherable to him. He, instead, had found a more simple way which made it possible to deal with not only continuous quantities, which could be represented by means of "regular" curves, but also discontinuous quantities. However, we shall not follow him in this direction, which would lead us to a well-known physical-mathematical problem,[50] and we shall continue to read his writings on music. However, we shall have to return to it in part further on, when we will talk about D'Alembert.[51]

The chord of the seventh, G, B, D, F, corresponded on Euler's scale to the numbers 36, 45, 54, 64, with the exponent $2^6 \times 3^3 \times 5$, giving the lowest common multiple 8,640. This number would be thus not at all so simple as to justify its frequent use in music, for a relatively less serious dissonance. Thus the ear was not in agreement with the harmonic theory of the famous mathematician, seeing that this number would have classified it among the worst dissonances. Consequently, he advanced a conjecture to explain this. First of all, he rejected how D'Alembert had treated this dissonance in his *Elémens de musique théorique et pratique suivant les principes de Mr. Rameau [Elements of Theoretical And Practical Music, In Accordance With the Principles of Mr. Rameau]*, on which we shall dwell in greater detail in the next section. The French physicist and mathematician had explained it in purely musical and tonal terms, as a way of underlining the tonality of *C major*. If, instead, it had been the perfect chord of *G major*, G, B, D, F, the F of the seventh would have had to be F ♯.

As usual, Euler preferred to trust whole numbers: precisely those which did not bear good fruit here. Consequently, as a true heir of the ancient Pythagorean tradition, he created a gap between them and the ear. "...l'oreille sent une proportion bien différent de celle qui subsiste effectivement parmi les sons. Dans la température égale ... la quinte y est exprimée par la proportion irrationelle de 1 à $(\sqrt[12]{2})^7$, qui est un peu différent de celle de 2 à 3. Cependant, quoiqu'un instrument soit accordé selon cette règle, l'oreille n'est pas blessée par cette proportion irrationnelle et entendant l'intervalle C à G ne laisse pas d'appercevoir une quinte, ou la proportion de 2 à 3; ..." ["...the ear hears a completely different ratio from the one that actually exists between the sounds. In the equable temperament, ...the fifth is expressed by means of the the irrational ratio of $1:(\sqrt[12]{2})^7$, which is slightly different from that of 2:3. However, although an instrument may be tuned in accordance with that rule, the ear is not offended by that irrational ratio, and when listening to the interval C–G, does not fail to perceive a fifth, of the ratio 2:3; ..."].

The brilliant Swiss mathematician could now (astutely?) transform the previous succession into 36, 45, 54, *63*, divide them by 9, and obtain the much more simple 4, 5, 6, 7. Now the number 7 had become equally as harmonic as 2, 3 and 5. Euler thus sealed the solution to his problem with practical musicians and their

[50] Boyer 1990, p. 567. Grattan-Guinness 1970, pp. 1–32. Bottazzini 1981, pp. 34–38.

[51] Euler 1759. See below, Sects. 11.2 and 12.1.

compositions, which had become tonal in their reference precisely to Leibniz. Our German mathematician and metaphysician "... a déjà remarqué que dans la musique on n'a pas encore appris à compter au delà de 5; ... Mais, si ma conjecture a lieu, on peut dire que dans la composition on compte déjà jusqu'à 7 et que l'oreille y est déjà accoutumée; ... Dans ce genre l'accord 4, 5, 6, 7 est la plus complette harmonie, ..." ["... has already noticed that nobody has yet learnt to count beyond 5; ... But if my conjecture is valid, we may say that in composition, we already count up to 7, and the ear is already accustomed to it; ... In this genre, the agreement 4, 5, 6, 7 presents the most complete harmony, ..."]. "Si les hommes avoient le jugement de leur oreille si exacte, qu'ils pussent distinguer les plus petites aberrations, c'en seroit fait de toute la musique; ..." ["If men confessed such a precise judgement of their ear as to be able to distinguish even the smallest aberrations, it would be the dead of all music; ..."].[52]

Euler continued to distribute, both in the Académie at Berlin and in that of St. Petersburg, his ideas about musical harmony. On the one hand, he would like to correct and extend the Pythagorean tradition, without denying its elements based on whole numbers, but at the same time he would wish to demonstrate its value, also with the main novelties of his period. As musicians were using the chord of the seventh, which was unknown to the ancients, he tried to absorb it into his theory. Some claimed to explain that G, D, F, G, B as a means to remain in the tonality of *C major*, instead of that of *G major*, but here, instead of d'Alembert, he quoted the original source of Rameau. He even added: "Je ne crois pas que cette explication ait besoin d'être réfutée." ["I do not believe that this explanation needs to be rejected."]

For him, the real cause of the chord should be another. After explaining the ancients by means of 2, 3, 5, for the moderns "... il faut recourir au nombre premier 7 ..." ["... we must have recourse to the prime number 7 ..."]. And here, in the article, the name of Leibniz come up again: "... nous pourrons dire avec feu Mr. De Leibnitz que la musique a maintenant appris à compter jusqu'à sept." ["... we can say, with the late Mr. Leibniz, that now music has learnt to count up to seven."] Having decided to make use of 7 as well, the preceding harmony of the seventh was adjusted not with the complicated, 'ugly' 36, 54, 64, 72 and 90, but with the numbers 36, 54, *63*, 72 and 90. Having transformed them by division into the 'elegant' 4, 6, 7, 8 and 10, they would become even better comprehensible and tolerable for the ears. According to the 'true' harmony of Euler, the seventh G–F should therefore have the ratio 4:7, which would discriminate the moderns from the ancients.[53]

Even though it was not easily transposable, the Swiss physicist-cum-mathematician now also calculated the interval of the seventh, starting from C, and therefore, C, G, E, B♭, but this last note was not exactly the one desired. Thus it deserved another ♯ sign, or an asterisk in the text (Fig. 11.4).

[52]Euler 1764a, pp. 511, 515 and 515.

[53]Euler 1764b, pp. 523–526.

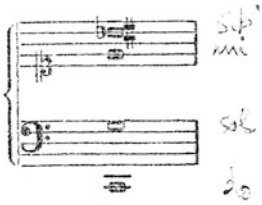

Fig. 11.4 The chord of the seventh according to Euler (1926, p. 535)

C, E, G, $B\flat$, C would be adjusted to elegant, easily comprehensible numbers. "C'est donc la simplicité et le bel ordre de ces nombres 4:5:6:7:8 qui rend ce nouvel accord agréable et lui procure une grace toute particuliere et inconnue à la musique ancienne." ["Thus it is the simplicity and the nice order of these numbers 4:5:6:7:8 that make the new harmony pleasing, and procure for it a thoroughly particular gracefulness, unknown to ancient music."] But this new Eulerian $B\flat$ would be different from the one obtained by compounding the two fourths of C, as in the tradition considered to be 'just' also by Euler, because the second is equal to 16:9 (ca. 1.777...). 7:4 sounds significantly lower (1.75) in the direction of A. It should be noted that the equable $B\flat$ would be much closer, on the contrary, to the old one, because it is equal to $1:(\sqrt[12]{2})^{10}$ (ca. 1.781...), but Euler did not recall this for us at all. As it was now possible to use the prime number 7, the octave was divided into 24 notes (Fig. 11.5).

The problem remained of musicians and singers operating in his period, who were by now accustomed to a completely different kind of music. The former were judged to be inclined to violate the 'true' rules of harmony. "...tout ce que les Compositeurs se permettent, n'est pas pour cela fondé dans l'harmonie; il semble plutôt qu'un pur caprice y ait souvent bien de la part." ["...not everything that composers permit themselves is for this reason founded on harmony; it seems rather that pure caprice often plays a part."]

It would thus be necessary to educate the ear in order to succeed in intoning the new ratio, too. "...la pluspart des sons sont arbitraires, pendant que les consonances dont je viens de parler sont fondées dans la nature même et accompagnées d'une certaine espece d'agrément qu'il est sur tout essentiel de faire bien sentir aux oreilles. [...] Il s'agit donc d'aborde d'accoutumer l'oreille à cette nouvelle consonance, afin qu'elle en comprenne bien la nature et l'agrément dont elle est accompagnée." ["...most of the sounds [used by musicians] are arbitrary, whereas the consonances presented above are founded on nature itself, and are accompanied by a certain kind of grace, which it is essential, above all, that the ears should hear clearly. [...] It is a question, then, of accustoming the ear to this new consonance from the start, so that it will understand clearly the nature and the pleasantness that accompany it."] Sure, some pages of the article were reserved for the education of the hearing.[54]

[54]Euler 1764b, pp. 537, 529 and 533.

Tons principaux	Nombres exposans		Tons ètrangers	Nombres exposans	
F	2^{12}	= 4096	F*	$2^6 . 3^2 . 7$	= 4032
Fs	$2^5 . 3^3 . 5$	= 4320	Fs*	$2^3 . 3 . 5^2 . 7$	= 4200
G	$2^9 . 3^2$	= 4608	G*	$2^7 . 5 . 7$	= 4480
Gs	$2^6 . 3 . 5^2$	= 4800	Gs*	$3^3 . 5^2 . 7$	= 4725
A	$2^{10} . 5$	= 5120	A*	$2^4 . 3^2 . 5 . 7$	= 5040
B	$2^3 . 3^3 . 5^2$	= 5400	B*	$2^8 . 3 . 7$	= 5376
H	$2^7 . 3^2 . 5$	= 5760	H*	$2^5 . 5^2 . 7$	= 5600
C	$2^{11} . 3$	= 6144	C*	$2^5 . 3^3 . 7$	= 6048
Cs	$2^8 . 5^2$	= 6400	Cs*	$2^2 . 3^2 . 5^2 . 7$	= 6300
D	$2^8 . 3^3$	= 6912	D*	$2^6 . 3 . 5 . 7$	= 6720
Ds	$2^5 . 3^2 . 5^2$	= 7200	Ds*	$2^{10} . 7$	= 7168
E	$2^9 . 3 . 5$	= 7680	E*	$2^3 . 3^3 . 5 . 7$	= 7560
f	2^{13}	= 8192	*f* *	$2^7 . 3^2 . 7$	= 8064

Fig. 11.5 How Euler divided the octave into 24 parts (Euler 1764b, pp. 533–535 and 539; 1926, p. 539)

Together with the repetition of his theory, the "Miroir musical [Musical mirror]" presented a little problem which recalled the one solved by Euler for the famous seven bridges of Königsberg. How was it possible to cross them all only once by continually going forwards? "... dans quel ordre il faut parcourir les douze sons de l'échelle musicale, pour qu'en passant toujours par des intervalles de quinte et de tierce majeure, et en ne faisant entendre chaque son qu'une seule fois, on revienne à celui d'où l'on est parti." ["... in what order must the twelve notes of the musical scale be crossed in such a way that, always moving by intervals of a fifth or a major third, and playing every note only once, the last note comes back to the starting point."] The ingenious mathematician found two solutions.[55]

Euler had faith that the laws of nature, of sounds and of music were revealed above all in prime numbers. The event that the many, and varied kinds of ears displayed by unfledged bipeds could even appreciate a piece of music he judged badly proportioned (with respect to his numbers) happened because they would be not very precise. Thus would be necessary to prepare ears better for listening to the 'just' tuning. Luckily for us, in the end, notwithstanding the unreliability of our (earthly?) senses, we would perceive the 'just' notes with pleasure, even though they might be tuned in a wrong way.

[55] Euler 1773, pp. 267–268. Kline 1972, p. 1164.

The event that Euler had taken such a great interest in music is generally overlooked by historians of the mathematical sciences. Instead, the same people cannot ignore how much of our modernity we owe to this physicist and mathematician who emigrated across Northern Europe. He wrote and published thousands of pages about almost every branch of mathematical science, clarifying their elements, organising their relationships and reforming them with new notions. He transformed Newton's fluxions and Leibniz's infinitesimals into mathematical analysis, to which he dedicated books. Here he discussed, with a hitherto unknown clarity, derivatives, integrals, infinite series, and differential equations, both of the ordinary kind and with partial derivatives. He mastered this new powerful instrument to such a point that he used it successfully to solve also problems of geometry and of number theory. He was one of the first whose textbooks can still without too much difficulty, be studied today, because his symbolic language has in part been transmitted to us, at least until the arrival of post-Hilbertian formalisms.

Already, he used his mathematical analysis to study also his beloved prime numbers. He was a master in the art of inventing the most unexpected relationships. He posited the one between the infinite series formed by the reciprocal of whole numbers and certain products of prime numbers, or another one between the same series increasing to infinity and the logarithm of the numbers. The latter arrived at infinity more slowly, but could be distinguished from the former by means of a constant known by his name, which still today remains fairly mysterious. My favourite relationships are those which are as surprising as they are relatively simple to enunciate. In a closed, convex polyhedron, the number of the Vertices, the Sides and the Faces are connected together by the simple equation $V - S + F = 2$. Euler thought of logarithms as exponents of numbers called bases. Among these, one in particular became useful for calculations in mathematical analysis. He indicated this by e, expressing its relationship with other famous numbers: $e^{\pi i} = -1$, that is to say $log_e(-1) = \pi i$, where i is the imaginary unit $i^2 = -1$.

Through the notion of "function", he was to change the face of mathematical analysis. Like Leibniz, he successfully went on to invent other symbols for the mathematical sciences: e, i, $sinx$ and above all $f(x)$. By the latter, our Swiss mathematician indicated that by changing any magnitude x under study, if there exists another, y, linked with the former for some reason and dependent on it, it will be possible to calculate the values $y = f(x)$. From this moment on, it is said that the second magnitude, y, is a "function" of the first one, x, specifying the various operations needed to obtain its numerical values.

Above all, after the preceding theory of music, we may legitimately think that Euler preferred the pure mathematics of numbers, whether prime, whole, rational, irrational, algebraic and transcendent, or imaginary: for all of which he displayed great capacities of calculation. Plagued by serious problems of eyesight, he was to fall a victim progressively to near complete blindness, and thus he had to use, above all, his mind and his spiritual imagination, getting somebody to help him to write things down. But we should not be too surprised if all these new ideal transcendent truths finally descended to earth. The son of pastors, he remained a devout Lutheran, with a staunch faith in Christian revelation. Although sublime and seeming an end

in itself, in his indefatigable, multiform production, he succeeded in incarnating mathematics in the most useful and concrete applications. The two main academies for which he worked undoubtedly encouraged him to do so.

He reaped success and received prizes for his writings dedicated to mechanics, illuminating the subject with his analytical style and opening up the chapter that studied rigid bodies. By means of mathematical analysis, Euler solved problems regarding the dynamics of fluids, and, of course, astronomy. And his theory was not to stop here. When he dealt with the movement of the Moon, our mathematician derived a method from it to calculate the longitude out on the open sea: he even received a part of the prize money destined for this purpose by the Royal Society. But he did not succeed in vindicating the ambitions of Newton in this field, because his (complicated?) calculation was to be rapidly substituted by the precise chronometers of Harrison. Furthermore, the movements of fluids were invented for, and stimulated by, the science of navigation, whereas the dynamics of heavy masses became the movement of shells for military ballistics.

For this mathematician, his own country Switzerland and his masters the Bernoullis had not been able to (or did not want to) create a job in his homeland. His physics took place in a world full of ether, following in the wake of Descartes and Leibniz. However, for religious reasons, he did not like the monads of the German metaphysician, and even became involved in disputes on the subject.[56]

When Euler dealt with music, the editors of his *Opera omnia* were forced to turn to Eduard Bernoulli and Martin Vogler for (external?) help. While the former recognized the roots in the ancient Greek traditions, the latter wanted to project him into the future for us, as a precursor of musical theories still to be developed. But, despite all his efforts, the theory of music proposed by the Swiss mathematician was not to find many followers, either in his period or in the following centuries, up to today. Musicians generally ignored him, and theoreticians, above all, criticised him. These included just Johann Mattheson, who was quoted by him, and who considered him to be a "... reflective algebraist, a fencer of relationships, a pedantic director of proportions and a merchant of Ratio."

However, that way of his of tuning the interval of the seventh to 7:4 was to receive the approval of a certain Johann Philipp Kirnberger (1721–1783), although Georg Joseph Vogler (1749–1814) considered it to be "intolerable" on organs. This seventh was also to be used in a choral composition by Karl Friederich Fasch (1736–1800). Without doubt, Rameau's system and the equable temperament were generally to prevail. The ideas of Euler left traces on music only in the experimental psychology of listening and in Helmholtz, as we shall see in Chap. 12 below. It is very difficult to conclude from this that our Swiss physicist and mathematician had achieved his desire of a practical application for music.[57]

[56] Youschkevitch 1970.

[57] Euler 1926, pp. XVI–XX. Vogler 1960. Other music theory is to be found in Euler's mathematical notebooks. See Knobloch 1987.

Among Euler's many writings, we consider here those dedicated to the models of vibrating strings. For some of them, it would be really not possible to ignore music, as is generally done. Clifford Truesdell realised that "Nearly everything specific concerning elasticity and vibration arose in the context of music." Unfortunately, he followed the dream of celebrating a mythical "Rational mechanics" of his, narrating its evolution as guided by an anachronistic reason. Thus the use he made of it was nearly only finalised to the emergence of his discipline, and consequently he often distorted history. For example, he confused the "strokes" of sound with "vibrations". Truesdell could therefore not understand, or explain accurately, a theory of consonances based on the coincidence of "strokes". Furthermore, he translated Mersenne and Galileo Galilei into modern formulas, and Vincenzio Galilei was completely ignored. In the case of Sauveur, however, rather than others like Mersenne, our scholar of rational mechanics from the United States recognized the merit of having fixed a clear terminology for phenomena of sound, that was destined to remain: fundamental sound, nodes, loops, harmonic sounds.

It should be noted, however, that this was developed, and initially remained, in the musical environment. Natural philosophers or geometricians of the period did not take an immediate interest in harmonic sounds, which first received due attention from Rameau. But nevertheless, the anachronistic Truesdell passed an implacable, highly scornful judgement, 'rational', on him: "... obscure and graceless writer ... confused medley of fact, error, and special pleading." Nor, living in the twentieth century, did this scholar of rational mechanics reserve any better treatment for his colleagues of the eighteenth century, who were members of the Académie royale des sciences, including d'Alembert: they would be guilty of confirming the experiments of Rameau, writing at the time that "... harmony ... has become a more geometrical science". Truesdell continued to be guided by the idea that above all Daniel Bernoulli and Euler offered reliable mathematical models of these phenomena.[58] The event that harmonic sounds could support the new tonal theory of musicians did not seem to interest him at all.

From Euler's *Tentamen*, Truesdell extracted only the properties attributed to sound, in order to register the criticism expressed at his time by Daniel Bernoulli. The latter conjectured that his younger colleague had not read Mersenne, as regards the tension to be assigned to the strings of musical instruments to obtain pleasant sounds from them. The Frenchman had written about a mean tension, whereas on the contrary, the Swiss scholar believed that they were to be stretched as much as possible without breaking. Daniel Bernoulli provided mechanical models of how heavy chains oscillated, and he developed analogies with the strings of musical instruments, arriving close to the series of frequencies. As we saw above, Galileo Galilei had in mind analogies between sounds and the oscillations of the pendulum. Instead, by making chains oscillate, perhaps would Daniel Bernoulli really have succeeded here in constructing approximate mechanical models of vibrations performed by a stretched string, with the relative nodes and series of

[58] Truesdell 1960, pp. 15–17, 121–125 and 154–236.

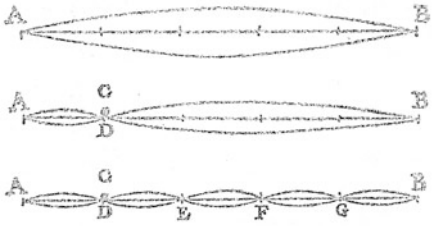

Fig. 11.6 Nodes and loops of a vibrating string as observed by Sauveur ("Nodes" in Euler 1960, p. 122)

harmonics? Our scholar of rational mechanics from the United States sustained that he would, notwithstanding that our Swiss scholar did not use any of the terms introduced by Sauveur. Anyway, both Daniel Bernoulli and Euler tried to calculate the oscillations of other bodies such as bars, hanging strings and flexible objects. As any differential form was still lacking, equations of movement were always elaborated by thinking of the pendulum.[59]

Music sometimes emerged explicitly, as in a work by Daniel Bernoulli about the various 'ways' of oscillating displayed by heavy bodies suspended on a flexible wire. Although these could initially be irregular, the compound oscillations became uniform in time. "Thus, for example, a musical string cannot make unequal [*i. e.* non-periodic] vibrations unless it does so from the start, such being the more non-regular, the swifter they are, while the string once set into vibration soon composes itself to the curvature necessary for isochronous motion." Also harmonics reappeared, now called 'modes'. "These remarks serve also for understanding the trembling motion of sounding strings: for the sound of one and the same string may be made up out of many tones." "Similarly, a taut musical string can produce its isochronous tremblings in many ways and even according to theory, infinitely many, though these are difficult to obtain [by experiments], and moreover in each mode it emits a higher or lower note." The 'modes' of oscillating were described as forming one, two or more arcs [that is to say, 'loops' between one 'node' and another], analogously to the figures drawn at the time by Sauveur (Fig. 11.6). "The next 'mode' demands that the string between two oscillations produces two arcs [loops] on opposite sides, and then the oscillations are twice as fast, and now it gives out the octave of the fundamental sound."[60]

In 1740, Daniel Bernoulli wrote to Euler: "I have also conducted experiments on the sounds of prismatic chimes such as are generally used in small carillons, and I think I have achieved this theory, too." He continued in 1741: "It is extraordinary that elastic bands give out different tones depending on how they are supported; that they have their 'nodes', upon which they must be supported in order to emit a clear tone . . . ". On returning to Basle after the Russian parenthesis at St. Petersburg, our Swiss scholar was conducting experiments and calculations on the elastic properties

[59]Truesdell 1960, pp. 154–155, 158–159 and passim.

[60]Truesdell 1960, pp. 122 and 179–180.

of bands, "... and this theory can be extended to all sounding bodies, and expecially to bells, as I conjecture." He seemed to take for granted, or to ignore, the behaviour of vibrating strings. For bands, he calculated 'modes' of vibration which produced irrational frequencies, like those proportional to $\sqrt{\frac{7}{2}}$.

Daniel Bernoulli (and, consequently, Truesdell) concluded that "... the harmony of these notes [of the string] ...is only a sort of accident, since the tones of struck bars are not only dissonant but also incommensurable ...". Could all this be interpreted as refuting conclusively what Rameau had tried to do in order to found his new harmony on the naturalistic vibrations of sounding bodies? Returning to his own country, our Swiss scholar thought so, and extended his criticism to one of the musician's followers, d'Alembert, who we shall shortly encounter. Truesdell followed in his footsteps, for the moment.[61]

It is no surprise at all to us that strings, bands or elastic bars, bells or *carillon* could not vibrate or make the same sound, or in the same 'modes'. On the contrary, it should sound curious to us that certain natural philosophers of the eighteenth century, who by now had almost become modern physicists and mathematicians, continued to equate irrational musical frequencies to dissonances, and these to a lack of harmony. We have been forced to suspect that also Rameau did so, but by now, in practice, musicians behaved differently, in order to fully exploit the advantages of the equable, irrational temperament. Thus Daniel Bernoulli (and Truesdell) would have had to come to deal not only with Rameau, but also with the ancient Pythagorean-style tradition, which believed that harmony was possible only on the basis of rational ratios. Might it be this, rather and not Rameau, that the 'modes' of vibrating in bands and bars (made of earthly materials) were disproving? Two ways of interpreting harmony were starting to divorce here: on the one hand, in music, the pleasing effect of notes for the ears, and on the other, in mathematical sciences, the movement of small regular oscillations as in the pendulum, which not all physical bodies could exhibit.

By making a needle vibrate suitably, Daniel Bernoulli could listen to a "... high sound at about the fifth in the double octave of the first." He attempted to construct the theory for these phenomena, but he limited himself to drawing some modalities in the oscillation. How did he verify the frequencies? "... by observing, as best I was able, the consonant sound on my harpsichord."

Euler worked on the same problems faced by the various Bernoullis, not only Daniel, perhaps with greater precision and completeness. "... si duae laminae elasticae tantum longitudine differant, erunt soni reciproce ut quadrata longitudinum; scilicet lamina duplo longior edet sonum duabus octavis graviorem. Corda autem tensa duplo longior sonum una tantum octava graviorem edit, si tensio maneat eadem. Ex quo patet sonos laminarum elasticarum longe aliam sequi rationem, atque sonos cordarum tensarum." ["... if two bands differ only in their lengths, the sounds will be in a reciprocal ratio to the squares of their lengths, that is to say that a band twice as long will emit a tone two octaves lower. However, a stretched string emits

[61] Truesdell 1960, pp. 193, 195–196.

a tone only one octave lower. This shows clearly that the tones of elastic bands follow a thoroughly different law from the tones of a stretched string."] Thus also the Swiss scientist in exile at Berlin measured the frequencies with which bands oscillated, using those of strings consonant with them.

Euler explicitly continued to interpret as musical intervals the frequencies that he was calculating, using logarithms. "Sonus ergo laminae priori modo oscillantis erit ad sonum eiusdem laminae hoc modo vibrantis, uti est quadratum numeri 1.8751040813 ad quadratum numeri 4.6940910795, hoc est ut 1 ad 6.266891 seu in minimis numeris ut 4 ad 25 seu 1 ad $6\frac{4}{15}$. Unde sonus posterior erit ad priorem duplex octava cum quinta et cum hemitonio fere." ["Thus the sound of the band oscillating in the preceding mode will stand to the sound of the same band oscillating in this way, as the square of the number 1.8751040813 stands to the square of the number 4.6940910795, that is to say, as 1 stands to 6.266891, or in smaller numbers, as 4 stands to 25, or 1 to $6\frac{4}{15}$. Consequently, the following sound will stand to the first one in a double octave and a fifth, with almost a semitone."].

In the following cases, he calculated other intervals. "Ratio ergo horum sonorum erit proxime ut 11 ad 70; horum ergo sonorum intervallum constituet duas octavas cum quinta et hemitonio. Sin autem posterior lamina libera duplo longior capiatur quam prior fixa, intervallum sonorum erit fere sexta minor." ["The ratio of these sounds will be close to 11:70; that is to say, the interval of the sounds makes up two octaves and a fifth, with a semitone. Instead, if the following free band reached twice the length of the fixed one, the interval of the sounds will be almost a minor sixth."] "... sonus autem edet multo auctiorem, quippe qui ad sonum praecedentem modo editum rationem tenebit fere ut 7^2 ad 3^2, hoc est, intervallum erit duarum octavarum cum quarta et hemitonii semisse. ... sonusque edetur plus una octava cum tertia minore acutior; ...". ["... [the band] will emit a much higher sound, which will undoubtedly stand to the sound emitted previously in a ratio of almost 7^2:3^2, that is to say, the interval will be two octaves and a fourth plus half a semitone. ... and the sound emitted will be one octave and a minor third more acute; ...".]

From the bands studied here, on other occasions, sounds two octaves more acute, or three octaves and one tone higher, were emitted. He probably verified them on his own keyboard instrument, which he enjoyed playing.[62] For anyone who still had doubts about the value assigned by Euler to his musical studies, I may underline that he published them together with his book on the calculus of variations: *Methodus inveniendi lineas curvas maximi minimive proprietates gaudentes [Method To Find Curves Which Enjoy the Property of Creating a Maximum and a Minimum]*.[63]

Treating the impulses that propagate in an elastic means as modelled by those between discrete punctiform particles, Euler surprisingly concluded that sound in the air could not be made of similar oscillations. "... sound may not at all be understood as propagated through the air, as some able men would have it, when

[62] Betti & Caparrini 2008, p. 14.

[63] Euler 1952, pp. 281, 283, 289, 292 and 294. Truesdell 1960, pp. 197, 199, 220, 222. Kline 1972, pp. 577–579. Boyer 1990, pp. 567–568.

they assert that, when a string or other sounding instrument is set in motion, there are in the air particles of this sort which take on an oscillatory motion and excite the organ of hearing."[64]

Continuing in our style, no sword will be used here to cut the Gordian knot formed by intricate priorities, letters or publications printed with a delay of years. These were often spiced with family rivalries between the various Bernoullis, with Euler kept prudently at a distance, not to mention those between nations who were fighting for supremacy in Europe.[65] I only wish to underline that music could not remain wholly on the sidelines, but it continued to supply that means of inspiration, or at least to investigate movements of vibration which the eye was not able to follow. We must also begin to think that, although written documents were so far missing, the Daniel Bernoulli quoted above might possess a certain practice with the harpsichord.

Both he and Euler seemed to be capable of calculating the series of frequencies of many sounds (sinusoidals) emitted by the most widely differing bodies. However, beyond the praiseworthy efforts of Truesdell, with his reconstructions in modern terms, they both remained prisoners of discrete mechanical models. They still lacked a mathematical symbolism more adequate to express the continuous movement of the vibrating string by means of a differential equation (with partial derivatives). But when this arrived with d'Alembert, it was to give rise to a heated controversy, which stimulated and separated the various formulations followed by contenders, both in the field of mathematics and in that of music.

The most experimental researcher and the one most attached to auditory phenomena, Daniel Bernoulli, tried to lead the question back to the 'modes' of vibration present in the strings, as already studied by him. "... every sounding body potentially contains an infinity of sounds and an infinity of corresponding ways of making its regular [periodic] vibrations; ... it can make a mixture of all these vibrations with all possible combinations; and moreover, all these new curves and new kinds of vibrations given by Messrs. d'Alembert and Euler are absolutely nothing other than a mixture of several kinds of Taylor's [Brook, 1685–1731]. If that is true, I could not approve the conglomeration of all these new curves, since then ... the string would not emit one and only one tone, but several at once." Instead, striking a steel bar, "... one hears at the same time a confused mixture of several sounds, ... extremely dissonant, ... Thus ... the harmony of sounds heard simultaneously in a sounding body is not essential to the matter, and should not serve as a principle for musical systems ... What proves best that the various undulations of the air do not interfere with one another is that at a concert one hears all the parts distinctly ...".

To an unexpressed addressee, perhaps during the years of the dispute about vibrating strings, this Swiss scientist seemed to recall the ancient music of the heavenly spheres, as it had remained in the heads of Kepler and Newton. "I admire more the physical treasure which was hidden: that natural motions, which seem to

[64] Truesdell 1960, pp. 229–230.

[65] Truesdell 1960, passim. Youschkevitch 1970. Straub 1981. Betti & Caparrini 2008.

be subject to no law, may be reduced to the simple isochronous motions which, it seems to me, nature uses in most of its operations. I am also convinced that the inequalities in the motions of the heavenly bodies consist only of two, three, or more simple reciprocal motions ... which can co-exist in one and the same body, while it moves subject to Kepler's laws; ...".[66]

In an article on the notes emitted by organ pipes, Daniel Bernoulli widened the gap that separated him from the mathematical considerations of d'Alembert and Euler. He would have more clearly explained through the small oscillations, how several different notes could be emitted by the same pipe. "...; c'est-là une preuve physique bien convaincante de ce que j'ai démontré sur la multiplicité des vibrations qui peuvent co-exister dans un même système, par où j'ai donné le dénouement & l'explication de plusieurs théorèmes sur les cordes vibrantes extrêmement paradoxes, auxquels *M.ssrs* d'Alembert & Euler avoient été conduits par un calcul fort ingénieux & sublime, mais en même temps trop abstrait ou métaphysique pour en connoître la juste application & le vrai sens." ["...; it is that a very convincing physical proof of what I demonstrated about multiple vibrations, which can co-exist in the same system, by means of [pages] where I solved and explained many extremely paradoxical theorems about vibrating strings; to them Messrs d'Alembert and Euler had been led by a highly ingenious, sublime calculation, which was, however, at the same time, too abstract or metaphysical to understand its correct application and its true meaning."]

This other Swiss physicist was rather absorbed by all his new multiple sounds, which he had reduced to combinations of basic sinusoidal elements (Fig. 11.7).
"J'ai pris la première pièce d'une flûte traversière que j'ai bouchée des deux côtés; ...; j'ai pu tirer de cette pièce successivement quatre tons par la variation de l'embouchure & de la force du souffle, sans pouvoir jamais en tirer un autre ton: ces quatre tons bien appréciés, m'ont paru parfaitement observer la proportion des nombres 1, 3, 5, 7, car après le ton fondamental, j'en tirois la douzième ou la quinte de l'octave, & puis la dix-septième ou la tierce majeur de la double octave, & enfin presque la vingt-unième ou à peu près la septième mineure de la double octave; d'où l'on voit que ces tons vont en proportion de 1, 3, 5, 7." ["I took the first piece of a transverse flute which I sealed at both ends; ...; I was subsequently able to obtain from this piece four notes by varying the mouthpiece and forcing my blow, without being able to obtain any other notes. Having carefully evaluated these four notes, it seemed to me that they observe the ratio of the numbers 1, 3, 5, 7 perfectly; because after the fundamental note, I emitted the twelfth, or the fifth of the octave, and then the seventeenth, or the major third of the double octave, and lastly the twenty-first, or the minor seventh of the double octave; whence we can see that the notes stand in the proportion 1, 3, 5, 7."]

[66]Truesdell 1960, pp. 254–259. Brook Taylor conducted an experiment which generated a note whose frequency was 383. He verified it on the harpsichord as an A; Wardhaugh 2008, p. 110. Music continued to make itself heard in studies on sound, which were about to be transformed into acoustics.

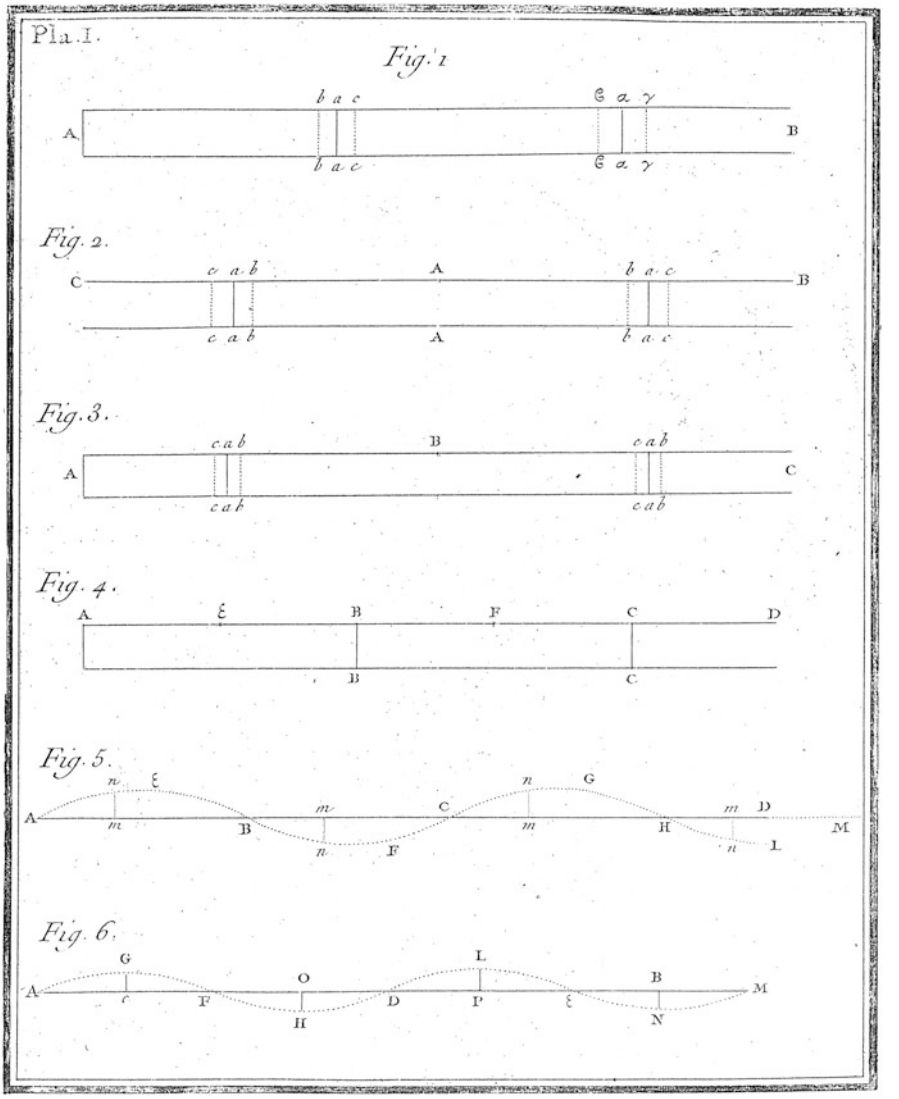

Fig. 11.7 How Daniel Bernoulli represented sinusoidal vibrations in organ pipes (D. Bernoulli, "Pipes", 1762)

Daniel Bernoulli got out from a pipe open at both ends octaves and other notes corresponding to the proportions 2, 4, 6, 8, 10. Thus he explained why trumpets, hunting horns and other wind instruments could emit a whole series of sounds by varying only "l'embouchure" ["the mouthpiece"]. He invented a device made of a stretched string and a cogwheel to set it vibrating. Thus he succeeded in making it possible to perceive "loops" and "nodes". "Tous ces phénomènes pourront être

remarqués aux yeux, & cependant ne feront encore aucun effet sur l'organe de l'ouïe; …". ["All these phenomena can be perceived by the eyes, and yet they will still not have any effect on the organ of hearing; …"]. Turning the wheel more quickly, the frequency of vibration increased, and then sounds began to be heard. "Il est naturel de dire que le petits brins de colophane attachés aux crins de l'archet font la fonction de ces dents, & que l'habileté d'un bon joueur de violon à en tirer un son net & agréable, consiste à manier si bien son archet que cette harmonie soit observée." ["It is natural to say that the small pieces of colophony attached to the horsehair of the bow have the function of these teeth and that the ability of a good violin player in producing a clear, pleasant sound consists of handling his bow so well that this harmony is observed."]

Like the turning of the wheel and the rubbing of the bow, this other Swiss researcher imagined that the quivering of the lips set the column of air contained in the pipes vibrating. "C'est ainsi qu'on presse plus ou moins les anches dans les hautbois & dans d'autres instrumens, pour rendre le ton naturel de l'anche harmonieux avec le ton qu'on prétend tirer du hautbois." ["It is thus that reeds are pressed more or less tightly in oboes and in other instruments to make the natural note of the reed harmonious with the note to be produced by the oboe."] Daniel Bernoulli calculated the vibrations of the string as small oscillations of an equivalent pendulum, and reduced the elastic oscillations of the air in the pipes to those of the strings, represented as forming motionless "nodes", and "loops" in movement (Fig. 11.7).

He also wrote equations where he represented, in the language of analysis, the movements guided by an elastic force proportional to the distance, in accordance with Newton's law, in which it was proportional to acceleration. There is little doubt that he had in mind the idea of being able to calculate them all in the same way, obtaining by means of integration the trigonometric functions (*sines* and *cosines*). However, as he like Newton was not particularly inclined to symbolism, he presented the results in a form that was not very effective. Although the essential was already here, from the pendulum to the vibrations of the various bodies, the abstract of a differential equation that unified all the harmonic movements had to pass through Euler.[67] In the end, the theory was applied to so-called '*a camino*' church organs and to … bottles of malvasia wine.[68]

"Pour peu que j'évitasse lesdits inconvéniens, le calcul ne me trompoit jamais au-delà d'un quart de ton; voilà une preuve bien convaincante de la vérité de nos hypothèses physique sur toute cette matière: …". ["Although I did little to avoid the said inconveniences, the calculation never deceived me by more than a quarter of a tone; this is a very convincing proof of the truth shown by our physical hypotheses in all this question: …"]. This other Swiss researcher would probably have obtained a better agreement if he had taken into consideration the effect on

[67] $m\frac{d^2x}{dt^2} = -kx; x = Acos(2\pi\nu t + \delta); \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}; \delta$ is the phase. Resnick & Halliday 1961, pp. 282–313.

[68] Bernoulli 1762, pp. 441–442, 436–440, 443–444 and 462–467. Cf. Truesdell 1960, pp. 262–263.

the notes due to the diameter of the pipes. Actually he heard notes that were deeper than those calculated, because the physical length of the open pipe is greater than the geometrical length, or vice versa if it was greatly narrowed. Anyway, he succeeded very well in demonstrating the formation of "nodes" and "loops" inside the pipes.

It thus appears to be curious that a similar experimenter, with a sensitive ear and eye, omitted to evaluate the effect due to the different diameters of pipes on the height of notes. "On sait d'ailleurs par expérience, & je le démontrerai ci-dessous, que les tons les plus graves des tuyaux bouchés de différentes longueurs, sont en raison réciproque de ces longueurs: …" ["Furthermore, we know by experience, and I shall demonstrate it below, that the deeper notes of pipes sealed at different lengths stand in a reciprocal ratio to their lengths."] "…les vibrations de l'air dans le même tuyau, sont absolument & en chaque point les mêmes que celles de chaque point analogue d'une corde uniforme de la même longueur L …cette seule consideration suffit pour déterminer les vibrations de l'air dans un tuyau cylindrique bouché par les deux bouts." ["…the vibrations of the air in the same pipe are exactly, and at every point, the same as those obtained at every analogous point by a uniform string of the same length L …this consideration alone is sufficient to determine the vibrations of the air in a cylindrical pipe sealed at both ends."].[69] However, some doubts about open pipes may have remained, seeing that he did not pronounce himself equally precisely on that subject.

Euler, on the other hand, was working on d'Alembert's new differential equation with partial derivatives, but he obtained different solutions, and therefore different consequences, from those of the Frenchman. He introduced the symbolism for differential calculus with several variables which was to remain in use until today.[70]

The famous dispute about vibrating strings should be narrated again, not as a dispute centred on the definitive truths that can be obtained in physics and in mathematics, believed by the contenders or publicised by historians and subsequent scholars, but as the development of the relative different ideas, notations and notions in the European context of the eighteenth century. However, secondary literature on the subject appears to be all too abundant, and we refer to this,[71] in order to return to our music. At the risk of repeating myself, this has in general been overlooked by everybody, even in this case where it was explicitly present with its intervals and its composers.

Despite heralding music in his initial declaration, Truesdell, too, did not practise what he preached. He actually expunged it, when he was forced to come across it in the writing of Euler: "…we have omitted Euler's interpretations of the calculated frequencies as musical intervals, since by this date such interpretations had become

[69] Bernoulli 1762, pp. 467, 469, 436 and 447.

[70] Truesdell 1960, pp. 259–262.

[71] Truesdell 1960; Truesdell 1968. Boyer 1990, pp. 507–539. Grattan-Guinness 1970, pp. 1–21. Kline 1972, pp. 478–483 and 502–522. Bottazzini 1981, pp. 19–51. David Speiser 1989. Kleinert 1996. Betti & Caparrini 2008.

straightforward."[72] Far from being "straightforward", the musical scale advocated by our Swiss physicist-cum-mathematician clashed with that of Rameau and above all with the widely adopted equable temperament. Might it not, perhaps, be that Euler's musical theory, so different from the practice of musicians, would make him as somewhat less engaged in the triumphant progress towards the truth, where, on the contrary, his successes in the mathematical sciences often projected him?

During the dispute about vibrating strings, he had written to Lagrange: "Pour les sons de Musique, je suis parfaitement de votre avis, Monsieur, que les sons consonans que M^r Rameau pretend entendre d'une même corde, viennent des autres corps ebranlés: et je ne vois pas pourquoi ce phénomene doit être regardé comme le principe de la Musique plutôt que les proportions veritables qui en sont le fondement. Je crois encore avoir bien determiné le degré d'agrement avec lequel on entend deux sons donnés, et de là deux sons en raison 8:9 s'apperçoivent plus aisément que s'ils etoient en raison 7:8. Mais je crois qu'ici il faut avoir egard à un prejugé, par lequel on suppose d'avance la proportion des sons, et alors une aberration est insupportable. Comme celui qui accorde un violon, si deux cordes se trouvent dans l'intervalle d'une sexte, il les juge fausses, puisqu'il pretend que leur intervalle soit une quinte." ["As regards musical notes, Sir, I agree with you perfectly that the consonant sounds [harmonics] that Rameau claims to hear in a single string derive from other vibrating bodies. I do not see any reason why this phenomenon should be considered to be the principle of music, rather than the true proportions which are its foundation. Furthermore, I believe I have determined correctly the level of pleasantness with which two given sounds are perceived; hence, two sounds in the ratio 8:9 are perceived more easily than if they stood in the ratio 7:8. But I believe that here, we should bear in mind a prejudice, which leads people to suppose the ratio of sounds in advance, and then an aberration becomes unbearable. Like a violin tuner, if he finds two strings with an interval of a sixth, he judges them to be false, because he expects their interval to be a fifth."][73]

Those who, like Daniel Bernoulli, listened more closely with their own ears to the sounds emitted by the vibrating strings, performing verifications on the harpsichord, would go closer to finding those solutions which were to receive an important and famous mathematical form from Joseph Fourier in the following century.[74]

It would not be at all difficult to trace again with Euler our historical coordinates towards a search for transcendence and absolute truth through dualistic separations, in order to reach the essence of the phenomena studied. The Protestant Christian marks of his Calvinist religious upbringing, which his family gave him, remained during his life. They led him to write a book which, in an Italian translation published in 1777 and edited by a Piarist father, was entitled *Saggio di una difesa della divina rivelazione [Essay On a Defence of Divine Revelation]*. As we can easily imagine, this will not have improved relations, already largely conflictual,

[72] Truesdell 1960, p. 222.

[73] Euler 1980, p. 425. Lagrange 1759, pp. 139–148. Christensen 1993, p. 155. See below, Sect. 11.2.

[74] See below, Sect. 12.1.

with his Enlightenment Parisian colleagues, or with the followers of Leibniz.[75] Through the works of this exile (perhaps self-exile[76]) from Switzerland and above all with his beloved numbers, must have passed the belief that the mathematical sciences were capable of revealing the essence of the world to us. Moreover they coalesced to demonstrate clearly the existence of a transcendent, omnipotent divinity. The dualism, soul vs. body, could not be solved for him as for Descartes, or as for Leibniz, but remained a mystery.[77]

As was dreamed by Leibniz, the best of possible worlds had been transformed by Euler into a real world, traversed by material bodies, fluids in movement, ships and cannons. Yet Euler did not trust the senses too much: the essential truth had to be made independent of them. How? Like the 'true' notes for music, which are fixed 'just' by whole numbers, although they are tuned by players in an opportunist way. These others were preferred to him by Frederick II, his king at the time when he was in Berlin. "A certain geometrician, who lost one eye performing calculations, decided to compose a minuet with *a* for *b*. If he had played it in front of Apollo, the poor soul would have been skinned alive like Marsyas."[78]

As regards the historical Russian context where Euler lived, we may recall the following interesting details. The term 'science' did not exist yet in Russian, and *HAÝKA* [nauka], was introduced subsequently to indicate it. Like Galileo Galilei and Johannes Kepler, also Euler was led to cast horoscopes for the family of the Czarina. Because an inauspicious one emerged, Euler and an astronomer presented another favourable one. When the destiny of the unlucky child came true, then Euler showed the authentic one which he had kept.[79] Our Swiss physicist-cum-mathematician was a remarkable character, rigorous and capable in every field, there is no denying it.

11.2 Jean le Rond d'Alembert, Jean-Jacques Rousseau and Denis Diderot

"At the age of twenty-four, there enters our scene now a talented but sinister personality, who is to make in six years a sequence of brilliant discoveries but thereafter will write endlessly in what seems today [1960] no more than a dogged attempt to confine the capacities of mathematics and to belittle the solid works of others." In this way, Clifford Truesdell presented d'Alembert. Of course, I shall take

[75] Betti & Caparrini 2008, pp. 9, 16, 75–77, 79 and 86–87.

[76] Betti & Caparrini 2008, p. 15.

[77] Betti & Caparrini 2008, p. 86.

[78] Betti & Caparrini 2008, p. 20.

[79] Betti & Caparrini 2008, p. 83.

the liberty of offering a slightly different image, undoubtedly a more musical one, of this illegitimate descendant of the French nobility.[80]

Actually, the famous article in which d'Alembert introduced his equation about waves should be read more carefully, right to the end. It was entitled "Recherches sur la courbe que forme une corde tenduë mise en vibration" ["Research on the curve formed by a stretched string set in vibration"], and in it he wrote that "... la force acceleratrice du point M ... est $-\frac{Fddy}{ds^2}$, si la courbe est concave ..., ou $\frac{Fddy}{ds^2}$ si elle est convexe." ["... the force of acceleration of the point M ... equals $-\frac{Fddy}{ds^2}$, if the curve is concave ..., or $\frac{Fddy}{ds^2}$ if it is convex."] He based his research on a result of Brook Taylor (encountered above) presented in the *Methodus incrementorum [Method of Increases]*, using a theorem of Euler and a lemma of Newton.[81] But without doubt, he was the first person to write this differential equation with partial derivatives and to succeed in finding the solutions. Today, naturally, following Euler, we would use other symbolic notations.[82] If Taylor thought that the string, vibrating, assumed only the form of a sinusoid, here d'Alembert demonstrated, instead, that other curves could give it infinite forms at will. We already know this was how the controversy began. Which limited characteristics the functions that solved the equation proposed by the Frenchman must have? In the various historical reconstructions already quoted above, it has been overlooked, as usual, that the original article went on to speak also about music.

Sure enough, this was followed in 1750 by an "Addition", at the end of which the dispute with Euler clearly emerged. "M. Euler a traité dans les Mémoires de 1748 le probleme des cordes vibrantes par une methode entierement semblable à la mienne, quant à la partie essentielle au problème, & seulement, ce me semble, un peu plus longue. ... est la même courbe que j'ay appellée *génératrice*. ... il faut de plus que cette courbe ait les conditions que j'ay exprimées dans mon mémoire, ..." ["Mr. Euler treated the problem of vibrating strings in his memoires of 1748 using a method wholly similar to mine, as regards the essential part of the problem, only it seems to me a bit longer. ... it is the same curve that I have called a *generator*. ... furthermore, it is necessary that this curve should satisfy the conditions expressed in my note, ..."].

The criticism was not only of a mathematical nature, regarding the solutions of the equation, smooth and differentiable those of the Frenchman, also with edges those of the Swiss scholar, but even of a musical nature. "Il est visible par les formules précedentes qu'à tension & à grosseurs égales le nombre de vibrations dans un même tems est en raison inverse de la longueur des cordes. Comme le son plus ou moins aigue des cordes depend du nombre de vibrations plus ou moins grand qu'elles font dans un tems donné, c'est sans doute pour cette raison

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[80] Truesdell 1960, p. 186. Cf. Euler 1980, p. 427: "... le souci évident de l'auteur de déprécier les apports de d'Alembert et de Lagrange." ["... the clear concern of the author [Truesdell] to denigrate the contributions of d'Alembert and Lagrange."]

[81] D'Alembert 1747, pp. 214–215.

[82] $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$; with v the velocity of the wave. Persico 1960, pp. 162–179.

que quelques Auteurs modernes très habiles ont cru pouvoir répresenter les sons par le logarithmes des rapports de la longueur des cordes. Cette idée est ingénieuse, & paroit même fondée sur la manière de parler usitée en acoustique & en musique, lors qu'on dit que si quatre cordes *a, b, c, d* sont en proportion Géométrique, l'intervalle des sons rendus par *a* & par *b*, sera égal à l'intervalle des sons rendus par *c* & par *d*; d'où l'on a cru pouvoir conclure que les logaritmes des rapports $\frac{a}{b}$, $\frac{c}{d}$, répresentoient les intervalles des sons. Mais on n'a pas prétendu sans doute que cette conclusion fût autre chose qu'une supposition purement arbitraire, les mots d'*intervalle* des sons, d'*égalité* ou de *difference des intervalles*, ne sont que des manières de parler abrégées, aux quelles il ne faut pas attribuer plus d'etenduë qu'elles n'en ont. Les sons ne sont que des sensations, & par conséquent n'ont réellement aucun rapport entr'eux; on ne peut pas comparer les sons plus que les couleurs, il ne faut qu'un peu d'attention pour le sentir; . . . "

["It can be seen from the preceding formulas that, tension and thickness being equal, the number of vibrations in the same period of time stands in a reciprocal ratio to the length of the strings. As the more or less acute sound of the strings depends on the more or less large number of vibrations they perform in the given time, it is undoubtedly for this reason that some very able modern authors [Euler, Rameau, . . .] have believed they can represent sounds by the logarithms of the ratios of the lengths of the strings. This idea is ingenious, and would seem to be likewise founded on the way of speaking in acoustics and music. Here, it is said that if four strings *a, b, c, d* are in a geometrical ratio, the interval of the sounds given by *a* and *b* will be *equal* to the interval given by *c* and *d*. Whence it has been believed it can be concluded that the logarithms of the ratios $\frac{a}{b}$ and $\frac{c}{d}$ represented the intervals of the sounds. But without doubt, it was not claimed that this conclusion was anything other than a purely arbitrary supposition: the words *interval* of sounds, *equality* or *difference of intervals* are only abbreviated ways of speaking, to which no further importance should be attributed than what they really have. Sounds are simply sensations, and consequently they have no real relationship between one another; sounds cannot be compared any more than colours can, it only takes a little attention to realise it; . . . "].[83]

Thus the French physicist and mathematician was interlacing with music the controversy that had developed on vibrating strings. The corpus callosum of his brain had not been severed, and the right hemisphere of music communicated perfectly with the left hemisphere of language. This was not difficult for him to believe, because harmonic sounds, posited by Rameau but also by d'Alembert himself as we are about to see at the basis of music, appeared to him to undermine seriously the millennial Pythagorean tradition based, instead, on numerical ratios. On the contrary, Euler appeared to wish to remain inside this tradition, although he was trying to extend its rigour and lead musicians to count up to 7! Thus the

[83] D'Alembert 1747, pp. 358–360.

dispute about the dynamics of strings marshalled on the battle field people who were contending at the same time in favour of rival musical theories.[84]

Our French mathematician considered it"…point important de l'Histoire des Sciences …" ["…an important point in the history of sciences …"] to clarify what perfection the ancients had reached, starting from Pythagoras, in music, an art which also was a science for him.[85] Pending the relative work of historians, he himself dedicated his attention to the discussions of music current at his time, that is to say, Rameau. d'Alembert was thus to become the main interlocutor of the composer. Actually, the mathematician and encyclopedist dedicated an entire book to music, *Elémens de musique théorique et pratique suivant les principes de M. Rameau [Elements of Theoretical and Practical Music Following the Principles of Mr. Rameau]* which was published in 1752, the same year as the letter to Euler seen above.

"Il a trouvé dans la résonnance du corps sonore l'origine la plus vraisemblable de l'harmonie, & du plaisir qu'elle nous cause: …il a réduit tous les accords à un petit nombre d'accords simples & fondamentaux, dont les autres ne sont que combinations & des renversemens." ["He [Rameau] has found in the resonance of the sounding body the most likely origin of harmony and of the pleasure that it procures us: …he has reduced all the chords to a small number of them, which are simple and fundamental, of which the others are only combinations and inversions."] In the wake of Rameau, also d'Alembert wrote that: "Tout corps sonore fait entendre, outre le son principal, la douzieme & la dix-septieme majeure de ce son." ["Besides the fundamental sound, all sounding bodies emit the twelfth and the seventeenth of this sound."] Hence the major and minor perfect chords could easily be deduced, together with many other properties, arriving at "…le principe & les lois du tempérament." ["… the principle and the laws of the temperament."]

But the French mathematician no longer followed the renowned composer (in the second edition of 1779), when he claimed that also the descending harmonics vibrated and divided. Above all, d'Alembert banned from his book "…toutes considérations sur les proportions & progressions géométriques, arithmétiques & harmoniques, …usage tout-à-fait inutile, & même, si nous l'osons dire, tout-à-fait illusoire dans la théorie de la Musique. …quand on n'y remarqueroit aucune progression ni aucune loi; quand ils seroient incommensurables entr'eux; la résonnance du corps sonore, & les sons multiples qui en dérivent, suffiroient pour fonder tout le systême de l'harmonie." ["…all considerations about geometrical, arithmetical, harmonic proportions and progressions, …a totally useless custom, and even, if we may dare to say so, totally illusory in the theory of music. …when we do not observe any progression, or any law; when they are incommensurable with one another; the resonance of the sounding body and the multiple sounds derived will be sufficient to found the whole system of harmony."]

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[84]Cf. Truesdell 1960, pp. 237–244, where the 'rational' scholar of the twentieth century completely ignored music.

[85]D'Alembert 1752a, p. ij; we are using the edition of 1779.

Sure, the vibrations could be counted, and the intervals classified. “Mais il faut bien remarquer que par ces expressions numériques on ne prétend point comparer les sons en eux-mêmes; car les sons en eux-mêmes ne sont que des sensations, …”. [“But it should be clearly noted that by means of these numerical expressions, we do not claim at all to compare the sounds between them; because in themselves, sounds are nothing other than sensations.”] Therefore, instead of putting them in large letters in the text, our French mathematician put whole and rational numbers at the foot of the page in small letters. It seems therefore that he did not go so far as to renounce completely the intervals represented in the usual way.[86]

D’Alembert set limits to the theoretical work of the musician, those limits which Rameau was to claim to go beyond. It was not necessary, therefore to seek to find there “…cette èvidence frappante, qui est le propre des seuls ouvrages de Géométrie, & qui se rencontre si rarement dans ceux où la Physique se mêle. Il entrera toujours dans la théorie des phénomenes musicaux une sorte de Métaphysique, que ces phénomenes supposent implicitement, & qui y porte son obscurité naturelle; …”. [“…that surprising evidence, typical only of geometrical works, which is encountered so rarely in those where physics is mixed with it. A sort of metaphysics, which these phenomena implicitly suppose, will always enter into the theory of musical phenomena, and this brings its natural obscurity; …”]. Among the obscure points, the French Enlightenment scholar included the minor scale. Nor could he offer at that time any “démonstration” [“demonstration”] of the principle followed, as Rameau had, on the contrary, tried to do, under the title *Démonstration du principe de l’harmonie [Demonstration of the Principle of Harmony]*. At that time, the Académie des Sciences had accepted it only as *Mémoires … d’un systême de Musique … [Memoires … of a System For Music …]*. Among the physical-mathematical sciences, like that of sounds, some needed more principles, and music was probably one of these. In the treatise of Giuseppe Tartini (1692–1770), it was perfectly possible to follow another different one, like that derived from the ‘third sound’.[87]

Criticism re-emerged here of “…très-habiles Physiciens …” [“…very able physicists …”] who had tried to explain the vibration of a musical string as a mixture of many individual vibrations. But why should only three of these be preferable to the others? D’Alembert doubted whether the phenomenon was due to an (unprovable?) presence of particles in the air “…tendues à différens tons, & que ces particules différemment ébranlées sont la cause de cette resonnance multiple.” [“…tuned to different notes, and that such particles shaken in a different way are the cause of this multiple resonance.”] He invited musician-philosophers not to get lost “…dans un labyrinthe de spéculations métaphysiques sur les causes du plaisir que l’harmonie nous fait éprouver.” [“…in a maze of metaphysical speculations about the causes of pleasure that harmony leads us to experience.”]

[86]D’Alembert 1752a, pp. vij, ix–xiij, 11.

[87]D’Alembert 1752a, pp. xiij, xvj, xix.

He rejected as a "... supposition gratuite ..." ["... gratuitous supposition ..."] the idea that the pleasure of consonances depended on the "... concurrence plus ou moins fréquente des vibrations; ... simplicité plus ou moins grande du rapport que ces vibrations ont entre elles." ["... more or less frequent coincidence of vibrations; ... the more or less great simplicity of the ratio that the vibrations have between them."] After all, how could the ear be so sensible to the simplicity of numerical ratios, when these are ignored by those who appreciate good music? Why could the interval of the fifth be altered without losing any of its pleasant qualities, or all its simplicity, while, on the contrary, the octave could not? Euler was not mentioned by name, but the criticism undoubtedly referred to him. Scholars then should refrain from any attempt to "... vouloir trouver entre la Musique & les autres Sciences des rapports chimériques." ["... find chimeric relationships between music and the other sciences."] Here, without naming him, Rameau was accused of talking about what he did not know.[88]

The famous French mathematician reduced arithmetic calculations to the minimum. "L'envie de donner à leurs productions un faux air scientifique, n'en impose qu'aux ignorans, & ne sert qu'à rendre leurs traités plus obscurs, & moins instructifs. En qualité de Géomètre, je crois avoir quelque droit de protester ici (...) contre cet abus ridicule de la Géométrie dans la Musique." ["The desire to give what they produce a false scientific air is only imposed on the ignorants, and only serves to make their treatises more obscure and less instructive. In my capacity as a geometrician, I believe I have some right to protest here (...) against this ridiculous abuse of geometry in music."] For him, the ratios of the ancient tradition like 1:2, 2:3, 4:5 "... ne sont peut-être par [sic! pas?] les vrais rapports de la nature; mais seulement des rapports approchés, & tels que l'experience les a pu faire connoître. Car l'experience donne-t-elle jamais autre chose que des *à peu près!*" ["... are not, maybe, the true relationships of nature, but only approximate ratios and such as experience has been able to make them known. Because experience never gives any other result than *roughly!*"]. Though not exactly true, these ratios served to tune the instruments.[89]

While our mathematician of the Enlightenment supported Rameau in justifying the major mode by means of the prime higher 'natural' harmonics C, E, G, for the minor mode, he abandoned him in 1779. For the minor chord, C, $E\flat$, G, he obtained the $E\flat$ as a note in resonance with G, because it was its major third or seventeenth. He based all his considerations on experiences and on the ear. Maybe he might be starting to harbour doubts about that "... frémiront sans résonner ..." ["... they will vibrate without resounding ..."] for the negative descending harmonics. Anyway, together with Rameau, he wanted to root the beginnings of musical harmony in "nature" and not at all in numbers, like Euler.[90]

[88] D'Alembert 1752a, pp. xxij–xxvj.

[89] D'Alembert 1752a, pp. xxx–xxxj.

[90] D'Alembert 1752a, pp. 22–24.

Having, in a footnote, re-obtained the traditional Pythagorean-Ptolemaic-Zarlinian ratios, d'Alembert faced up to the inevitable difference between the minor third D–F, the ratio of which is 32:27, and the minor third E–G, the ratio of which is 6:5. He concluded that "…la vraie tierce mineure est celle qui est donnée par la nature entre *mi&sol*, dans la résonnance du corps sonore, …" ["…the true minor third is the one given by nature between E–G, with the resonance of the sounding body, …"]. The ear would not have paied attention to an interval "…pas parfaitement juste …" ["…not perfectly just …"], because it followed the harmonics of C by fifths G, C, G, C, F, C, F, which were called "basse fondamentale" ["fundamental bass"] "…vrai guide de l'oreille …" ["…the true guide of the ear …"]. It would not succeed in perceiving the differences given by the *comma* of Didymus 81:80.[91]

With the temperament, the mathematician of the Enlightenment got out of trouble by invoking the needs of the harpsichord. And yet, in spite of the new light, he too with his notes at the foot of the page, seemed to get lost in the maze of *commas*. He emerged from it by keeping the octaves fixed, in other words: "…plus de point fixe dans l'harmonie ni dans la mélodie." ["…no more fixed points in harmony or in melody."] Consequently, he altered the fifths to make them coincide again with the last octave. Like Rameau, he made all the semitones equal, and inserted in a note their irrational mathematical ratio, 1: $\sqrt[12]{2}$. This last type of temperament was "…necessaire pour passer d'un mode dans un autre sans que l'oreille soit choquée; …" ["…necessary to pass from one tonality to another without the ear being irritated; …"]. In contrast with those musicians who did not like "…cette uniformité dans les modulations …" ["…this uniformity in modulations …"] he again invoked "…l'intention de la nature, …Le caractère d'un air vient principalement de l'entrelacement des modes; …" ["…the intention of nature. The character of an aria derives principally from the intertwining of tonalities; …"].[92]

D'Alembert remembered that at the beginning, with his first treatise, Rameau had preferred to alter the fifths rather than the thirds, basing himself on an experience of resonating strings, but he, too, was subsequently to be converted to the equable temperament. For his part, he conceded the choice to the reader between different possibilities. The ear, guided in any case by the fundamental bass, would tolerate them, without noticing them, supplying what was missing by itself. The dissonant interval of the seventh was justified by our French mathematician because it was capable of distinguishing the tonality. As we have seen above, this was what Euler criticised. The minor mode appeared to offer greater variety to d'Alembert. Above all, "…le mode majeur est-il l'ouvrage de la nature seule, au lieu que le mineur est en partie l'ouvrage de l'art." ["…the major mode is only the work of nature,

[91]D'Alembert 1752a, pp. 33, 32, 34.

[92]D'Alembert 1752a, pp. 50, 56–57.

whereas the minor one is partly the fruit of art."] The same could be said of the seventh chord, which was equally capable of providing new variety for harmony.[93]

Having observed that the melody itself may be changed by modifying only the underlying harmony in the fundamental bass, our natural philosopher of the Enlightenment concluded: "...la mélodie a son principe dans l'harmonie; ..." ["...the melody has its principle in harmony; ..."]. With the result that some able musicians started to write the bass. "Cependant cette méthode paroit en général plus propre à produire une Musique harmonieuse & savante, qu'une Musique d'enthousiasme & de génie." ["However, this method in general would seem to be more suitable to produce a harmonious, cultured music than an exciting, of genius kind."] Would this be also a veiled criticism of Rameau's musical style? And why? In any case, in the dualism that had been created between harmony and melody, "...il ne faut pas cependant en conclure avec quelques Musiciens célèbre, que les effects de l'harmonie doivent être préférés à ceux de la mélodie. L'expérience prouve le contraire; ..." ["...we should not conclude, however, with certain famous musicians that the effects of harmony are to be preferred to those of the melody. Experience proves the opposite; ..."]. D'Alembert quoted himself in an article that we find immediately below: "La liberté de la Musique" ["The freedom of music"].[94]

Although he repeated in a footnote (useless?) calculations about major and minor, chromatic and enharmonic semitones, up to four different quarters of a tone, in the end he will find himself with our usual (tempered) 12 semitones and 24 tonalities: one major and one minor for every note of the scale. Then he was finally able to dedicate himself to the composition of music. Without unfortunately venturing to intervene himself, this fixed in rules for the fundamental bass and the continuous bass, with their due licences, "...un extension de la regle, qui à la vérité n'a point de fondement dans la résonnance du corps sonore, mais que l'expérience & l'oreille autorisent." ["...an extension of the rule, which, to tell the truth, has no foundation in the resonance of the resounding body, but which experience and the ear authorise."] In reality, this was based on those (inexistent?) descending harmonics which could not resound, but only "fremir".[95]

Our mathematician often referred to compositions of his period, above all those of Rameau, such as *Pygmalion*, *Zoroastre*, *Hippolyte et Aricie*, *Les Indes galantes*. Repeating it at the end, with the notes on the stave in support of the book, he copied the same monologue from the *Armide et Renaud* by Lulli, which had been the object of a commentary by Rameau at his time, and which later became the object of a dispute with Rousseau, as we shall soon see (Fig. 11.8).[96] In a polemical tone, d'Alembert referred a couple of times to the entry FONDAMENTAL of the famous *Encyclopédie*,[97] printed shortly before together with Diderot. It offered various

[93] D'Alembert 1752a, p. 88. See Sect. 11.1.

[94] D'Alembert 1752a, pp. 115, 117–118.

[95] D'Alembert 1752a, pp. 159.

[96] D'Alembert 1752a, pp. 76, 194, 205, 202. Rameau 1980, pp. 169–191.

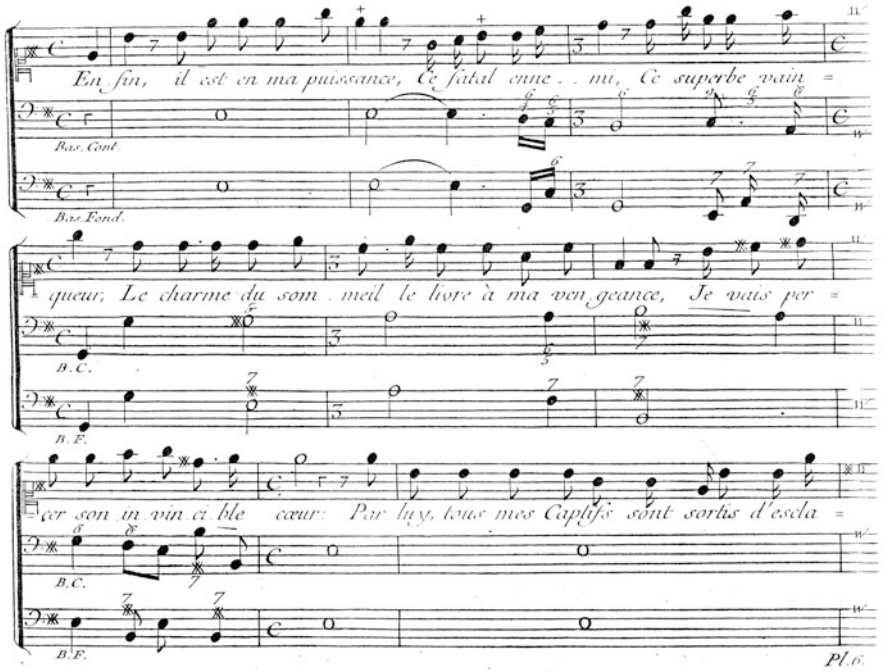
[97] D'Alembert 1752a, pp. xxj and 117.

En fin, il est en ma puissance, Ce fatal ennemi, Ce superbe vain
Bas. Cont.
Bas. Fond.
queur, Le charme du som meil le livre à ma ven geance, Je vais per
B. C.
B. F.
cer son in vin ci ble cœur: Par luy, tous mes Captifs sont sortis d'escla
B. C.
B. F.
Pl. 6.

Fig. 11.8 D'Alembert commented on Lulli's *Armide et Renaud* (D'Alembert, "Lulli", 1752a, pl. 6)

entries about music, some written by himself, others by Rousseau or together with him, but marked by the letters Ue.

A small amount of music already appeared in the "Preliminary Discourse" (work of the French mathematician) here and there: seen as capable of arousing sentiments and passions, and also connected with sensations. "We owe the fine arts, good taste, and numerous models of incomparable perfection mainly to Italy. [...] Among the arts, music is perhaps the one which has made most progress among us in the last fifteen years. ... Mr. Rameau, bringing the technique of his art to such a high level of perfection, has become a model and the same time an object of jealousy of numerous artists, ... But his superiority consists ... of having reduced to more certain and simple laws a science which, before him, was wholly entrusted to arbitrary rules or dictated by blind experience."[98]

However, it was above all in the ten pages dedicated to FONDAMENTAL that d'Alembert reserved his explanations about musical notes, harmonics and the sounding body. Here, he dealt with the fundamental sound and the first harmonics which form the perfect chord of the major mode. He also criticised the relative

[98]D'Alembert 1968, pp. 31, 52 and 79.

experiments considered by Daniel Bernoulli and by Euler. Thus, one of those figures who were inventing modern mathematical physics interwove it with music, and not only, as we have seen, in this frivolous entry for the *Encyclopédie*. "J'accorde d'abord à M. Bernoulli ce que je ne crois pas, & ce que M. Euler me paroît avoir très-bien réfuté ...; savoir, qu'une corde en vibration décrit toujours ou une trochoïde simple, ou une courbe, qui n'est autre chose que le mélange de plusieurs trochoïdes." ["At the beginning, I concede to Mr. Bernoulli what I do not believe, and what Mr. Euler seems to have refuted very effectively ...; that is to say, that a vibrating string always describes either a simple sine, or a curve that is nothing other than a mixture of many sines."] But if this were true, why would it possible to hear the octave, the twelfth and the seventeenth, instead of all the other sounds? Our French mathematician denied that experience showed the formation of loops and nodes in the vibrating string, except at the fixed extremities. Furthermore, if the string emitted a similar mixture of sines, "...ne devroit former qu'une harmonie confuse & une espece de cacophonie. Pourquoi cela n'arrive-t-il pas?" ["...only a confused harmony and a kind of cacophony should be created. Why does this not happen?"][99] D'Alembert also rejected the hypothesis that elastic particles of air stretched differently started vibrating. In this case, also the lower harmonics should have been heard to vibrate, which, on the contrary, only "trembled". However, even if it had vibrated, only a confused cacophony would have been obtained.

Having classified the chords, our French Enlightenment scholar began to distinguish himself more decidedly from Rameau, although to a large extent, he had been inspired by him. Actually the entry of the *Encyclopaedia* was thought to be very close to *Elements of Music*, seeing that similar musical examples could be seen. And yet, he called the contents of the notes $F-A-B-D\sharp$ a chord of *the excessive sixth* or *the Italian sixth*, a case ignored in Rameau's book and by his theory. Together with Rousseau, d'Alembert criticised the renowned composer for using ratios in justifying dissonances: a useless idea, he repeated here, too, seeing that a different and better explanation could be obtained from the same other principles of Rameau. The event that the Italians successfully used the chord of the excessive sixth, without it entering into the fundamental bass meant "L'oreille est ici le vrai juge, ou plûtôt le seul; ..." ["The ear is the real judge here, or rather the only one; ..."] Theory could only intervene afterwards. "...la plûpart des musiciens, les uns aveuglés par la routine, les autres prévenus par les systèmes, n'ayent pas tiré de l'harmonie tout le parti qu'ils auroient pû, ..." ["...most musicians, some blinded by habit, the others prevented by the systems, have not derived from harmony all the utility that they could have done, ..."]. Even Rameau was to violate his own rules in the *Fêtes de l'Hymen et de l'Amour [Celebrations of Nuptials and Love]*.[100]

Do these laws all exist in the nature of the vibrating sounding body? Some could be derived from it, like the major perfect chord, but most of them "...sont uniquement l'ouvrage de la réflexion ... & nullement l'ouvrage de la nature: ..."

[99] D'Alembert 1751d, pp. 98, 99.

[100] D'Alembert 1751d, p. 101.

[“. . . are only the work of reflection . . . and not at all the work of nature: . . . ”]. Daniel Bernoulli had proposed experiments, finding bodies which emitted dissonant sounds with the fundamental one. “. . . nos adversaires n’en pourroient tirer aucune conclusion, puisque cette expérience iroit à infirmer toute la théorie sur laquelle la *basse fondamentale* est appuyée. Aussi M. Daniel Bernoulli prétend-il . . . qu’on ne peut tirer de la résonnance du corps sonore aucune théorie musicale. Je crois cependant cette conclusion trop précipitée: . . . elles sont apparemment fort rares, & viennent sans doute de quelque structure particuliere des corps, qui les empêche de pouvoir être véritablement regardés comme des corps sonores.” [“. . . our adversaries could not deduce any conclusion from this, because this experiment would go to invalidate the whole theory on which the *fundamental bass* is grounded. Mr. Daniel Bernoulli also claims . . . that no musical theory can be obtained from the resonance of the sounding body. I believe, however, that this conclusion is too hasty . . . they [the exceptions that do not generate sounds with harmonics] are clearly very rare, and without doubt come from some particular structure of bodies, which prevents them from being considered truly as sounding bodies.”][101]

To decide about the relationship between the harmony of the fundamental bass and the melody now superimposed on it, our French mathematician suggested first listening to “. . . un grand nombre de beaux airs italiens; . . . tout chant qui paroîtra également susceptible de plusieurs basses, est un chant de pure fantaisie, un chant metif, si on peut parler ainsi.” [“. . . a large number of beautiful Italian airs; . . . every song that will seem equally susceptible to many basses is a song of pure imagination, a hybrid song, so to speak.”] If authority counted in the case of science, he went on, the violinist from Lucca, Francesco Geminiani (1687–1762, then at Paris) and of course Rameau would have committed themselves in favour of harmony in the bass. But d’Alembert repeated, word for word, what we have already read in the *Element of Music*: an (Italian?) piece of music, full of enthusiasm would hardly be likely to start from the bass.[102]

He immediately criticised those who preferred to work too much on harmony: “. . . s’exposeroient à tomber dans le défaut qui n’est que trop ordinaire à nos musiciens françois, de tout sacrifier à l’harmonie, de croire relever un chant trivial par une basse fort travaillée & fort peu naturelle, & de s’imaginer, en entassant parties sur parties, avoir fait de l’harmonie, lorsqu’ils n’ont fait que du bruit. . . . son plaisir continue d’être un, . . . c’est ce qui fait surtout le charme de la bonne musique italienne; & c’est-là cette unité de la melodie dont M. Rousseau a si bien établi la nécessité dans la ‘Lettre sur la Musique françoise’. . . . *Les Italiens ne veulent pas qu’on entende rien dans l’accompagnement, dans la basse, qui puisse distraire l’oreille de l’object principal,* . . . Préférer les effects de l’harmonie à ceux de la mélodie, sous ce prétexte que l’une est le fondement de l’autre, c’est à-peu-près come si on vouloit soûtenir que les fondemens d’une maison sont l’endroit le plus agréable à habiter, parce que tout l’édifice porte dessus.” [“. . . they would risk falling

[101] D’Alembert 1751d, p. 102.

[102] D’Alembert 1751d, p. 103.

into the defect, which is only too common with our French musicians, of sacrificing everything to harmony. They believe that it is possible to improve a banal song by means of a highly elaborate, and not very natural bass. They imagine, piling up parts together one on top of the other, that harmony has been created, while nothing has been produced except noise. ... Its pleasure [of song] continues to be one, ... it is this, above all, that creates the fascination of good Italian music. It is that unity of melody, of which Mr. Rousseau has so clearly established the need in his 'Letter on French music': *In the accompaniment and in the bass, the Italians do not want anything to be heard that can distract the ear from its main aim, ...* Preferring the effects of harmony to those of melody, under the pretext that one is the foundation of the other, would be almost like wishing to maintain that the foundations of a house were the most convenient place to live, because the whole building rests on them."][103]

Having written these observations as a supplement to the *Elements of Music*, d'Alembert repeated here how useless ratios were for the theory of musical art, where the prime harmonics were sufficient. And as Rousseau had shown, considering the numerical ratios of sounds could not explain the pleasure in listening to them. Only a false scientific air was obtained, as he also wrote in the *Elements of Music*. No misuse was to be made of Geometry in music, therefore, as it was not to be abused in physics, or in metaphysics. Nor could reasonings based on physics be of any use to the theory of musical art, as claimed by Rameau, who only advanced hypothetical conjectures about the elasticity of air particles. "Des faits, & point de verbiage; voilà la grande regle en Physique come en Histoire." ["Facts and not hot air; this is the important rule in physics, as in history."] For this reason, the long article in the *Encyclopédie* ended by referring the experience of Giuseppe Tartini (1692–1770), from the *Treatise on Music*, about the third different sound obtained in appropriate cases by playing only two others.[104]

At the entry CORDES (*Vibrations des*) ["Strings (Vibrations of)"], our French mathematician repeated that Taylor was wrong, because the string can assume an infinity of other figures, besides the sine, and that Euler had solved the problem after him analogously, but by a longer method.

At the entry GAMME ["Gamut"], he, as one of the judged operating in the Académie des Sciences, renewed his criticism of Rameau: "... cette compagnie n'a jamais pretendu approuver le systême de Musique de M. Rameau, comme renfermant une science *démonstrée* ..., mais seulement comme un systême beaucoup mieux fondé plus clair, plus simple, mieux lié, & plus étendu qu'aucun de ceux qui avoient précedé; ... les consequences qu'on tire de cette résonnance n'ont point & ne sauroient avoir l'évidence des théorêmes d' Euclide; elles n'ont pas même toutes un égal degré de force & de liaison avec l'éxperience fondamentale. ... il ne range sa théorie musicale que dans la classe des probabilités." ["... this assembly has never claimed to approve of the Mr. Rameau's musical system as

[103] D'Alembert 1751d, p. 104.

[104] D'Alembert 1751d, p. 105.

containing a *proven* science…, but only as a much better grounded, clearer, more simple, better connected and more extended system than all the preceding ones; …the consequences that can be derived from this resonance do not have at all, and could not have, the evidence of Euclid's theorems. They do not all have the same degree of force and connection with the fundamental experiment. … he only sets his musical theory within the class of probability."][105]

Under MÉCANIQUE, d'Alembert ratified that, as they were not algebraic, the 'mechanical' curves of Descartes had been termed 'transcendent' by Leibniz. 'Mechanics', with Newton, had become a science of forces. And yet, for him "…le philosophe doit pour ainsi dire, détourner la vue de dessus les causes motrices, pour n'envisager uniquement que le mouvement qu'elles produisent; il doit entiérement proscrire les forces inhérentes au corps en mouvement, êtres obscurs & métaphysiques, qui ne sont capable que de répandre les ténèbres sur une science claire par elle-même." ["…the philosopher must, so to speak, turn his attention away from the driving causes, to consider only the movements produced; he must totally eliminate the forces inherent in bodies in movement: obscure, metaphysical beings which are only capable of expanding darkness over a science which is, in itself, clear."][106]

The French follower of the Light continued to recognize the merits of Rameau, without, however, he could keep quiet about the excessive claims invading his own field. "L'expérience seule est donc la base de l'Acoustique, et c'est de là qu'il en faut tirer les regles. Un célebre Musicien de nos jours a déjà frayé cette route, … sur un grand nombre de points importans ne paroit pas susceptible de démonstration, …a été pour nous le Descartes de la Musique." ["Only experience, therefore, is the basis of acoustics, and it is from here that we must derive the rules. A renowned musician of our days [Rameau] has already opened up this route, …[although] regarding a large number of points, demonstration does not seem possible, …he has been for us the Descartes of music."] Compared with painting, music revealed greater possibilities for d'Alembert. "Une succession de couleurs, …, ne pourroit représenter ni rappeller une succession de sons, comme une certaine succession de sons peut nous retracer l'idée ou le souvenir de la lumiere; …" ["A succession of colours, …, could not represent, or recall a succession of sounds, as a certain succession of sounds could track down the idea and the memory of light."][107]

It is clear that d'Alembert took part with pleasure in the musical life of Paris, and in his considerations, the echoes could be heard of the great hubbub aroused by the happening that was to open up a new epoch in all Europe. Why else would he have quoted so often and so readily Italian musicians? While the music composed by the heirs of Lulli and Rameau on subjects of mythology or antiquity dominated at court, some theatres in the city presented Italian companies with a completely different repertoire, linked with the daily life of the bourgeois classes. Until, on

[105]D'Alembert 1758a, pp. 193–194. D'Alembert 1758b, p. 469.

[106]D'Alembert 1778, p. 329.

[107]D'Alembert 1759, pp. 163 and 292.

August 2nd, 1752, the work by Giovanni Battista Pergolesi (1710–1736) *La serva padrona [The Bossy Maid]*.[108] was re-presented with great success. Between those favourable to it, and those who were contrary, the conservatives of the old pompous style, the *querelle des bouffons* ["jesters' dispute"] broke out. The cultural and political world divided into two, the *coin du Roi* [King's Corner] against the *coin de la Reine* [Queen's Corner][109]: d'Alembert amused himself more listening to Pergolesi in the Queen's Corner. We can easily imagine our illegitimate, yet noble, mathematician shining among the ladies in the sitting-rooms and the theatre boxes. And yet he was involved in music at a deeper and more direct level than we might suspect.

In about 1752, in an autobiographical fragment, he wrote: "Je parle par expérience. J'ai examiné plusieurs scènes de récitatif; je les ai chantées à l'italienne, en supprimant les longues cadences et les tenues: il en résulté un chant assez semblable au récitatif italien, et j'ose le dire, qui ne déplaîroit pas pour peu qu'on voulût s'y prêter. J'en ay fait l'essay devant quelques personnes qui en on jugé comme moy. ... Je crois au moins que ce seroit une trés bonne méthode pour tous les musiciens d'essayer ainsi leur récitatif." ["I speak from experience. I have examined many scenes of recitative; I have sung them in the Italian manner, cancelling the long cadences and the prolonged notes from it: the result was a song very similar to the Italian recitative, and I dare say that it would not be displeasing, even if one is little inclined. I carried out the test in front of some people, whose judgement was the same as mine. ... I believe at least that this would be a very good method for all musicians, to put their recitative to the test in this way".][110] Thus he had not only limited himself to the theory, but he had also verified it in practice with his voice and his ears in front of a public!

A passion for Italian music returned in various writings of Monsieur d'Alembert. On the contrary, he found singing in French "... défectueux pour ne rien dire de pis." ["... defective, not to say anything worse."] The advantage of the Italians was that their music conformed to the language, whereas this could not happen with French composers. With them, actually, the final mute *e′s* were stressed, sung and thus they were perceived. "... notre nation en général est peu faite pour la musique, ... Le génie des Italiens est vif et prompt, le génie françois, lent et paresseux pour la musique. ... les François n'aiment pas la nouveauté en musique." ["... our nation in general is not really made for music, ... The genius of the Italians is ready and lively, the French genius is slow and lazy as regards music. ... the French do not like novelties in music."][111]

The work by Rameau that our mathematician of the Enlightenment appreciated most was the comedy-ballet, *Platée*, because it was intermediate between Italy and France. "Peut-être la *Serva padrona* auroit-elle été moins goûtée si *Platée* ne nous

[108] Pergolesi 1733.

[109] Rousseau 1966, pp. 492–495. Fubini 1971, pp. 92–95.

[110] D'Alembert 1752b, p. 181.

[111] D'Alembert 1752b, pp. 168–169, 171–172 and passim.

avoit accoutumés à cette musique." ["Maybe *La serva padrona* would have been appreciated less if *Platée* had not accustomed us to this kind of music."] Even if the French opera surpassed Italian opera in its choruses and its dances, it had lost people's interest. "Dans l'état où il est ce ne sont que de grandes marionnettes pour des enfants décrépits." ["In the state in which it stands, they are only large marionettes for decrepit children."] He would have preferred recitatives to take *La serva padrona* as their model, with little harmony and a simpler melody. "La musique n'est pas simplement un bruit agréable, … c'est une langue ou une espèce de langue faite pour produire quelque sentiment dans l'âme." ["Music is not simply a pleasant noise, … it is a language, or a kind of language made to produce some sentiments in the soul."][112]

"Nous devons aux opéras-bouffons italiens, que nous entendîmes à Paris en 1751, les premiers rayons qui nous ont éclairés sur l'imperfection de notre musique et la lumière qui depuis y a succédé." ["We owe to the Italians of comic operas, which we first heard in Paris in 1751, the first rays that enlightened us as to the imperfection of our music, and the light that came after it."] Some, instead, were to feel offended because in the noble theatre of the Opéra "… des bouffons, des baladins, à peine dignes des trétaux d'une guinguette!" ["… buffoons and acrobats, hardly worthy of the stage boxes of a tavern!"] had been presented. For a citizen from Geneva, [Rousseau], who had written the *Lettre sur la musique françoise* ["Letter on French music"], another "lettre de cachet" ["letter of arrest"] had already been prepared. Fortunately, it was never sent, otherwise the jesters' war would have finished as Tacitus had said: "*ubi solitudinem fecere, pacem appellant; où ils ont fait un désert, ils croient avoir fait la paix*." [" *Ubi … appellant*; where they have made a desert, they believe that they have made peace."][113] The comparison between the absolutist French regime, with its censure always impending, and the Roman empire, with its threatening legions, clearly renders the idea of how deeply the controversy about music had touched French society. For d'Alembert, it appeared less frivolous and insignificant than a host of following historians of sciences were to imagine.

Compared with the restrained movements of courtly music, the lively notes of *La serva padrona*, with their clear, and malicious overturning of social hierarchies, met with the favour of a public that was delighted and open towards similar prospects. Among those who looked in this direction, there was Jean Jacques Rousseau (1712–1778), who not only orchestrated the return of the Italian comic interludes, but also ventured to take part himself, with a composition of his own: *Le devin du village [The Village Fortune-teller]*[114] However, the imitation made all too many concessions to the dominant French taste of the period. While the *libretto* related how the little shepherdess re-conquered, by means of a stratagem, her enamoured shepherd-boy, led astray by a powerful city lady. It was to be expected without doubt

[112] D'Alembert 1752b, pp. 178, 180 and 182.

[113] D'Alembert 1887, pp. 162–163.

[114] Rousseau 1752.

from the famous citizen of Geneva that natural simplicity would triumph over the capricious, vain temptations of riches.

> Ici de la simple nature
> L'Amour suit la naïveté.
> [Here from simple nature
> Love follows the spontaneity.]

His recitative, however, proceeded in a slow, dignified, conventional manner, with the defects criticised by d'Alembert and by himself in other French people. And we would not find among his arias any of the expressive force like the one sung by Serpina, who, as a maid, really behaved like a mistress. The rhythm of the steps forward taken in *The Village Fortune-teller* remained that of the palaces inhabited by the *ancien régime*, because music did not succeed yet in coming out into the open, in spite of the seeming liberating subject.

The notes do not seem to me to flow in a perfectly consistent manner with the words and the theory. To find something somewhat lively, it is necessary to wait for a "forlana" dance. While the best musical expression is to be found in the Chorus, and in the final concerted piece, where the moral of the story was celebrated, dancing the minuet.

> L'Amour ne sai guère
> Ce qu'il permet, ce qu'il défend;
> C'est un enfant, c'est un enfant.
> [Love does not know at all
> What it permits, what it forbids;
> It's a child, it's a child.]

To the initial contrasts and passions between the two lovers happened a mournful music without any *verve*.[115]

Although only momentarily successful, even the marchioness of Pompadour had once played the part of pastor Colin, the moral fable in music was to be better substantiated by the theory expounded in the *Lettre sur la musique françoise [Letter on French Music]*. Rousseau argued with phonetic details about why the French language was not suitable for music. Sometimes he and d'Alembert used the same words, but with a curious inversion. The philosopher from Geneva attributed to the orchestra players of the Opéra in Paris, as a defect, the event that they were "worthy of the stage boxes of a tavern". The same expression instead, according to the mathematician, had been attributed, by their adversaries, to the Italian strolling players. On other points, the two were in agreement. "…s'il y a en Europe une langue propre à la musique, c'est certainement l'italienne; car cette langue est douce, sonore, harmonieuse, et accentuée plus qu'aucune autre, et ces quatre qualités sont précisément les plus convenables au chant." ["…if there is a language suitable for music in Europe, it is undoubtedly Italian; for this language is sweet, resonant,

[115]Rousseau 1752, p. 203.

harmonious and stressed more than any other; these four qualities are precisely the most appropriate ones for singing."][116]

Our Enlightenment philosopher from Geneva compared Lulli and Rameau with many Italian composers, such as Leonardo Leo (1694–1744), Pergolesi, Baldassarre Galuppi (1706–1785), Nicola Porpora (1686–1768), Niccolò Jommelli (1714–1774), David Perez (1711–1778), Gioachino Cocchi (1715–1804), Arcangelo Corelli (1653–1713), Giovanni Bononcini (1670–1747) and Leonardo Vinci (1690–1727), to whom he also added the Catalan composer Domingo Terradellas (1713–1751). He compared elaborate counterpoint, double fugues or mirror fugues (in the Flemish style) to the portals of Gothic cathedrals, barbaric relics of bad taste. As the model of a duet that did not violate the rules of naturalness, he indicated, from *La serva padrona*: "Lo conosco, a quegli occhietti, furbi, ladri, malignetti", [...I know him, with those cunning, stealing, mischievous little eyes], calling up for Pergolesi even a Homer. The disparaging term "ariettes", foisted on the elegant Italian "arias", for him betrayed the ignorance of the conceited French towards them. As regards French composers, Rousseau found their slow movements monotonous: "...ton lamentable qui règne perpétuellement dans notre opéra, ...ils voudroient être tristes, et ne sont qu'ennuyeux; ..." ["...a mournful tone that dominates perpetually in our Opéra, ...they [the monologues] intend to be sad, and they are only boring; ..."].[117]

From Lulli's *Armide et Renaud*, Rameau took as an example of a successful modulation the famous monologue of the witch "Enfin il est en ma puissance" ["He is finally in my power"]. But, for the musicologist-musician-writer from Geneva, the commendation would change into a "...véritable satire, ...régularité scolastique ..." ["...veritable satire, ...scholastic regularity ..."]. With its modulations, Lulli's music was not capable of representing that storm of passions experienced by such a character in her heart, torn between opposing sentiments: who had passed from the desire for *vendetta*, cutting her enemy's throat and stabbing him, to loving adoration. She changed tonality when it was not appropriate, whereas she maintained the same one where there was a dramatic sentimental transformation in the lines.[118] It was curious that he was rebuking Lulli for the fault that he himself was to reveal in *The Village Fortune-teller*.

In the musical entries written for the *Encyclopédie*, Rousseau made reference to Rameau without arguing excessively, but recognizing his importance. He rather criticised, without naming them, those who justified consonances with the concordance of vibrations-strokes in the sounds. Among the first scholars to notice the flaw, he advanced the objection "...que la première vibration de chacun des deux corps sonores commence exactement avec celle de l'autre, car si l'une précédoit un peu,

[116]Rousseau 1839, *Correspondance* t. I, pp. 193–194 and 208–209; 1839, *Beaux-Arts* t. I, pp. 177, 182–183 and passim. Rousseau 1966, pp. 706–707. Rousseau gave Colin an acute 'female' voice, despite his presumed naturalism.

[117]Rousseau 1839, *Beaux-Arts* t. I, pp. 205, 212, 214.

[118]Rousseau 1839, *Beaux-Arts* t. I, pp. 222–231.

elles ne concourroient plus selon le rapport déterminé ou peut être ne concourroient jamais, & par conséquent l'intervalle devroit changer, …" ["…the first vibration emitted by each of the two sounding bodies must begin exactly together with that of the other one, otherwise, if one preceded the other, even slightly, they would no longer agree in accordance with the determined ratio, or perhaps they would never agree, and consequently the interval should change, …"]. Nowadays, we would say that the two sounds would be out of phase, even if they have the same frequency. Besides, why did organs and harpsichords that altered the numerical ratios, with the temperament, not produce a "…cacophonie …effroyable …" ["…fearsome …cacophony …"]? Our Enlightenment scholar from Geneva seemed somewhat better disposed towards those who explained consonances by means of their greater simplicity. Perhaps he had read it in Euler, but in any case, it did not agree with experience, and he did not take it as proven. Thus the ear remained the best judge for him, too.

In describing a rational chromatic scale, he let it slip (printing error? distraction?) that this divided tones into two equal parts. But then, as it is right, he reserved the 12 "perfectly" equal semitones only for Aristoxenus. In order to be able to modulate and transpose, having thus arrived at the need to temper notes, he registered the conversion of Rameau to the equable one, with the 11 proportional means in the octave; but he also referred the criticism brought against it by certain other musicians and instrument makers. While he did not follow Rameau's construction of dissonances, regarding his equable temperament he wrote: "…ce chapitre a été fait par deux hommes qui ne s'entendoient pas même l'un l'autre, savoir un mathématicien & un musicien." ["…this chapter [of *Génération harmonique*] was written by two [different] men, who could not even understand each other, in other words, a mathematician and a musician."][119] The musicologist Rousseau does not seem to have appreciated here the greater advantages offered to musicians in general by the equable temperament.

To these attacks against French music made by our philosopher from Geneva, Rameau answered with his "Observations sur notre instinct pour la musique, et sur son principe" ["Observations on our instinct for music and on its principle"]. The composer and theoretician always put harmony in the first place, as capable of expressing passions and sentiments best, whereas the Italians, according to him, would prefer to imitate the noises and the movements in their comic genre. In the end, also Rameau wrote at length on Lulli's *Armide et Renaud*, driving back Rousseau's charges. Sure, the latter had adopted the principles of the former, but in reality, he did not know how to use them properly, and fell into several contradictions. In the scene (five, act II) of the monologue sung by Armida, he did not recognize the interplay of modulations between the weak minor tonality and the strength of the major. "Le mode change à chaque phrase, …" ["The mode changes with every sentence, …"].

[119]Rousseau 1751a; 1751b, p. 306; 1751c; 1751d; 1751e, p. 11.

In the prosody, our Enlightenment scholar from Geneva, would confuse short syllables and long ones. "…le goût, le sentiment, et l'oreille ne sont pas de trop pour y réussir et l'on n'est pas toujours également favorisé de ces dons. Il est vrai qu'ils sont presque inutiles dans le récitatif italien, où l'art a beaucoup plus de part que la nature: plus on y heurte désagréablement l'oreille par du chromatique et de l'enharmonique, mieux on croit y réussir, telle est la manie de l'École italienne, il n'est point du tout fait pour l'oreille: …" ["…good taste, sentiment and the ear are not redundant in order to succeed, and not all people are equally favoured by these gifts. It is true that they are almost useless in Italian recitative, where art plays a much greater role than nature: the more the ear is unpleasantly offended by the chromatic and the enharmonic, the better they think they succeed, such is the mania of the Italian school, it is not made for the ear at all: …"]. Lulli, instead, Rameau continued, though still ignorant at that time of the fundamental principles of harmony, was guided by sentiment, and would follow them instinctively.

With (too?) many details of sharps and flats, tonics, dominants and subdominants, cadences and dissonances with their presumed rules, the famous French composer rejected the accusations raised against the musician from Florence. The latter had been transferred to Paris by the knight Di Guisa to found, together with Louis XIV, the French Opéra filling with ballets. Rousseau, instead, would fall into contradiction, "…on y reconnaîtra l'ascendant de la passion sur la raison, …" ["…in him the ascendant of passion will be recognized over reason, …"]. And yet, in order to demonstrate that all the drama of scene five, full of contrasts and changes, was rich in the suitable (Italian?) chromatisms, Rameau was forced to add to the bass sharps and flats that Lulli had not written. "…on ne voit aucun de ces dièses ni bémols dans le chant, dans la basse, non plus que dans le chiffre: aussi faut-il plus que des yeux pour juger en pareil cas (…)." ["…none of these sharps can be seen, nor any flat in the song, in the bass or even in the figured bass: it takes more than the eyes to judge a similar case (…)."]

Sentiments were expressed effectively in the harmony chosen by the composer. "Il y a là un jeu chromatique qui ne paraît point effectivement dans le chiffre de Lully, mais qui paraît si bien être le fondement des différentes expressions, …" ["There exists here a chromatic interplay which effectively does not appear in the figured bass of Lulli, but which seems to be the foundation of the different expressions, …"]. However, if in a score not completely written the 'harmonic' ear of Rameau had to make up for the ellipses in accordance with his own theory of harmony implied in the song, was he not begging the question? What reason (or passion) was the reader to follow, then? The logical errors of Rameau, the rationalist, or the contradictions of the passionate Rousseau? The latter was accused by the former of being capable of seducing with his fine style. "…mais on ne doit s'attacher qu'à la vérité. L'homme sage craint et néglige des fleurs dont le parfum est souvent empoisonné." ["…but the only thing to which we should attach ourselves is the truth. The wise man fears and avoids flowers, the scent of which is often poisonous."][120]

[120]Rameau 1980, pp. 145, 179–180, 182, 185, 187, 189 and passim.

In the Italian style, we will judge with our ears which, however, are contaminated and prejudiced by the subsequent post-Pergolesian music. Armida's monologue sounds above all driven by tones of war and *vendetta*, rather than those of compassion and love. At least this sentiment, which starts uncertainly in the scene, without any great hopes, appears to be overwhelmed in the music by a sequence of events which lead inescapably to his death. In the interpretation that I listened to, noise is certainly not missing and the dramatic singing is accompanied by a heavy orchestra.

In that period, the critique would have been: "…un poignard à la main, prête à transpercer la poitrine de Renaud, endormi sur un lit de verdure! La fureur l'animait à sa vue, l'amour vint et s'empara de son coeur; l'une et l'autre l'affectèrent tour à tour. La pitié et la tendresse s'ensuivirent, et l'amour remporta la victoire." ["…dagger in hand, [Armide], ready to pierce Renaud's heart as he lies sleeping on the bed of branches! At his sight, the fury animated her, and love came to take control of her heart; both, in turn, stir her. Compassion and love followed on, and love will bring the victory."] The allegory represented the victory of Catholicism over the Protestant Huguenot heresy; this might have been imagined, seeing that the staging, the costumes and the dances must have helped considerably.[121] Also the musical entries of the *Encyclopédie* were placed in contradiction by the French composer with the affirmations contained in Rousseau's *Letter on Music* and criticised analytically, both here and in another booklet dedicated to them.[122]

On the spot, the musicologist and writer from Geneva put together another rejoinder, which, however, he was to make public only much later, at the time of his own *Dictionnaire de musique [Dictionary of Music]*. Here, he revised the entries that he had already written for the *Encyclopédie* and completed them in his own way and at his own time. For various reasons, he was effectively not satisfied with them.[123] In his "Examen de deux principes avancés par M. Rameau" ["Examination of two principles proposed by Mr. Rameau"], Rousseau rejected the idea that the melody could be subordinated and reduced to harmony, as the other one claimed. In the countryside songs of which he had experience, harmony did not play any part. "Quoique le principe de l'armonie soit naturel, …, le sentiment qui la développe est acquis et artificial, et c'est surtout en cette partie de la musique qu'il y a, comme dit très-bien M. d'Alembert, un art d'entendre comme un art d'exécuter." ["Although the principle of harmony is natural, …, the sentiment that develops it is acquired and artificial, …; it is above all in this part of music that there is, as M. d'Alembert quite rightly says, an art of listening as well as an art of executing."]

"Leur variété ne vient donc pas de l'harmonie, mais seulement de la modulation, qui appartient incontestablement à la melodie." ["Their variety does not come from

[121] Lully 2008, p. 17.

[122] Rameau 1980, pp. 190–191 and passim.

[123] Rousseau 1839, *Dictionnaire de musique*. Having finished it, he asked Alexis Claude Clairaut (1713–1765) to correct the mistakes. 1839, *Correspondance* t. V, pp. 124 e 164; t. VII, pp. 127–128.

harmony, therefore, but only from modulation, which undeniably belongs to the melody."] The soul would be touched by the accents emitted by a single voice, and not by the chorus, because they expressed passions better. If something could be aroused by harmony, it would be thanks to the great noise, "…comme l'ardeur des soldats par les instruments militaires, …" ["…like the ardour of soldiers with the military band instruments, …"]. Of the second principle, that harmony represented the sounding body, the Enlightenment scholar from Geneva cast doubts precisely on the physical and experimental part. The chords of the minor mode and the dissonances, which all musicians were using, could not come from the phenomena of resonance, as given in the experiences of Sauveur. Lower harmonics, actually, could not be heard. "…c'est ce que l'observation dément et que la raison ne peut admettre. […] Que M. Rameau prenne la peine de nous expliquer ce que c'est qu'une corde sonore qui vibre et ne résonne pas." ["…[lower harmonics] are what observation denies and reason cannot admit. […] Mr. Rameau should take pains to explain to us what a sounding string that vibrates and does not resonate means." He concluded "…pour l'honneur de la vérité …" ["…for the sake of truth …"], that Rameau was responsible for "…la mauvaise musique dont son école inonde le public depuis nombre d'années; …son système harmonique est insuffisant, mal prouvé, fondé sur une fausse expérience." ["…the bad music with which his school has been deluging the public for several years; …his harmonic system is insufficient, badly proved, and founded on a false experiment."][124]

Although he was moving in other directions, which were to make him much more famous with historians of philosophy and politics, Rousseau was to continue to occupy himself with music.[125] Albeit in different contexts, he was to maintain the attitude that had been graven in the clash with Rameau. Without this original stimulus, it would have been difficult to understand some of his subsequent writings, such as the *Essay On the Origin of Languages Where People Speak of Melody and Musical Imitation*. It is interesting that, as they knew how to sing better, here the ancient Greeks took the place of the Italians. Did he know, perhaps, the thesis of Vincenzio Galilei, who had worked back to the Greek monodies in order to renew Italian melodies? In his dispute against the analogy between sounds and colours (for example, in Newton), he let it slip that the number of vibrations in sounding bodies was *non* measurable, when, on the contrary, from Mersenne onwards, someone was to succeed somehow.[126]

In the *Encyclopédie* entry for "Genève" ["Geneva"], d'Alembert had criticised its religious music: "Le chant est d'assez mauvais goût; et les vers français qu'on chante, plus mauvais encore." ["The singing is of extremely bad taste; and the

[124]Rousseau 1755, pp. 12, 14, 18 and 27–29.

[125]Rousseau 1989; 1926; 1839, *Dictionnaire de musique*.

[126]Rousseau 1989, pp. 79–85, 93 and 95. In the heavy commentary of the editress, only an excessively small, insignificant reference is made to Rameau. The bad habit of cutting music, for academic reasons, is not found, unfortunately, only among historians of the sciences. Cf. Fubini 1971.

French verses are sung even worse."] Furthermore, he invited the inhabitants of the Swiss city to make up for the lack of theatrical representations, linked with that "... âme atroce, aussi bien qu'un esprit éclairé." ["... atrocious soul, and at the same time illuminated spirit."] of Jean Calvin (1509–1564). After praising the Republic for its laws, its culture, its almost deistic religion and the peace regained with respect to the tormented surrounding nations, the French mathematician proposed finding a remedy for the dreaded licentiousness of actors and actresses, with severe laws regarding their conduct, instead of rigorously preventing their access.[127]

Touched on the raw as regards his own city of origin, and in an attempt to reject the advice, Rousseau answered with a particularly long open letter, which among other things started to confirm in public the break down with the encyclopedists. We may wonder why he had not been commissioned to write about Geneva, his own city. With a wealth of details (too many?) and classical references, which were not always precise, our Swiss-French citizen argued that actors were bound to be appreciated by the public. In every culture, it had its characteristics inclinations, and in order to be successful, it was their passions that had to be seized. "Il n'y a que la raison qui ne soit bonne à rien sur la scène." ["It is only reason that is of no use on the stage."] The French therefore were hated in London, talk was of pirates in Tunis, at Messina, they staged "... une vengeance bien savoureuse ..." ["... a very tasty *vendetta* ..."], at Goa Jews were burnt. "Un peuple voluptueux veut de la musique et des danses. Un peuple galant veut de l'amour et de la politesse. Un peuple badin veut de la plaisanterie e du ridicule." ["A voluptuous people wants music and dancing; a gallant people wants love and kindness; a humorous people wants jokes and the grotesque."]

A famous pastor in Geneva wondered: "... si l'on débite leur morale incontrollée, si les personnes qui exercent cette profession mènent une vie licencieuse et servent à corrompre les autres, si de tels spectacles entretiennent la vanité, la fainéantise, le luxe, l'impudicité, ... il vaut mieux renoncer à cette sorte d'amusement." ["... if we charge them with uncontrolled morals, if the people [actors and actress] who exercise this profession lead a licentious life and only serve to corrupt others, if such spectacles nourish vanity, laziness, luxury, wantonness, ... it is better to renounce this sort of amusement."] Rousseau did not spare a dig at his old enemies. "Noble acteurs de l'Opéra de Paris, ah! si vous eussiez joui de la puissance impériale, je ne gémirais pas maintenant d'avoir trop vécu!" ["Noble actors of the Opéra in Paris, ah! If you had enjoyed the imperial power, now I would not moan that I have lived too long!"][128]

If he had not been protected by the ladies at court, as at the time of Nero, would the philosopher from Geneva perhaps have been killed by those French singers that he denigrated? By invoking their banishment, would he have come dangerously close to the Plato of the *Republic*, who would have liked to exclude all poets and lascivious music from it? "Platon bannisait Homère de sa république, et nous

[127]Rousseau 1926, pp. 294, 292, 102–105 and passim.

[128]Rousseau 1926, pp. 124, 123, 122, 128–129 and passim.

souffrirons Molière dans la nôtre!" ["Plato would banishe Homer from his Republic, and we will suffer Molière in ours!"] "... I looked at myself as a member of Plato's republic", he was later to confess.[129]

We shall cut the story short, so that we can return as quickly as possible to music, and to the mathematical sciences. Perhaps as a (Sicilian?) *vendetta* or maybe out of Calvinistic rigour, Rousseau tried to demonstrate in several hundred pages that moral beauty was innate in us, whereas, among many others, the spectacles (above all, those organised for the city?), would have irremediably ruined this for us. On this whole subject, every reader of our great epoch, rich in cinema, newspapers and television, will have his or her own regular more or less indulgent opinions. According to the prophet from Geneva, the theatre would succeed in changing public opinion. "Un des infaillibles effects d'un théâtre ... sera de changer nos maximes, ou, si l'on veut, nos préjugés et nos opinions publiques; ..." ["One of the infallible effects of a theatre ... will be to change our principles, or, if you prefer, our prejudices and our public opinions; ..."].

We will limit ourselves here to gleaning a few other small pearls, if we can find some. His judgements on women were implacable. "... elles ne savent rien, quoiqu'elles jugent de tout; ..." ["... they do not know anything, although they express judgements about everything; ..."]. "... ils auront tant d'esprit que vous voudrez, jamais d'âme; ..." ["... they [the writings of women] have as much spirit as you want, but never soul; ..."]. Romanticism was still to come. "... l'esprit général de la galanterie étouffe à la fois le genie et l'amour." ["... the general spirit of gallantry extinguishes genius and love in a single blow."] "... celui qui connut le véritable amour et l'a su vaincre, ah! pardonnons à ce mortel, s'il existe, d'oser prétendre à la vertu!" ["... he who has known true love and has succeeded in defeating it, ah! We forgive this mortal, if he exists, for daring to aspire to virtue!"] On this page, Rousseau was disguising himself, but he was not hiding that he would have liked to subtract this passion from young men, because he like Plato considered it responsible for making them effeminate.[130]

At home on the Swiss Alps, our Enlightenment scholar from Geneva remembered how they played the flute and sang psalms in four-part harmony. "... l'harmonie forte et mâle de Goudimel, depuis si longtemps oubliée de nos savants artistes." ["... the strong, masculine harmony of Goudimel, forgotten for so long by our learned artists."][131] Claude Goudimel (1514–1572) was a composer of masses, motets, and polyphonic psalms, who ended up by being killed at Lyon, because he was a Huguenot. Could this Rousseau be the same one as the *Letter on French music*? The same one who had praised the singing and the recitative of the blandishments in *La serva padrona*? The silence regarding music is so complete in this text, more pertinent than numerous other digressions, that it sounds deafening.

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[129]Rousseau 1926, p. 256. At other points, pp. 247, 260 and 272, he praised its laws. Rousseau 1966, p. 459.

[130]Rousseau 1926, pp. 201, 169, 239, 240 and 258.

[131]Rousseau 1926, p. 185.

Was he perhaps ashamed to have to admit that he had changed his mind about Pergolesi's farce? Would the melodic revolution introduced by the Italian language at Paris have become a representation of female indecency at Geneva, only suitable to corrupt young men? Did it disturb the matrimonial order of nature? Besides the essay finished up amid the (masculine?) dances of sturdy Swiss soldiers to the sound of pipes and drums "... esprit martial convenable à des hommes libres." ["... martial spirit suitable for free men."] Anyway, he certainly did not lack the ability to justify himself, distinguishing in this case the irredeemable corruption of Paris from the presumed purity of Swiss maidens. While the blatant contradiction of his behaviour was not his problem. In a note of *The Social Contract*, he was later to appreciate how d'Alembert had described appropriately the classes of inhabitants at Geneva. "As far as I can see, no other Frenchman has understood the true meaning of the word *citizen*".[132]

Luckily, d'Alembert took a lot fewer pages for his answer. Unfortunately, he did not return to the subject of music, either. And we can easily imagine how often the sophisticated Enlightenment scholar from Paris overturned the implacable judgements inflicted by the severe philosopher of natural spontaneity, above all those about women. But he could not avoid a remark about the lyrical *intermezzo* of the latter. "... vous aurez longtemps la douleur de voir *Le devin du village* détruire tout le bien que vos écrits contre la comédie auraient pu nous faire." ["... for a long time you will have the sorrow of seeing *The village fortune-teller* destroy all the good that your writings against the comedy have achieved."][133]

Like a true modern intellectual, Rousseau offered opinions with fluency, passion and good style: by now a member of the bourgeoisie, free to do and say anything he wanted, including sadly contradicting himself (it seems that he did not like laughing and joking). Years later, he listed the entries of an independent *Dictionnaire de musique [Dictionary of Music]* of his own. In what way would this be different from the *Encyclopédie*?

Here, at last, we find the term ACOUSTIQUE. "L'*acoustique* est proprement la partie théorique de la musique; c'est elle qui donne ou doit donner les raisons du plaisir que nous font l'harmonie et le chant, qui détermine les rapports des intervalles harmoniques, qui découvre les affections ou propriétés des cordes vibrantes, etc. ..." ["Strictly speaking, acoustics is the theoretical part of music; it is this that provides, or should give, the reasons for the pleasure created by harmony and by singing, which determines the ratios of the harmonic intervals, discovers the effects or the properties of vibrating strings, etc. ..."].[134] This entry was missing, if I am not mistaken, in the first edition of the *Encyclopédie*, but it appeared in others, for example, the version printed at Leghorn in 1770. To my surprise, it corresponds word for word, with very few, insignificant variants, to that of Rousseau.

[132]Rousseau 1926, pp. 280–281, 43–53 and passim. Rousseau 1945, p. 60. Rousseau 1966.

[133]Rousseau 1926, p. 327.

[134]Rousseau 1839, *Dictionnaire de musique*, t. I, p. 53. Rousseau 1966, p. 659.

Nowadays, we find acoustics as a branch of physics,[135] but for Rousseau and the *Encyclopédie*, it was "the theoretical part of music". Whereas for us, the theoretical part of music was to become the theory of harmony according to Rameau. We shall understand better in the following chapter when, where, how, and why this change took place.

In the other entries, our agitated Swiss writer, by now persecuted and wandering around Europe, copied those of the *Encyclopédie*: always criticising Rameau as regards the lower harmonics, and denying the theory of coincidences for consonances. However, he expanded at will, and added some interesting details. We shall refer here the most significant ones. Following d'Alembert, he would not want to base his justifications on ratios between numbers, but then he used them for dissonances. After much searching back and forth, between d'Alembert, Rameau, Tartini and himself, in the end he stopped at "... le texte de la nature ..." ["... the text of nature ..."]. He took advantage of the entry DITHYRAMBE ["Dithyrambic", Dionysiac] to continue to move away from other Enlightenment scholars. "C'est fort mal fait sans doute de s'enivrer, surtout en l'honneur de la divinité; mais j'aimerois mieux encore être ivre moi-même que de n'avoir que ce sot bon sens qui mesure sur la froid raison tous les discours d'un homme échauffé par le vin." ["Obviously, it is very wrong to get drunk, above all in honour of the divinity, but I myself would prefer to be still tipsy rather than having only this crass good sense, which measures on the basis of cold reason all the discourses of a man heated up by wine."][136]

Under GÉNIE ["Genius"], the passion and the 'jesters' dispute' re-emerged. "Veux-tu donc savoir si quelque étincelle de ce feu dévorante t'anime; cours, vole à Naples écouter les chefs-d'oeuvre de *Leo*, de *Durante*, de *Jomelli*, de *Pergolèse*. Si tes yeux s'emplissent de larmes, si tu sens ton coeur palpiter, si des tressaillements t'agitent, si l'oppression te suffoque dans tes transports, prends le Métastase et travaille; son *génie* échauffera le tien, ... Que t'importeroit de le connoître? tu ne saurois le sentir: fais de la musique françoise." ["Do you want to know, then, whether some spark of this devouring fire possesses you? Run, fly to Naples to listen to the masterpieces of Leo, Durante [Francesco, 1684–1755], Jommelli, Pergolesi. If your eyes start brimming with tears, if you feel your heart beating, if tremors agitate you, if oppression suffocates you in your transport, take Metastasio [Pietro, 1698–1782] and start working; his genius will heat up your own, ... Aren't you interested in knowing him? You will not be able to hear him; then make music in the French style."][137]

It remains obscure why Rousseau did not want to make the hearts of Swiss maidens beat strongly. Perhaps they were a hunting reserve for his novels. On the contrary, the reason why the famous "*rans-des-vaches*" ["cattle call"], played by Swiss herdsmen on their bagpipes, had been severely prohibited for Swiss troops

[135] Resnick & Halliday 1961, pp. 392–447.

[136] Rousseau 1839, *Dictionnaire de musique* t. I, pp. 260–263.

[137] Rousseau 1839, *Dictionnaire de musique* t. II, p. 67.

was clear: the soldiers would have died of nostalgia, or they would have deserted to return home. Our writer from Geneva reproduced an 'aria' from China, an 'aria' from Persia, and two songs of the savages from America. "On trouvera dans tous ces morceaux une conformitè de modulation avec notre musique, qui pourra faire admirer aux uns la bonté et l'universalité de nos règles, et peut-être rendre suspecte à d'autres l'intelligence ou la fidélité de ceux qui nous ont transmis ces airs." ["In all these pieces, a conformity of tonality with our music will be found, which may lead the ones to admire the goodness and the universality of our rules, but will perhaps lead the others to suspect the intelligence and the faithfulness of those who have transmitted the pieces of music."] (Fig. 11.9).[138] To some extent rightly, Claude Lévi-Strauss considered him to be a master of anthropology.[139] May we also extend the idea to ethnomusicology?

In his *Dictionnaire*, Rousseau reprinted the pieces of music chosen for the *Encyclopédie*, changing, however, in some cases, the notes, and above all the rhythm and the bars. The *Air Persan* became the *Chanson Persane*, the *Air des Sauvages de l'Amerique* took as its title *Chansons des Sauvages du Canada*. Clearly, 'Aria' led people to think too much of civilised, corrupt Europe, whereas 'Chanson' sounded much more spontaneous. As no indication was added, we have to attribute the comments compiled for the "Planches" [Tables] on music to the person who, in 1765, had sent a letter, quoted, to the *Mercure de France*, proposing to substitute seven vowel sounds instead of the syllables of Guido D'Arezzo for the notes: "D. L." To him, less than partial information might arrive from the writings of missionaries, regarding Chinese music: "…c'est sans regle, sans science & comme par hazard, …" ["…it is without rules, without science, as if composed at random, …"]. As regards the music of the Turks, the Arabs and the Persians, he recognized the link with Greek theory, but curiously, he made reference rather to that of Aristoxenus. Those interested in the system of notation by means of figures, invented by Rousseau for music, will find a description here.[140]

Among the many others known to us, Rousseau made reference by name also to a Wallis, to a Descartes and even to a "Galilée". The latter absolutely might be Vincenzio Galilei senior, rather than Galileo junior. Because, under the entry SON ["Sound"] in this *Dictionary of Music*, he rightly attributed to "…Vincent Galilée, que les *sons* sont entre eux, non comme les poids tendants, mais en raison

[138]Rousseau 1839, *Dictionnaire de musique* t. II, pp. 192. In the 1839 edition, the table quoted with the pieces of music does not exist. But it is again found in the 1995 edition, v. V, pp. 1190–1191, which takes up, with variants, the "Planches Musique" of the *Encyclopédie*.

[139]Rousseau 1989, pp. XXVIII–XXX. Aligned with Rameau, Thomas Christensen has failed to notice the clear doubt of Rousseau regarding the reliability of missionaries; Chistensen 1993, p. 296.

[140]Diderot & d'Alembert 1751–1776, "Planches musique" pp. 21, 3–5. Pl. IV. Judging by editorial convention, but the problem appears to be much more complex, the entries not signed were generally compiled by Diderot himself, even if not always; Diderot 1976, pp. 2–12. Diderot [?] 1765. Did "D. L." stand for Diderot, as certain indications suggest? Or did it stand for a D'Alembert Le Ronde, perhaps corrected by Diderot?

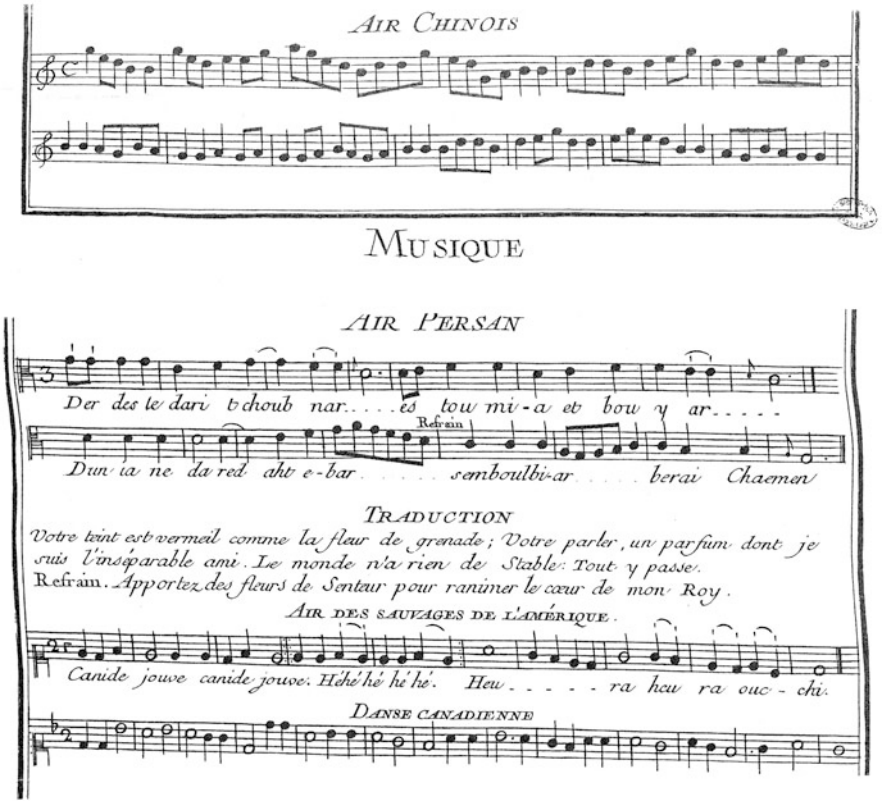
AIR CHINOIS
MUSIQUE
AIR PERSAN
Der des le dari b choub nar. . . . . es tow mi-a eb bow y ar. . . . .
Refrain
Dun ia ne da red aht e-bar . . . . . semboulbi-ar . . . . . berai Chaemen
TRADUCTION
Votre teint est vermeil comme la fleur de grenade ; Votre parler, un parfum dont je suis Vinséparable ami. Le monde n'a rien de Stable: Tout y passe.
Refrain. Apportez des fleurs de Senteur pour ranimer le cœur de mon Roy.
AIR DES SAUVAGES DE L'AMÉRIQUE.
Canide jouve canide jouve. Hehé hé hé hé. Heu - - - - - ra heu ra ouc - chi.
DANSE CANADIENNE

Fig. 11.9 How Rousseau transcribed 'arias' from China, from Persia and those of the American savages (Rousseau, "Planches musiques", in *Encyclopédie*, pl. III, Fig. 4, pl. IV, Figs. 1–3; Diderot & d'Alembert 1751–1776)

sous-double de ces mêmes poids." ["... Vincenzio Galilei, that the sound stand in a ratio not like weights which stretch [the strings], but in a ratio with the square root of the weights themselves."] For a possible explanation of simultaneous sounds, here he also quoted the name of Pietro Mengoli (1625–1686). Curious that, among the former, it was the almost musician and passionate Rousseau who made a clear distinction between frequency, intensity, timbre, and speed of a sound, also correcting an error made by father Mersenne. He offered a good presentation of the idea that sounds formed a *continuum* limited in its frequency between 30 and 7,552 vibrations a second, as Euler sustained. That he had read in the *Principes d'acoustique [Principles of Acoustics]* by Denis Diderot.[141]

[141]Rousseau 1839, *Dictionnaire de musique* t. III, pp. 122 and 125–1301. Diderot 1975, pp. 251–252.

He praised Tartini, and wrote about Rameau that nobody had read him, but that everybody knew of him, thanks to d'Alembert. He continued, naturally, to talk about himself.[142] And yet he was also to arrive at the point of accusing the French mathematician of theft and plagiarism. "In his [d'Alembert's] *Elements of Music*, I had found many things taken from what I had written in the *Encyclopaedia* on this art, which I gave him many years before the publication of his *Elements*. I ignore what role he may have had in a book entitled *Dictionary of fine arts*; but I found some articles copied word for word from mine; and this happened long before these articles were printed in the *Encyclopaedia*."[143]

In the *Confessions*, Rousseau often dealt with music in various guises. "My passion for this art made the idea of making me a musician come into her [Madame de Warens'] mind." "I was so incapable of reading a tune at first sight that, in the brilliant concert, ..., it was not possible for me to follow the execution for one moment, to know whether they were playing correctly what I had in front of my eyes, which I had composed myself." "I speak of music. Undoubtedly I must have been born for this art, seeing that I started to love it from my infancy, and it has been the only one that I have loved constantly at all times. ... after a whole lifetime of practice, I have never succeeded in singing extempore with confidence." "... I devoured my treatise on harmony [by Rameau]; but it was so long, so verbose, so badly organised that I understood that I needed a considerable length of time to study it and make it clear." In the period in which he tried to become a musician, it was particularly curious that he was taught to fence by means of music. Our unfulfilled musician and successful man of letters wrote that he had invented a new system to note down music by means of numbers. He submitted it to the Académie in Paris, who criticised it as already known and defective, while Rameau rejected it because it was unsuitable for a reading convenient to be plaied.[144]

While he related the success of his *The Village Fortune-teller*, he could not remain silent about Pergolesi's *La serva padrona*. "When I composed my intermezzo, my mind and soul were full of Italian operas; these gave me the idea, and at that time I was very far from expecting that they would hear it beside the others. If I had been a plagiarist, what a lot of thefts would have become evident, and what a lot of people would have taken pains to point it out! [...] Although [Italian buffoons] were detestable, and the orchestra, still extremely inexperienced, spoilt, as if this gave them delight, the pieces that they presented, all the same they brought damage to the French Opera which has never been remedied." At this point, Friedrich Grimm (1723–1807) entered into the dispute, with his *Little prophet*. "But the *Little prophet*, which they insisted for a long time on attributing to me, despite myself, was taken as a joke, and never created the slightest trouble for its author. On the contrary, *The Letter On Music* was taken seriously, and stirred up the whole nation against

[142]Rousseau 1839, *Dictionnaire de musique* t. II, p. 195.

[143]Rousseau 1966, p. 776.

[144]Rousseau 1966, pp. 23, 161–162, 194–197, 236, 240–247, 260–261, 284–285, 308, 350–351, 364–369 and passim. See above.

me, making them think that they had been offended in their music." His tone now became tragicomic. "…my life itself was in danger. The orchestra of the Opéra devised the honest plot of assassinating me, while I was leaving. I was informed about it; I was simply more assiduous at the Opéra." By way of compensation, …, *The Village Fortune-teller* earned him a lot of money.[145]

"Firstly, women. […] For me, music was another less violent passion, but no less tiring for the ardour with which I dedicated myself to it, for the assiduous study of the obscure books by Rameau, for my invincible obstinacy in insisting on impressing them on my memory, which was always loath, for my continuous excursions, for the immense compilations that I accumulated, often spending whole nights copying them out." Everyone knows how the Swiss Enlightenment scholar treated women. He wrote to d'Alembert: "…it should not be thought that the taste for *vendetta* belongs exclusively to philosophers, and that when they will choose to become women, women will become philosophers."[146]

He had confessed that music was only the second passion of his life. Carnal sins had been punished *in re ipsa* [in the same matter] in the case of Peter Abelard (1079–1142) with bleeding castration. Those of his lover, Eloise (1101–1164), were expiated, on the contrary, *in anima* [in the soul], by the continuous ineradicable memory of an earthly paradise lost for ever.[147] Now it was Jean-Jacques Rousseau's turn to confess his own sins *in libris* [in his books], having received the punishment of their bonfires in the squares of Paris and Geneva. All the wine in Europe would not have been enough to make him forget an unlimited sense of guilt. Put together in his last few tormented years, and generally written during his exile in England, these pages might even pass for a romantic conscience. But, not having a heart of stone, the *Confessions* seem to me rather a form of English humorous novel, where he had to include everything, above all gossip about the nobles that he had met, friends and enemies, except the creation of a faithful diary.[148]

"Où y fait plus, où n'y fait nulle chose. [Where more is done, there are the places where nothing is done at all.] …It is at the same time the [laziness] of a child who is always in movement so as not to do anything, and that of a raving idiot whose head wanders as soon as his arms are at rest."[149] At last, we can now abandon Rousseau, the hyperactive, to return to Rameau, who in the meantime had not remained calm at all. Actually he too wrote a "Lettre à M. d'Alembert, Sur ses opinions en Musique, insérées dans les articles FONDAMENTAL & GAMME de l'*Encyclopédie*" ["Letter to Mr. d'Alembert on his opinions about music, included in the entries FUNDAMENTAL and GAMUT of the *Encyclopaedia*"], which,

[145]Rousseau 1966, pp. 480–498, above all 492–493.

[146]Rousseau 1966, pp. 284–285 and 686.

[147]Abelardo & Eloisa 2008.

[148]My comment was written before reading the contemptuous judgement that our man from Geneva had expressed, towards the end of the book, on the English writer, Samuel Richardson (1689–1761); Rousseau 1966, p. 697.

[149]Rousseau 1966, pp. 815–816.

fortunately, was short and dealt with music. But with a series of interminable rhetorical question marks, the composer repeated what he had already written elsewhere, and the mathematician had tried to refute: the importance of ratios for the theory of music, so great that even geometry had its origin in them, and in it, ever since the times of Pythagoras and Eudoxus.[150]

Rameau defended French musicians, who he had seen attacked, also by d'Alembert, in the wake of Rousseau, in his article "De la liberté de la musique" ["On the freedom of music"]. "…en voulant tout interdire à la Musique françoise, pour ne pas dire aux Musiciens François, comme si j'imaginois, par example, de vous prouver que vous n'existez pas, & que si vous existiez, ce seroit tant pire pour vous." ["…you [d'Alembert] wish to take away everything from French music, not to say from French musicians, as if I imagined, for example, demonstrating to you that you do not exist, and that if you existed, it would be so much the worse for you."] Besides thinking that he was influenced by the Swiss writer, he also considered him to be under the leverage of the Catalan, Terradeglias. Only his lack of experience, the event that he had not "…encore écouté que des chansons, même dans un âge avancé." ["…yet listened to anything but songs, and what's more, at an advanced age."] might justify the idea that it was the melody that suggested the harmony of the fundamental bass, and not vice versa.[151]

As referred by Rousseau and d'Alembert, Rameau's judgement on the experience of Tartini sounded definitely *trenchant*. "Je crois fort que l'imagination y a plus de part que l'oreille." ["I firmly believe that the imagination plays a greater part than the ear." The famous third sound of the Italian musician was reduced to a "…bourdonnement…" ["…buzzing…"] among female voices. "…c'est un pur effect de l'Air sur l'oreille (…) qui n'y fait entendre qu'un son extrêmement frèle: prérogative qui peut bien n'être que du ressort de l'oreille, …S'il n'y avoit pas de la bonne foi dans l'excés où M. Tartini a porté son expérience, on verroit en lui que vous & moi ne sommes pas les seuls qui voulions *en imposer aux ignorans*. Si j'en impose, du moins je donne les moyens de s'instruire sur la vérité; mais vous, M. vous semez la confusion partout: vous niez, vous critiquez, vous proposez des doutes, & n'éclaircissez rien, …" ["…it is purely an effect of the air on the ear (…) which only makes you hear an extremely fragile sound, a prerogative which may very well only be the elasticity of the ear, …If the excess to which Mr. Tartini pushed his experience were not in good faith, you would see in him that you and I would not be the only ones who want to *deceive the ignorant*. If I lead people to believe something, at least I supply the means to obtain instruction about the truth; but you, Sir, you sow confusion everywhere: you deny, you criticise, you advance doubts and you do not clarify anything, …"].

"…votre sçience est décidée, on vous croit sans aller au scrutin: la mienne est toute neuve, & l'on ne pourra me croire qu'après l'avoir approfondie: hélas!" ["…your science is established, you are believed without any verification: mine

[150]Rameau 1762a.

[151]Rameau 1762a, pp. 272–273 and 278. D'Alembert 1762a.

is completely new, and I can only be believed after investigation. Alas!"] As well as arithmetic and the geometry of Pythagoras, the French composer extended the comparison with science to the astronomy of Kepler, but he did not make reference to anything precise.[152]

Rameau thought: how could it be a coincidence that the first ratios between numbers were precisely those of music? He sure wanted to arrive at a even stronger hypothesis: "Origine des sciences" ["The origin of the sciences"]. In ancient histories, he went in search of those traces, which were sometimes hidden and masked (also Newton had written that), where he could read that music had acted as a midwife for the sciences. He was referring to the work of Jean Étienne Montucla (1725–1799).[153] To establish the properties of the sounding body, from which everything began, he invoked a "SUPERBISSIMUM AURIS JUDICIUM" ["Supreme judgement of the ear"]. "Voyez-les tous, dans l'Europe entière, même à la Chine, s'obstiner à des recherches sans bornes & sans fruit sur la Musique, pendant une infinité de siécles, jusqu'à la rendre l'arbitre des différens effects qu'on éprouve dans ce vaste Univers." ["Look at all of them, in the whole of Europe, and even in China, obstinately carrying out research on music, without limits and without fruit, for an infinity of centuries, ending up by making it the arbiter of the different effects experienced in the vast universe."] As regards China, perhaps he quoted father Kircher.[154]

To support his thesis, the French composed invoked nature, history and facts. 3, 4, and 5 were not only numbers suitable for musical proportions, but also those of the sides in order to obtain a right-angled triangle [$9 + 16 = 25$]. "Ce début en Géometrie parle déjà beaucoup en faveur de la Musique." ["This start in geometry already speaks clearly in favour of music."] "...les Chinois proposent la progression de quintes, dite triple, jusqu'à son treizième terme, dont ils suivent l'ordre dans leur système de Musique, bien plus régulièrement que ne l'a fait Pythagore; ils prennent date même avant l'établissement des Egyptiens." ["...the Chinese propose the progression by fifths, called triples, until its thirteenth term [twelve notes!], and they follow this order in their system of music, with much greater regularity than Pythagoras did; their date goes back to even before the Egyptians."] But in the heat of the argument, Rameau seemed not to realise that the Chinese scale based on fifths, 3:2, will never come to an end on those octaves that are indispensable to him for his harmonic system. "S'il y a des *idées innées*, peut-on les refuser à la Musique?" ["If *innate ideas* exist, can they be refused to music?"]

In the eyes of his adversaries, above all those present in the Académie des sciences in Paris, the renowned theoretician of music worsened his situation by insisting that the ratios between the notes, generated by the vibrations of the sounding body, supplied the fundamental model of "...toutes les sciences possibles." ["...all the possible sciences."] Lacking qualifications and up-to-date

[152]Rameau 1762a, pp. 278–279.

[153]Cf. Montucla 1799.

[154]Rameau 1762b, p. 288.

scientific competence, he was unable to succeed in imposing his own musical supremacy on the European scientific community. Not only but above all, he did not realise that by now the mathematical sciences were abandoning the theory of ratios, in favour of the much more powerful means of differential equations (with partial derivatives). "Quand M. d'Alembert auroit prouvé, ce qu'il lui est impossible de faire, que l'on a mesuré des surfaces avant d'avoir combiné des nottes, il n'en seroit pas plus avancé, parce qu'il n'en seroit pas moins vrai que le sens de l'ouïe a dû présenter plus nettement qu'aucun autre sens à l'ame de celui qui s'en occupoit, …" ["When Mr. d'Alembert will have demonstrated, a thing that it is impossible for him to do, that surfaces have been measured before combining notes, we would not have made much progress, because it would be no less true that the sense of hearing has had to impress itself in the soul, for the one who dedicated himself to it, with greater incisiveness to every other sense, …"].[155]

Increasingly irritated by the introduction that the mathematician placed at the beginning of the new edition of *Elements of Music* seen above, Rameau even went so far as to challenge his qualification of *geometer* [at the time, a synonym of mathematician]. "…(qualité qui cependant est bien plutôt due aux Inventeurs des régles, qu'à celui qui ne fait que les suivre." ["…(a quality, however, which is far more deserved by the inventors of rules than by those who only follow them[)]."] Poor co-inventor of differential equations with partial derivatives, reduced to an epigone of Euclid! "Tous les premiers principes de la Musique, de l'Arithmétique & de la Géometrie, pour ne rien dire de plus, ne se trouvent-ils pas ici réunis dans un object unique, dans un seul corps sonore en un mot?" ["All the prime principles of Music, of Arithmetic and of Geometry, not to mention others, are they not reunited here in a single object, in a word, within a single sounding body?"][156]

His adversaries, who were contrary to "…un principe aussi évident …" ["…such a clear principle …"], were invited by our composer to demonstrate "1°. Que …les Sciences ne sont pas fondées sur les proportions. 2°. Que le corps sonore ne produit pas toutes les proportions …3°. …que notre instint a pu suffire pour faire naître en nous les idées qui nous on conduit à ces grandes & sublimes découvertes …De quelle utilité on donc été nos sens en pareil cas?" ["1st. That …the sciences are not based on ratios. 2nd. That the sounding body does not produce all the ratios …3rd. …that our instinct was sufficient to develop in us the ideas which have led to these great and sublime discoveries …What use have our senses been, therefore, in a similar situation?"] Instead of providing these proofs, they opposed useless stupidities and errors. "Depuis quand le Philosophe ne sacrifie-t-il plus ses opinions à la vérité? Ne doit-il pas se taire quand la Nature parle?" ["Since when does a philosopher not sacrifice his own opinions to the truth? Is he not forced to keep quiet when Nature speaks?"][157]

[155]Rameau 1762b, pp. 290–291 and 294.

[156]Rameau 1762b, pp. 314–315 and 323.

[157]Rameau 1762b, p. 326.

Poor Rameau, those natural philosophers transformed into mathematical-physicists already had all the demonstrations he wanted ready. Most musicians would have preferred to follow the route opened up by Pergolesi. Anyway, he was to be left with the merit of having founded harmony, even though not on a presumed absolute naturalistic base.

D'Alembert replied brusquely, "Je ne pourrai jamais me résoudre, …, à vous prouver sérieusement que la *Géométrie n'est pas fondée sur la Musique*; …" ["I will never be able to resolve, …, to demonstrate to you seriously that *Geometry is not founded on Music*; …"]. Effectively, in that same year, our French mathematician had reopened the "…guerre civile …" ["…civil war …"] about the "Italian buffoons", to the great disappointment of Rameau, though he called his comic opera *Platée [Platea]* "…son chef-d'oeuvre et celui de la musique français." ["…his masterpiece and that of French music."]. D'Alembert defended the paladin of Italian music, Rousseau, from the accusation of being a "…perturbateur du repos public, …mieux nommé *perturbateur du* bruit *public*, attendu que la musique française en fait beaucoup." ["…disturber [terrorist] of the public peace …better defined as the disturber of the public *noise*, seeing that French music makes a lot of din."][158]

But freedom in music, to prefer that Italian music, had become a synonym for more to come other, different kinds of freedom. In the end, people would become Republicans, atheists or materialists. "La dioptrique est la science des propriétés des lunettes; les lunettes supposent les yeux; les yeux sont un des organes des nos sens; l'existence des nos sens suppose celle de Dieu, puisque c'est Dieu qui nous les a donnés; l'existence de Dieu est le fondement de la religion chrétienne; nous allons donc prouver la vérité de la religion pour première leçon de dioptrique." ["Dioptrics is the science that studies the properties of lenses; lenses presuppose eyes; eyes are one of the organs of our senses; the existence of our senses presupposes that of God, because He is the one who gave them to us; the existence of God is the foundation of the Christian religion: we thus arrive at demonstrating the truth of religion as the first lesson of dioptrics."][159] The one (a Jesuit?) who sustained this argument may undoubtedly have aroused the irony of an Enlightenment scholar who defended reason. But it should serve to remind us that the *enjeu* [stakes] had remained the same as those of the war, still being fought, about the truth, which had started at the time of Galileo Galilei, between exponents of religion and natural philosophers.

In the comparison between French and Italian operatic recitative, d'Alembert wrote that he had repeated his vocal experiment: "…je chantai le récitatif à l'italienne, …" ["…I sang the recitative in the Italian style …"]. Sometimes the effect would be better, at other times worse, depending on whether the French opera was more or less capable of linking well with the phrase. In the opera *Dardanus* by Rameau, he found verses "…qu'un acteur intelligent les débite à l'italienne, en se conformant à la note, …" ["…that an intelligent actor …declaims in the Italian

[158]D'Alembert 1762b, p. 2; 1762a, pp. 517, 515.

[159]D'Alembert 1762a, p. 521.

style, conforming to the note, …"]. Interpreted in this way, the Enlightenment mathematician felt in the scene that "…vérité du sentiment et la ressemblance du chant avec le discours." ["…truth of sentiment and correspondence of the singing to the phrase."][160]

He reconsidered the monologue from Lulli's *Armide*, already seen above, which had been defended by Rameau against Rousseau's attack. He, too, noted that in order to justify him, the French composer had to modify the bass. He discovered that Lulli had imported the Italian music of his time into France, and ended up by sustaining the excellence of Philippe Quinault (1635–1688) in composing the *libretto*, compared with the defects of the musician. Nor were things to go better with the 'arias' in the French style. Italian music was capable of linking up with words better suited to the dramatic situation. "Le chant français a le défaut le plus contraire à l'expression; …" ["French singing presents the defect that is most contrary to expression; …"]: he used the same style to express different sentiments. Referring to Rousseau, d'Alembert continued to prefer a melody set in the foreground and backed up by harmony, without being noisily covered up in the French manner.[161]

Supposing for argument's sake that a French music existed, it would be better in symphonies only with instruments, without any voices, and for ballets like those in the *Indes galantes [Gallant Indies]* by Rameau. Only in this could Italians be surpassed, whose sonatas did not encounter the musical aesthetics of the mathematician. "Toute cette musique purement instrumentale, sans dessein et sans object, ne parle ni à l'esprit, ni à l'âme, et mérite qu'on lui demande avec Fontenelle, *sonate que me veux-tu?* […] …on ne sent toute l'expression de la musique, que lorsqu'elle est liée à des paroles ou à des danses. La musique est une langue sans voyelles; c'est à l'action à les y mettre." ["All this music, which is only instrumental, without any design and without any object, does not speak either to the spirit, or to the soul, and deserves to be asked the same question as Fontenelle [Bernard le Bovier (1657–1757), secretary of the Académie, asked]: *sonata, what are you trying to tell me*? …all the expression of music is only heard when it is linked with words and with dancing. Music is a language without vowels; it is the task of action to provide them."] If he made an exception for a sonata by Tartini, d'Alembert had to ignore completely the hundreds of sonatas for harpsichord composed by Domenico Scarlatti.[162]

Anyway we may take this expressive pseudonaturalism, it would be difficult for me not to agree with his conclusions. "Voilà bien des réflexions …qui, bonnes ou mauvaises, ne valent pas à coup sûr un bel air de musique. …Raphaël n'a point faites de dissertations, mais des tableaux. En musique *nous ècrivons* et les Italiens *exécutent*." ["Here are some reflections, then …which, good or bad as they may be, are definitely not worth a fine aria of music. …Raphael did not compose any dissertations, but pictures. In music, *we write*, the Italians *perform it*."][163]

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[160]D'Alembert 1762a, pp. 533–534.

[161]D'Alembert 1762a, p. 539.

[162]D'Alembert 1762a, pp. 544.

[163]D'Alembert 1762a, p. 546.

All this greatly angered the King's musician, who, however, continued to write about it, undaunted. To the mathematician's reflections, Rameau added his own, which were likewise a bit too repetitive. The latter vainly tried to enrich his model of music based on ratios, by varying the details, when the former had already been enraptured by the Italian melodies. Recalling also the Chinese system every now and then, sanctifying the interval of the fifth by means of the number 3 in the triple ratio 1, 3, 9, Rameau seemed to have almost forgotten the octave. Even the temperament was traced back to fifths. "…certaines consonnances sont altérées, …nécessairement, pour faire connoître & sentir le Mode dans la plus parfaite justesse des produits d'une seule quinte, …" ["…certain consonances are altered, …necessarily, to let the Mode be known and heard with the most perfect precision in the products of a single fifth, …"]. Now the composer accused the mathematician of having started to attach him, taking back the favourable report of the Académie on the *Encyclopédie*. Subsequently, he insisted even more on the 'demonstration' of his own principle of harmony. "La Géométrie n'est-elle pas fondée sur le calcul; & le calcul n'est-il pas fondé sur les proportions? […] …par consequent que la Géométrie ne peut, sans injustice, contester à la Musique une primauté, en faveur de la quelle la Nature dépose d'une manière si évidente." ["Is geometry not founded on calculation [with numbers]?; and is calculation not founded on proportions? […] …consequently, geometry cannot, under penalty of the charge of injustice, challenge the primacy of music, in favour of which Nature so clearly disposes."][164]

The illustrious Académie des sciences had in due time given him its approval. Thus it was necessary to reply to the accusation that he felt was levelled at himself, "…de donner à mes productions un air scientifique, air qui vous appellez faux, & *qui n'en impose qu'aux Ignorans*: …" ["…of giving my results a scientific air, an air that you call false, and *which only deceives the ignorant*: …"]. Showing a certain polemical ability, Rameau answered : "…c'est donc vous aussi qui êtes l'ignorant à qui j'en ai imposé come à toute l'Académie des Sciences." ["…then you too are ignorant, because I have deceived you together with all the Academy of Sciences."][165] During the self-styled Age of Enlightenment and Reason, effectively, that Academy had hoped to be able to extend its dominion also to music, thanks to the naturalistic-arithmetic theory of Rameau. But it was to be deceived, because now the composer claimed supremacy for music, and not submission.

D'Alembert left us another article on the theory of music. It was even read in front of the Academy of sciences. Through it, in Rameau's face was thrown the finding the two sexes of animals in the major and minor modes, and the Catholic Trinity in the triple resonance of the sounding body. But then the geometers, like himself, would have put order in "…ses idées un peu informes et confuses, …" ["…his ideas, which were a bit vague and confused, …"]. As regards Euler, he wrote that he founded his theory of music on arithmetic ratios, searching for the source of pleasure, "…mais toute cette physique ou métaphysique, comme

[164]Rameau 1765, pp. 26, 30–31.

[165]Rameau 1765, p. 36.

on voudra l'appeler, est encore bien imparfaite, bien vague et bien précaire." ["...but all this physics or metaphysics, anyway you choose to call it, is still very imperfect, very vague and very precarious."] He preferred to quote extensive extracts from his introduction to the new edition of the *Elements of Music* published in 1772. He also made reference to Kepler, but it was clear that he had not read the *Harmonices Mundi Libri Quinque*. "...trouva la loi astronomique qui l'a immortalisé, c'est d'aprés quelques idées chimériques dont il étoit prévenu sur certaines perfections pythagoriques des nombres, ..." ["...He found the astronomic law, which made him immortal, ...behind certain chimeric ideas, being prejudiced on these, regarding certain Pythagorean perfections of numbers."] We know, instead, that the German had rather been guided by geometry and by music.[166]

In any case, the French mathematician and physicist found some utility in all these errors because, albeit indirectly, they were to lead to the truth. Philosophers were thus to be left free to choose their favourite hypothesis. "L'imperfection de toutes les théories musicales vient de la même cause que la futilité de presque tous les systèmes physiques." ["The imperfection of all musical theories derives from the same cause of the uselessness revealed by almost all the physical systems."] Waiting for the true theory, still to come, as for "...le progrès de la science. ...aucune nation peut-être n'est plus propre en cet instant que la nôtre à faire et à recevoir ces nouveaux essais d'harmonie." ["...the progress of science. ...perhaps no other nation nowadays is more suitable than ours to produce and to receive these new essays on harmony."][167] If he had still been alive, this was a sentence that Rameau, for once, would have appreciated.

Also Denis Diderot (1713–1784) had something to say: "La musique a donc des principes invariables et une théorie. C'est une vérité que les Anciens ont connue. Pythagore posa les premiers fondements de la science des sons." ["Music has invariable principles, therefore, and a theory. This is a truth known also to the ancients. Pythagoras laid the first foundations of the science of sounds."] In search of these principles of his, he commented on the results of Brook Taylor, bearing in mind, however, those achieved by d'Alembert for vibrating strings. He also knew Euler: "...le plaisir musical consiste dans la perception des rapports des sons." ["...the pleasure of music consists in the perception of the ratios between sounds."][168]

He fiddled around with the logarithms of ratios, but concluded, "Du reste, il n'y aurait pas d'inconvénient à ne prendre ces expressions logarithmiques que comme une hypothèse. Il n'y a pas même d'apparence que M. Euler, qui nous les propose, prétende les faire valoir davantage. Car on ne peut guère calculer ou comparer les sons en tant que sensations. Les longueurs des cordes et les nombres des vibrations qui les constituent sont les seules choses comparables." ["On the other hand, there will be no harm in taking these logarithmic expressions only as a hypothesis. Indeed,

[166] See above, Sect. 8.3.

[167] D'Alembert 1777, pp. 138, 144, 139–141. Cf. Fubini 1971, pp. 185–193.

[168] Diderot 1975, pp. 237, 244 and 236.

it is only an appearance that Mr Euler, who proposes them to us, claims to give them greater value. Because it is quite impossible to calculate or compare sounds, seeing that they are sensations. The length of strings and the numbers of the vibrations of which they are made up are the only things that can be compared."][169] He cast doubt on the affirmation [which we know came from Leibniz[170]] "L'âme a-t-elle ces connaisances sans s'en apercevoir, ...? REPONSE Nous ne déciderons rien là-dessus; ...est d'expérience que les accords les plus parfaits sont formés par les sons qui ont entre eux les rapports les plus simples; ..." ["Does the soul acquire its knowledge without being aware of it...? ANSWER: There about it, we do not decide anything; ...it is by experience that the most perfect chords are formed by sounds which have the most simple ratios between them; ..."]. In this, Diderot followed Euler, as in other problems read in the *Tentamina musicae [Essays On Music]*, including the vibrations of the air in an organ pipe or in a flute.[171]

Through Fontenelle, he carefully quoted the experiments of Sauveur regarding harmonics. The editor-author of the *Encyclopédie* also proposed a new experiment with incommensurable [irrational] lengths between them, convinced that irregular sounds would be obtained "...sans aucune proportion sensible, sans degrés successifs et contre toutes règles connues. ...les grands instruments à vent sont pareillement sujets à ces irrégularités; ..." ["...without any sensible ratio, without successive degrees, and against all known rules. ...large wind instruments are likewise subject to these irregularities; ..."]. Evidently, whether through Euler or not, the Pythagorean prejudices about incommensurable ratios in music had arrived as far as him. We cannot expect much, therefore, regarding the equable temperament. Thus Diderot let, rather, return the theory that commensurable vibrations meet every now and then.[172]

In the field of music, the French philosopher of the *Encyclopédie* possessed a certain competence. He ironised as regards the "Jesters' Dispute", and dedicated a few essays to the art of the harpsichord, taking pains to give the relative education to his daughter Angélique. Among other people more or less well-known, he maintained relations with Rameau, François-André Philidor (1726–1795), Carl Philipp Emanuel Bach (1714–1788) and Johann Christian Bach (1735–1782).[173] In *The Dream of d'Alembert*, Diderot caused various sensibilities to be developed, from matter, in living beings, arriving as far as ideas, comparing them to vibrating strings rich with harmonics, which emitted notes. "Our senses are as many keys played by the nature that surrounds us, which often play by themselves; this, in my opinion, is all that happens in a harpsichord organised like you and like me." Two of these living harpsichords would be able to communicate by means of their sounds. "There is a moment of delirium in which the sensible harpsichord thought it was the

[169] Diderot 1975, p. 254.

[170] See above, Sect. 10.1.

[171] Diderot 1975, pp. 258–259. See above, Sect. 11.1.

[172] Diderot 1975, pp. 275 and 277–278.

[173] Diderot 1983, pp. VII–XXII.

only harpsichord that existed in the world, and it believed that all the harmony of the universe resided in him. . . . The analogy [the relationship], in the most complex cases, is only a rule of three performed on the sensible [musical] instrument. . . . The string, whose resonance the animal expects, is a fourth harmonic one proportional to the other three; . . . ".[174]

"It was the signs of language that gave rise to the abstract sciences. . . . Every abstraction is simply a sign devoid of idea. Every abstract science is simply a combination of signs. The idea has been excluded, separating the sign from the physical object, and only by reuniting the sign with the physical object will science return to being a science of ideas; . . . ". We have no way of knowing how the philosopher of the *Encyclopédie* succeeded in reconciling this declared materialism with remnants in the music of ancient Pythagoreanism. Undoubtedly he did not lack faith in reason, in the truth and in the (transcendent?) mathematical sciences, although his works are rich in impulses to overcome the Cartesian dualism.[175]

I did not have to make any effort to set some of the most famous and important musicians of the eighteenth century in our history of sciences. The natural philosophers, who were undergoing transformation to modern academic disciplines, wrote about them explicitly. I know that I should invest greater energy to interest colleagues of today. We shall see immediately in the next section that one of the most ancient *Histories of mathematics*, published at the end of that century, also dealt spontaneously with music. Now, therefore, it may be useful to dedicate a little more emphasis to it.

From Lulli to Pergolesi, with or without the accent on the *a* and on the *e*, the Opera gradually passed from the courts to the theatres. The musician from Tuscany, by now Frenchified, has started from "recitar cantando" [reciting in song], and now joined this more with the tragedies of Jean Racine (1639–1699) than with the comedies of Molière, obtaining the solemn gait of accompanied recitative, as in the *Proserpine*. The orchestra contributed to the drama. "One of the finest examples that can be quoted is the wonderful nocturne of the *Triomphe de l'Amour [Triumph of Love]*. A hazy, imprecise murmur of muffled violins, on which a song of a sweet serenity comes to rest." From dances to festivities, from arias to symphonies, everything had to be inserted into the tale of a mythological tragedy or into a pastoral love comedy. It was better if it could render in music "those war scenes of a youthful, victorious progression, which Louis XIV must have liked." Although, when collaborating with Molière at the beginning, he even displayed a comic *verve*, Lulli was forced to give it up in the following lyrical dramas that met with greater success at court. As already clear from the *querelle* between Rameau and Rousseau, his favourite tone was not passion, but an abstract transfiguration, distant, detached in a calm, composed declamation which advanced with slow steps. Even in the most tragic case, love did not suffer much with him; it was only a " 'delightful

[174]Diderot 1994, pp. 20 and 24–26.

[175]Diderot 1994, pp. 95–96. Cf. Fubini 1971, pp. 133–183.

trouble' ... There is sometimes a desire for one of those exclamations of animal pain which burst forth so convincingly in the works of ... Cavalli."

Lulli presented his music to a world which may have been pretty, but was largely definitely boring, made up of well-kept, square-shaped Italian gardens, far away from dark, spine-chilling woods. His orchestra constructed stereotypes: tender laments with the flute, warlike clashes with trumpets, lullabies for heroes with violins. Jean de la Fontaine (1621–1695) wrote: "His instrumental concerts have the sound of thunder". Neither the polyphonic recent past, nor the harmonic immediate present were within the capacities of the Florentine emigrant who had integrated well in France.[176] For several aspects, Rameau was his heir, while he was to develop the harmonic chord structure much more, partly through his pieces for harpsichord. He even invented a new way of theorising it, but without being too capable of using it effectively, because it was still weighed down by the theatrical conceptions of Lulli. It was not enough to modulate better and more often in the interplay of tonalities; it was necessary that they should express the ferments that were already present in the world of the *ancien régime*, but still suffocated.

The novelty was to come from far away, from outside the royal court circles: Pergolesi. Enlightenment scholars like Rousseau and d'Alembert felt that the new life, where male and female bodies were realistic, was coherent and useful for their aspirations for more general and deeper changes. Above all, they did not get bored, and took greater pleasure in listening to the lively comic scuffles sung by Italian strolling players than in contemplating the controlled steps on the scene of gods, heroes or shepherdesses. Restoring to the senses of the body a central role in life, also the ear could free itself from all those fictitious bonds, fixed by ratios between whole numbers. In the new conditions, there was only to be a better music, more exciting, more expressive, and passionate.

To understand how notes worked in music, in the place of traditional ratios, d'Alembert put the force that acted on the violin string, called the sounding body. Or better, seeing that he considered Newton's notion of force obscure, he described the perceivable movement of it, the acceleration, by means of a new differential equation never written before. From this, he succeeded in obtaining solutions that were large classes of curves, with few constraints on their forms. When he abandoned those four, five or six numbers, a whole new world opened up to everyone, in which physicists and mathematicians were to work, and squabble, in the following centuries: the equation of the vibrating strings.

The violin string in movement generated many sounds, and thus reserved other surprises. That epoch now counted numerous people who, from it, were capable of creating an abundance of notes. Violinists joined together to form orchestras, often mixing with the other instruments. At Cremona, in Italy, the famous models of the instrument were constructed: Amati, Stradivari, Guarneri. Composers, like

[176] Prunières 1950, pp. 97, 95, 102, 109 and passim.

Geminiani mentioned above, Pietro Antonio Locatelli (1695–1764), Francesco Maria Veracini (1690–1750) and others, wrote *Sonatas* for 2, 3, or 4 violins.[177]

Also Giuseppe Tartini was a *virtuoso* of the violin, and the author of *Sonatas* and *Concertos*. As mentioned above even in France, he is the one who hears that "third sound" which is destined to clarify better the nature of the material vibrations that propagate as waves: this is what is nowadays called the phenomenon of beats. Two vibrating strings, with only a small difference in frequency, generate another third audible sound, which is added to the two others. Our musician from North Italy composed restless, dynamic, dramatic music, which expressed a turbulent, adventurous personality. He related that the devil himself had appeared to him in a dream, dictating to him his piece that has remained the most famous of his compositions: *The devil's trill*. Tartini dedicated manuals and technical studies to the violin.[178] Like Rameau, he tried to explain the harmonic relations between notes, not only playing and composing, but also writing books: *Trattato di musica secondo la vera scienza dell'armonia* [Treatise of Music In Accordance With the True Science of Harmony].

However, our Italo-Dalmatian musician continued to try to fit the sounding novelty that he heard with the third note into the ancient Procrustes' bed of Pythagoreanism. The equable temperament was simply ignored, while Tartini laboured (in vain?) to prove that the three arithmetic, harmonic and geometric means of the ancient Greeks represented the basis of the musical system for his period. "Dati due suoni di qualunque strumento musicale, che possa protrarre e rinforzare il suono per quanto tempo si voglia (trombe, corni da caccia, strumenti d'arco, oboe ecc.) si ha un terzo suono prodotto dall'urto de' due volumi di aria mossi dalli due dati suoni. ... Da un suonatore di violino si suonino equitemporaneamente con arcata forte e sostenuta i seguenti intervalli perfettamente intonati. Si sentirà un terzo suono affatto distinguibile e sarà il sottoposto segnato in note chiuse musicali. .... Data la sesquiquinta [6:5], o sia praticamente terza minore, il terzo suono è in decima maggiore grave con la nota grave della terza minore."[179] [Given two sounds of any musical instrument, which can protract and reinforce the sound for however long is desired (trumpets, hunting horns, strings, oboes, etc.), there is a third sound produced by the clash of the two volumes of air moved by the two given sounds. ... Let the following intervals be perfectly intoned by a violin player simultaneously with a strong, sustained bow. A third clearly distinguishable sound will be heard, and the one placed inferior will be marked in musical closed notes. .... Given the sesquififth [6:5], or practically speaking the minor third, the third sound is deep in the major tenth, with the deep note of the minor third."]. That is to say, for example, playing the interval $E - G$, also the C of the lower octave is heard. Consequently, the triad $C - E - G$ of the major mode is generated. In this way, the violinist and

[177] Careri 1994. Grossato 1994.

[178] Di Benedetto and Nicolodi 1994. Grasso Caprioli 1994. Angelucci 1994.

[179] Tartini 1996, pp. 54–55.

theoretician believe that they have provided a physical basis for the major tonalities. With the minor ones, it was less easy and less successful.

"Dovendo io essere corrispondente ai miei principî, l'esame di sentimento e di ragione deve cadere sopra l'armonia, e non sopra la cantilena [melodia], perché quella è cagione e radice, questa effetto e prodotto. Per sentimento comune di tutte le nazioni, appresso alle quali si coltiva la nostra musica, l'armonia di terza maggiore è forte, allegra, ardita; l'armonia di terza minore è languida malinconica e dolce. Riducendo il sentimento comune a ragione, null'altra può assegnarsi in genere fisico-dimostrativo, se non la natura armonica della prima, l'aritmetica della seconda: …".[180] ["As I must be consistent with my principles, the examination of sentiment and reason must fall on harmony, and not on the *cantilena* [melody], because the former is the cause and the root, the latter the effect and the product. According to the common sentiment of all the nations where our music is cultivated, the harmony of the major third is strong, joyful, and courageous; the harmony of the minor third is languid, melancholic and sweet. Reducing the common sentiment to reason, nothing else can be assigned of a physico-demonstrative nature, except the harmonic nature of the former and the arithmetic nature of the latter: …"]. When he played the violin, composed music, and thus followed his own ear, Tartini offered surprising novelties, destined to mark his epoch. However, if he tried to justify them with demonstrations that he believed to be scientific, the route he followed did not lead to convincing results. He did not abandon Zarlino's pattern, but rather claimed to provide physical and mathematical reasons for it. Thus, he too rejected irrational quantities and "…dell'infinito non vi è, né vi può essere scienza." ["…of the infinite, there is not, and cannot be, any science…"].[181]

From this harmony of his, our Italo-Dalmatian musician derived the scale. And yet he knew very well that he had only obtained a succession of theoretical notes. "La scala, che veramente si usa, è la suddetta in precisione di note musicali, ma non in precisione di ragioni [rapporti]. La cagione si è l'organo e clavicembalo costituiti dall'uso direttori e regolatori dell'armonia musicale. …Ma organo e clavicembalo (se non si moltiplichino i tasti a dismisura) non hanno altro intervallo perfettamente accordato, se non l'ottava; e quasi tutti gli altri intervalli di quinte, quarte, terze maggiori e minori, tuoni e semituoni sono accordati per discretivo temperamento, e non secondo la ragione [rapporto], o sia forma dell'intervallo rispettivo. …Io nel mio violino, dove suonando a doppia corda posso incontrar fisicamente la forma dell'intervallo, di cui è segno fisico dimostrativo il tal terzo suono che deve risultare, ho il vantaggio per me e per i miei scolari della sicura intonazione, e in conseguenza dell'uso reale della scala suddetta in precisione di ragioni [rapporti]." ["The scale, which is actually used, is the above-mentioned one as regards the precision of musical notes, but not as regards the precision of reasons [ratios]. The cause is the organ and the harpsichord, which have been elevated by use to the status of directors and regulators of musical harmony. …But organ and harpischord (if they do not

[180]Tartini 1996, p. 249.

[181]Tartini 1996, p. 108.

multiply their keys excessively) do not have any other interval perfectly tuned, except the octave; and almost all the other intervals, of fifths, fourths, major and minor thirds, tones, and semitones, are tuned by discretionary temperament, and not in accordance with the reason [ratio]: in other words with the form of the respective interval. ... In my violin, by playing double strings, I can physically encounter the form of the interval, of which the resulting third sound is the demonstrative physical sign that must result. Here, therefore, I have the advantage, for myself and my students, of the sure intonation, and consequently of the real use of the above scale in the precision of the reasons [ratios]"].[182]

As Rameau tried to do, as well, Tartini constructed arguments and mathematical demonstrations directed at the natural philosophers of his time. Kepler and Descartes had acted similarly, and also the Italian musician used the circumference to obtain musical rules which distinguished between consonances and dissonances, chords and tonalities. The *Treatise of Music In Accordance With the True Science of Harmony* became full of circular figures variously re-elaborated to generate the desired whole numbers, which were always the usual 1, 2, 3, 4, 5, 6 of the arithmetic progression, or 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ of the harmonic one.

Through Francesco Algarotti (1712–1764), then at the court of Frederick the Great of Prussia, Tartini also contacted Euler, who replied with a manuscript of his. The renowned Swiss mathematician noted that the third sound did not always give the correct fundamental bass. Tartini himself "sovente s'allontana da questa regola, come veder si può negli esempi riportati alla fine del suo *Trattato*". ["frequently moves away from this rule, as we can see in the examples presented at the end of his *Treatise*"]. Hoping to be favourable received at that German court, our Italian musician had looked for analogies with the calculations of Euler. But then, faced with the objections of the mathematician, he ended with a peremptory "È dunque assolutamente falsa la suddetta formola fondata sui divisori integrali, perché non reggendo alla sesquiquinta [6:5], ragione integrale del consonante sistema, non regge all'intero sistema". ["The above formula based on integral divisors is therefore absolutely false, because if it is not valid for the sesquififth [6:5], the integral ratio of the system of consonances is not valid for the entire system"]. Even though they both played with whole numbers, the two figures of the eighteenth century did not find a sufficient agreement to be able to collaborate on a common theory.[183] The one based it on prime numbers, the other on connecting Zarlino's whole numbers with the third sound.

Nor did things go better for Tartini with another physicist and mathematician closer to him, Giordano Riccati (1703–1790). Son of the more famous Jacopo (1676–1754), he challenged the musician's theory just based on the harmonic circle, to which he opposed another, derived by sectioning a parabola. "... si rende totalmente inutile e superfluo il principio del circolo, che ha tanto a che fare coll'accordo de' suoni quanto coll'accordo de' colori, dei sapori e degli odori."

[182] Tartini 1996, pp. 173–174.

[183] Barbieri 1994, pp. 340, 341.

[“. . . the principle of the circle becomes totally useless and superfluous, since it is as pertinent to the agreement of sounds as it is to the agreement of colours, of flavours and of smells.”]. Actually, while the geometric and arithmetic means were easily visible on the circle, the harmonic one was not so clear in the musician’s arguments. Fundamentally, this Riccati preferred the direct experience of his sounds to arguments with numbers and geometry. “E se questa dimostrazione non ci fosse? Io son persuaso che il numero 5 tra gli impari sia l’ultimo numero consono, perché tal e tanta, e non più, è l’attività dell’orecchia umana.” [“And if this demonstration did not exist? I am convinced that the number 5, among odd numbers, is the last consonant number, because such, and only so much, is the activity of the human ear.”].

Between the two, the dispute produced a substantial exchange of correspondence, in which the musician challenged the physicist to do better with another “harmonic figure”. Riccati replied: “Se la parabola le dà tutto quello che le dà il circolo, ed anche qualcosa di più e con maggior uniformità ed eleganza, non deve ella ripudiar il circolo ed adottar la parabola? L’assicuro che, se fossi persuaso del suo metodo e dei suoi principî, non tergiverserei un momento. Io, che batto un’altra strada, non posso dichiararmi per la parabola se non a confronto del circolo, essendo mia ferma opinione che e l’una e l’altro siano basi troppo deboli e mal atte a sostenere il musicale sistema.” [“If the parabola gives you everything that the circle gives you, and even something more, with greater uniformity and elegance, then should you not repudiate the circle and adopt the parabola? I assure you that if I were convinced of your method, and your principles, I would not prevaricate for a moment. I, who follow another route, cannot declare myself in favour of the parabola, except in comparison with the circle, since it is my firm opinion that the one and the other are bases that are too weak and unsuitable to sustain the musical system.”]. Heedlessly, Tartini replied “. . . il detto cerchio è il vero principio dell’armonia, perché nel detto cerchio si risolvono tutt’i dati premessi.” [“. . . the said circle is the true principle of harmony, because all the above data are solved in the circle.”][184]

It is curious that neither the one, nor the other remembered that Pappus had succeeded in representing in a different way also the harmonic mean in Tartini’s controversial circle.[185] Anyway, Giordano Riccati is to be included among the physicists and mathematicians of the post-Newton era who took an interest in music. He went beyond the “Esame del sistema musico del Sig. Giuseppe Tartini. Dissertazione acustico-matematica”, [“Examination of the musical system of Mr. Giuseppe Tartini. Acoustic-mathematical dissertation”], in compiling his *Le leggi del contrappunto [The Laws of Counter-point]*, for which, in turn, he solicited the comments of his professional interlocutor. But the physicist and mathematician of the eighteenth century did not give any reply to them. “. . . non per mancanza di stima per la sua [di Tartini] persona e del suo sapere, ma perché queste controversie io le stimo inutilissime, non servendo ad altro che a gittare il tempo senza frutto

[184] Barbieri 1994, pp. 325, 330, 333–334.

[185] See Part I, Sect. 2.7.

e restando per lo più ciascuno nella sua opinione. ... Il Mondo poi, senza tanti contrasti, giudica se la ragione stia o dalla parte del critico, o dalla parte del criticato." [... not out of a lack of esteem for your [Tartini's] person and your knowledge, but because I consider these controversies to be completely useless, since they only serve to waste time fruitlessly, with everyone generally remaining of the same opinion. ... Then the World judges, without so many contrasts, whether it is the critic who is right, or the person who is criticised"].[186]

Effectively, the "World" would have done better, abandoning all these discussions made useless by the musicians, who chose, independently of their mathematical theories, the equable temperament as the most suitable to their way of playing and composing. Thus, also Giuseppe Tartini was to offer his best things by playing the violin and composing for it, rather than by elaborating a theory that started its existence already old and defective. In his person, we can clearly observe, nowadays, the dilemma of the eighteenth century: between the practice of musicians and a theory in search of physical and mathematical bases.

> Les historiens sont comme les commentateurs; ils disent tout ce qu'on ne leur demande pas, et rien de ce qu'on voudroit savoir.
> [Historians are like commentators: they say everything that they are not asked, and nothing that we would like to know.]
>
> Jean le-Rond d'Alembert
>
> ... to study man, it is necessary to learn to look into the distance; first it is necessary to observe differences in order to discover properties.

> L'opinion, reine du monde, n'est point soumise au pouvoir des rois; ils sont eux-mêmes ses premiers esclaves.
> [Public] opinion, queen of the world, is not subject to the power of kings; they themselves are her first slaves.]

> On verra les aspirants aux charges briguer leur faveur pour obtenir les suffrages; les élections se feront dans les loges des actrices, et les chefs d'un peuple libre seront les créatures d'une bande d'histrions.
> [We shall see those who aspire to [public] office jockeying to have their favour [of strolling players], to obtain votes; the elections will be held in the actresses' dressing-rooms, and the leaders of a free people will be the marionettes of a band of actors.]

> Je ne dirais pas, de peur d'obscurité, que le beau consiste dans l'imitation du vrai, mais dans le vrai de l'imitation.
> [As I am afraid of being obscure, I would not say that the beautiful consists of the imitation of the true, but in the truth of the imitation.]

> [... and it seemed to me that solving a problem of geometry by means of equations was like playing an aria by turning a handle round.]
>
> Jean-Jacques Rousseau

> *Omne tulit punctum qui miscuit utile dulci* [He who unites the useful to the enjoyable obtains approval of everybody]: the supreme merit consists in uniting the useful to the enjoyable.
>
> Denis Diderot

[186] Barbieri 1994, pp. 343, 335–336.

11.3 Counting, Singing and Listening: From Rameau to Mozart

> . . . il latino barbaro, l'italiano semi-barbaro, le formule matematiche, il caos d'un libro pieno di *cioè*, di *citazioni* e di *note*, che non possono stare né col testo né senza testo, sono come i carciofi vecchi - spine di sopra - barbaccia irta di sotto - spicchi foglia per foglia -. Chi ha fame ne sfogli un migliaio. Per così poco io non uso di pigliarmici tanto fastidio.
> [. . . barbarised Latin, semi-barbarised Italian, mathematical formulas, the chaos of a book full of *that is*, of *quotations* and of *notes*, which cannot stand, either with the text, or without any text, are like old artichokes - thorns on top - a bristly beard underneath - segments leaf by leaf -. Whoever is hungry can pull off a thousand. For so little, I am not used to taking so much trouble.]
>
> Ugo Foscolo

> *Newton* des sons, astre de l'Harmonie,
> . . . . . .
> A déchirer le bandeau d'Uranie.
> La vérité sur les ailes du tems,
> . . . . . .
> *Rameau* paroit, & la nuit se dissipe,
> . . . . . .
> [*Newton* of sounds, star of harmony,
> . . . . . .
> to rend the veils of Urania.
> Truth on the wings of time,
> . . . . . .
> *Rameau* appeared and the night vanished
> . . . . .]
>
> Jean-François Marmontel

In the "Quatrième supplément [Fourth Supplement]" of his *Histoire des Mathématiques [History of Mathematics]*, Jean Montucla (1725–1799) dealt with the "Histoire de la Musique" ["History of Music"]. He preferred the ancient Pythagorean scale based on fifths to the one of Ptolemy, and modern. For some unknown reason, he called the event that the numerical ratios of the fifth, the fourth, and the octave were not reconcilable between each other, and consequently every theoretician chose one of his own liking as fundamental, ". . . un paradoxe musicale auquel le célèbre J. J. Rousseau n'étoit pas éloigné d'adhérer." [". . . a musical paradox to which the famous J. J. Rousseau would not be far from adhering."] He continued to give importance to fifths: ". . . examiner l'exactitude de la quinte par la tierce, c'est vouloir mesurer la justesse d'une pendule en y employant une montre de poche." [". . . examining the precision of the fifth by means of the third is like wanting to measure the correctness of a pendulum by means of a pocket watch."] For him, the temperament was ". . . un remède peut-être pire que le mal, . . . " [". . . a remedy perhaps worse than the disease, . . . "]. He has the merit of remembering Vincenzio Galilei: ". . . le père du célèbre Galilée. Son ouvrage est rare et estimé; . . . " [". . . the father of the famous Galileo [Galilei]. His work is rare and highly

esteemed; ...”].[187] Our French historian testifies that in that period, music was still part of the evolution imagined for the mathematical sciences.

Through Montucla, we can get an idea of how the European scholars of the eighteenth century imagined the mathematical sciences cultivated in the Orient. On the basis of (distorted?) information received from missionaries, the judgement continued to be that Chinese mathematicians “... étoient bien inférieurs aux Européens, ...” [“... were considerably inferior to Europeans, ...”]. In compensation, they granted that the Chinese empire was rather old, on condition that the presumed dates of the Biblical flood were maintained. However, the years indicated in Chinese books were generally challenged. The French historian was doubtful about how Leibniz had identified the diagrams of the *Yijing* with dyadic arithmetic “... conjecture, à tout prendre, plus ingénieuse que solide.” [“... a conjecture, altogether, more ingenious than solid.”] He recorded that an ancient sovereign had invented “... un nouvel instrument de musique, où se trouvoient les douze tons chromatiques de l'octave.” [“... a new musical instrument, where the twelve chromatic notes of the octave are found.”] The event that the Chinese operated in the opposite way to the Europeans could not be missing. One form of behaviour was considered to be privileged, however. Their sovereigns “... commandent en quelque sorte des découvertes, comme si des découvertes se commandoient.” [“... in some way, order certain discoveries, as if certain discoveries could be ordered.”][188]

As regards Chinese music, Montucla described the 12 pipes for the *lülü*, with the numbers of the relative ratios, but he took them as if they were equal to those of the Greeks. Not finding the octave, he commented “... c'est sans doute une omission, qui fait qu'on ne trouve pas ici la réplique ou l'octave du premier son, qui doit être $4\frac{1}{2}$; ...” [“... is it without doubt an omission that the repetition or the octave of the first sound is not found, which should be $4\frac{1}{2}$; ...”]. He judged it “... un systême musical fort imparfait, je veux dire une échelle musicale à laquelle manquent deux des sons que comporte l'étendue de l'octave diatonique; ...” [“... a very imperfect musical system, I mean, a musical scale where are missing two of the sounds involved in the extension of the diatonic octave .”] Apart from the *Annals* of Lübuwei, the bibliography included only the books written by missionaries. “... la concision et l'ambiguité de la langue Chinoise.” The [“... concision and the ambiguity of the Chinese language”] was recognized in the text, but contradicted in the bibliography. Here, unfortunately, the proportion of the Chinese text to Western translations was 10 to 1.[189]

Not having occupied himself either with music or with mathematical sciences, even though he was a rival of Rousseau and a friend of Diderot, as well as a collaborator of the *Encyclopédie*, Paul Thiry d'Holbach (1723–1789) would not enter into our history. He would serve only as a signal that, however radical, no profession of materialist, anti-religious faith would be sufficient to free oneself from

[187] Montucla 1802, pp. 650–652.

[188] Montucla 1802, t. I, Seconde partie Livre quatrième, pp. 449, 457–458, 461.

[189] Montucla 1802, t. I ... pp. 477–478 and 454. Cf. above, Part I, Chap. 3.

a transcendent God. Actually all of his criticisms, levelled against a similar infinite divinity, do not seem to extend to the transcendent infinities of infinitesimal calculus. Consequently, there was the risk that what had been thrown, with great emphasis, out of the front door would come back in again through the upstairs windows. Independently of his positions adopted, I will avoid repeating for the Baron of the Enlightenment what has already been discussed previously[190] concerning 'being', 'law', 'truth' or 'dualism', which continually recurred in his incessant arguments. Clearly, he could not create oppositions between the body and the soul, because the latter did not exist for him, but those between truth and error, good and evil, abounded.

Blinded, perhaps, by the Sun of his truths, this polemist of the eighteenth century manifested a scanty, superficial knowledge of the works written by the physicists and mathematicians of his period. "Men are always in agreement with the others and with themselves about proven truths, whatever the conditions may be in which they find themselves: unless they are crazy, all will recognize that two and two makes four, that the Sun gives light, that the whole is greater than any of its parts, . . . ". ". . . could he thus persuade me that two and two does not make four, that one is equal to three, and three to one, that a God who fills the whole universe with his immenseness, was able to incarnate himself into the body of a Jew, . . . ?"[191] If we wished to adopt his faith, loudly proclaimed, but published anonymously, in absolute natural truths, it would not be difficult to demonstrate that such affirmations could not stand up to rigorous scrutiny, whether historical or otherwise. It would be sufficient to re-read some of the preceding pages.

Whether ardent anti-Christian and atheistic materialists like the Baron d'Holbach, or otherwise, there are not many historians who have extended the mathematical interests cultivated by d'Alembert so far as to include music. Most of them insisted on his contributions to the mechanical sciences, following a different conception from that of Newton. The Frenchman surely avoided 'force', which was considered to be a covert, and therefore unreliable quantity for an illuminated geometrician inspired by sensations. Instead of following a particle of it, for the motion of fluids, he also made use of the notion of a 'field', which could cancel the obscure action at a distance used by Newton for the gravitational attraction, with the aggravation of having to have recourse to his God.[192]

D'Alembert recognized the limits of the mathematical sciences. "Geometry owes its certainties to the simplicity of the things that it deals with; when the phenomena become more complicated, the results become less sure." Yet what a lot of faith in progress by the use of reason transpired from the "Discours préliminaire" to the *Encyclopédie*! Our French physicist and mathematician of the Enlightenment liked to deal with spectacles, Opéra, literature, and with political-cum-religious questions (including the expulsion of the Jesuits from France), mixing all these

[190] Above all in Chap. 10.

[191] D'Holbach 1985, 112, 121, XXX, XXXVI and passim.

[192] Briggs 1981. Hankins 1970, p. 49.

with the sciences. The historians of our great epoch, who are generally great-grandchildren of Eulerian specialist notations, have not succeeded in following him in this, and end up by condemning, more or less, his general polyhedric commitment, transformed into a brilliant libertine superficiality. "... it is impossible today to swallow a single line of the mathematics of d'Alembert, whereas most of the works of Euler can be read with pleasure, ..." the German mathematician Carl Gustav Jacob Jacobi (1804–1851) immediately sustained, already in the nineteenth century. Yet d'Alembert was starting to overcome the contrast between Newton and Leibniz about infinitesimal calculus, by using a new notion of his: 'limit'. But he only gave an ambiguous definition of it, as a potential infinity, while the majority of the scientific community was following, by now, the notations of Leibniz and Euler.

Voltaire (François-Marie Arouet, 1694–1778) repeatedly displayed his appreciation for Chinese customs, although his knowledge of them was to prove to be spoiled, like that of others. In the Country-at-the-centre, the "bonzes jésuites" ["Jesuits bonzes"] would have been tolerated like all the others, up to a certain point, thanks to their sciences. Through their behaviour in China, he made fun of religious principles. Without church dogmas, the French man of letters put into the mouth of a Chinese mandarin his own faith in universal reason and in the supreme Being, who was the creator of the universe, infinite, omnipotent, and just. The dispute continued to be the one about the truth, a truth that the Jesuits cancelled in favour of illusions and opportunistic lies. Our fictitious Chinaman rejected the astrologers, and followed the true astronomers. "Ce qui vient de Dieu est universel et immutable; ce qui vient des hommes est local, inconstant, périssable. ... Voyage d'un bout de la terre à l'autre, tu verras que partout deux et deux font quatre, que Dieu est adoré partout; ..." ["What comes from God is universal and unchanging; what comes from men is local, inconstant and perishable. ... Travel from one end of the earth to the other; you will see that everywhere, two plus two is four, and God is worshipped everywhere."][193]

Above all thinking of its social regulations, however, Voltaire's admiration for China came to an end at a certain point. "Ce qui étonne plus, c'est qu'ayant si longtemps cultivé toutes les sciences, ils soient demeurés au terme où nous étions en Europe aux X^e, XI^e et XII^e siècles. Ils ont de la musique, et ils ne savent pas noter un air, encore moins chanter en parties. Ils ont fait des ouvrages d'une mécanique prodigeuse, et ils ignoraient les mathématiques. Ils observaient, ils calculaient les éclipses; mais les éléments de l'astronomie leur étaient inconnus. ... Aujourd'hui les Chinois commencent à oser faire usage de leur esprit, grâce à nos mathématiciens d'Europe." ["After cultivating the sciences for a long time, what is most surprising is that they stopped at the point where we had arrived in Europe in the X, XI and XII centuries. They make music, but they do not know how to write an aria, and even less how to sing with separate voices. They have carried out prodigious mechanical achievements, and they ignored mathematics. They observed

[193] Voltaire 1879, t. VI, pp. 33–34. Voltaire 1962, v. I, pp. 463–465, 556–557; v. II, pp. 88–108, 139–143. Voltaire 1966, pp. 86–88, 203–230.

and calculated eclipses, but the elements of astronomy were unknown to them. . . . Today, the Chinese are beginning to dare to make use of their genius, thanks to our mathematicians in Europe."] To the extent that, when an emperor banished all the European religious figures in the seventeenth century, "La cour ne garda que deux ou trois mathématiciens, parce que d'ordinaire ce ne sont pas ces gens-là qui bouleversent le monde par des arguments théologiques." ["The court only kept two or three mathematicians, because it is not usually people like that who upset the world with theological arguments."][194]

At that point, might not the suspicion have occurred to our famous Frenchman illuminated by reason that something did not add up in his picture? Why would the mathematical sciences develop in the religious and superstitious Europe, but not in a deistic, perhaps atheistic China? Wouldn't a relationship have been posited between religion and the mathematical sciences? Or had these been developed by the Chinese in other directions? And then, would reason and the mathematical sciences not be either universal, or one in form? Yet our ironic French man of letters must have had some doubts, if he concluded: ". . . mais qui sait si le temps ne viendra pas où les Chinois auront des Cassini et des Newton?" [". . . but who knows if the time will come when the Chinese will have some Cassinis [Giandomenico Cassini, 1625–1712] and some Newtons?"][195]

"It is surprising that this people of inventors has never succeeded in going beyond the initial elements of geometry. . . . more than three thousand nine hundred and sixty years ago, the emperor Yu had applied the properties of the right-angled triangle to make a geographical plan of a province; and the same [European churchman] quotes a book written one thousand one hundred years before our [C.] era, which says the famous demonstration attributed in the West to Pythagoras had been among the best-known theorems for a long time. We wonder: why the Chinese, who were so far ahead in such remote times, have still remained at this point; why astronomy is so ancient and so limited with them; and why in music they do not know of semitones yet." We are pleased to see that, at least in this form, our glorious *Zhoubi* reappeared during the eighteenth century in France, but we are sorry that the news about Chinese music, on the contrary, was still so rudimentary and inexact.[196] As a result of his philosophy of progress, the presumed backwardness of the Chinese in the sciences and music was explained by the Enlightenment scholar by that faraway Confucian people's preference for tradition compared to continual improvement. But it suffered from another equally illuminating defect: consisting ". . . of the nature of their language, the start of all knowledge."[197]

These illustrious and competent progressive people had been able in using China for their commendable purposes, but fundamentally they could not understand it

[194] Voltaire 1879, t. VIII, pp. 470, 473. Voltaire 1966, p. 85. Cf. Sabattini & Santangelo 1989, pp. 556–557.

[195] Voltaire 1879, t. VIII, p. 474. See above, Chap. 3.

[196] See above, Part I, Chap. 3, and Part II, Sect. 8.1.

[197] Voltaire 1966, pp. 220–221.

very well. Voltaire was not only speaking about the Western disputes on atheism in China or not, he was also describing himself when he had written: "The great misunderstanding about rites in China came about because we judged their customs on the basis of our own, because we carry the prejudices of our quarrelsome spirit to the other end of the world."[198]

Through the emperor of China, Voltaire mocked Rousseau, who, in his *Projet de paix perpétuelle [Project of a Perpetual Peace]*, had forgotten that tiny, insignificant country. Yet, curiously, in a page against war (and against the laws established by the victors) our French writer did not remember how contrary the Confucian Chinese had also been.[199] Perhaps his referents of the Company of Jesus did not write to him about it. As to China, he rather criticised the Taoists. "KU. But the sect of Laokium [Laozi] sustains that there is neither virtue nor vice. CU-SU. Does this sect sustain that neither good health nor illness exists? KU. No, it does not profess such a serious error. CU-SU. It is an equally serious, harmful error to think that neither good health nor illness of the soul exists, neither virtue not vice. ... KU. ... Who will be able to fix with certainty the limits that separate good from evil? What rule will you give me to discern them. CU-SU. Those of my master K'ung fu-tzu [Confucius]: ...".[200]

It is even more curious that Voltaire described China as a country of tribunals and laws. "Or s'il y eut jamais un État dans lequel la vie, l'honneur, et le bien des hommes, aient été protégés par les lois, c'est l'empire de la Chine." ["Now, if ever there existed a state in which life, honour, and the welfare of men have been protected by the laws, this is China."] As there was no separation between powers, the idea of Montesquieu (Charles-Louis de Secondat, 1689–1755) was diametrically opposite. Everybody knows that the deformations about China concealed criticism, which was risky if explicit, levelled at the French absolutism of the period.

Again based on Jesuit sources, Voltaire wrote about the war between Ming and Qing (Manciù). "Le P. Adam Schall, natif de Cologne, avait tellement réussi auprès de cet empereur par ses connaissances en physique et en mathématiques, qu'il était devenu mandarin. C'était lui qui, le premier, avait fondu du canon de bronze à la Chine; mais le peu qu'il y en avait à Pékin, et qu'on ne savait pas employer, ne sauva pas l'empire." ["Father Adam Schall, a native of Cologne, enjoyed such great success with this emperor, as a result of his knowledge of physics and mathematics, that he became a mandarin. He was the one who first cast bronze cannons in China; but the few existing at Peking did not save the empire because nobody knew how to use them."][201] Apart from the Chinese pupils converted by Matteo Ricci, we know that it did not go exactly like that, however, the version offered by the famous Frenchman does not seem to be among the worst. If nothing else, it seizes the role played by the Jesuit scientists in the war. It is to his merit too that he understood the Chinese did not separate the astronomy of eclipses from their terrestrial history.

[198] Voltaire 1966, p. 226.

[199] Voltaire 1961, pp. 411–413. Voltaire 1962, v. I, pp. 624–632.

[200] Voltaire 1962, v. II, pp. 91–92.

[201] Voltaire 1963, pp. 786 and 789.

"…the Chinese have united together the history of the sky and of the earth, and have thus reciprocally justified them."[202]

However, Voltaire avoided commenting on the *Yijing* and quite rightly doubted whether the bonfire of books, ordered by the first Qinshi Huangdi, had been very effective. Like almost all my students, this philosopher of the Enlightenment believed that printing had been invented by Johannes Gutenberg (1400–1470) before the Chinese, but he conceded them the primacy for paper. As regards gunpowder, he helped to spread the too imprecise commonplace that it was true the Chinese had invented it, but they only used it for fireworks. Again, therefore, there were no cannons in China, for him. "It was the Portuguese, in these recent centuries, who taught them the use of the artillery, and the Jesuits taught them to cast cannons. If the Chinese did not apply themselves to the invention of these instruments of destruction, it is not right to praise their virtue, because they waged war all the same."[203] The event that Confucius preferred music was completely ignored.

Voltaire found more truth in Newton than in Descartes, conceding to the latter only the Method of reasoning and geometry.[204]

In the secondary literature, the well-known rivalry, studied in detail, between Daniel Bernoulli, Euler and d'Alembert rarely involved music.[205] In one of the few cases where they did not ignore it, we can find sentences like the following. "Ce surcroît de complexité théorique ne traduit pas fidèlement la réalité." ["The [Euler's] excess of theoretical complexity does not reproduce reality faithfully"], because Euler's whole numbers distinguished octaves that the ears consider to be almost equivalent. "…la notion d'*ordre* fonde le principe du goût. Cet ordre se mesure, comme dans l'ancienne théorie de la coïncidence des coups. …ces choix n'expliquent pas vraiment les successions harmoniques réellement utilisées par les musiciens." For our Swiss mathematician, ["…the notion of *order* founds the principle of taste. This order is measured as in the ancient theory of the coincidence of strokes …. these choices do not truly explain the harmonic successions used by musicians in reality."]

Unfortunately, here too in an anachronistic manner, Euler's musical theory was judged through the subsequent theory of beats sustained by Helmholtz. The same thing happened again with d'Alembert and Rameau. "Contrairement à Euler, notre mathématicien français ne 'divague' pas; il fait d'aborde oeuvre de musicien, du moins de théoricien da la musique. Il n'invente pas de nouvelles gammes ou de nouveax accords pour satisfaire aux calculs. Il respecte les données de la musique tonale de son temps. …le statut qu'il accorde à la science de la musique. Il l'assimile à la physique." ["Unlike Euler, our French mathematician does not 'digress'; first of all, he does the work of a musician, at least of a theoretician of

[202] Voltaire 1966, p. 212.

[203] Voltaire 1966, pp. 213, 218–219.

[204] Voltaire 1962, v. I, pp. 51–57.

[205] Hankins 1970, pp. 42–65. Truesdell 1984, pp. 218–222. Delsedime 1972. Boyer 1990, pp. 517–532.

music. He does not invent new gamuts or new chords to satisfy his calculations. He respects tha data of the music of his times. . . . the statute that he [d'Alembert] assigns to the science of music: he assimilates it to physics."][206]

Patrice Bailhache did not take the liberty, alas!, of investigating the more or less close relationships of the main mathematical activities cultivated by Euler and d'Alembert with their relative musical models. In these historical reconstructions, there is a total disregard for the grand *finale* added by the renowned Enlightenment scholar to his articles on the equation for vibrating strings. It is strange that such a scrupulous scholar as the Frenchman, ready to follow Euler attentively in all his useless numerical acrobatics, should have overlooked an essential point. When the prime numbers of the Swiss scholar gave solutions opposed to those of the ears, too tolerant as our ears are with the equable temperament, Euler asserted that in reality, the ears did not hear the irrational frequencies, but their whole numbers. Thus we would be using the "covert" perception in which Leibniz believed.[207] Instead, Bailhache wrote for Euler: ". . . contrairement au maître d'Hannovre, il ne s'aventure pas à parler de perception inconsciente; . . . " [". . . unlike the master from Hannover [Leibniz], he does not venture to speak of unconscious perception; . . . "].[208] When he, too, ended up in Berlin, the renowned mathematician undoubtedly did not appreciate the philosophy of the monads proposed by the followers of Leibniz, but on the contrary, was perfectly happy with the "exercitium arithmeticae occultum", even if he did not understand the irony as regards the number seven (like Bailhache, as well). It is useless to insist on the clear belief of the historic Messieur in progress, which was achieved, according to him, also in musical theories, from Euclid to Helmholtz. It is a pity that he overlooked Aristoxenus together with Vincenzio Galilei, and remained silent about the cultural differences with the Chinese.

It has been pointed out, with a certain justification, that when in the eighteenth century musicians in general abandoned the old counterpoint style to dedicate considerable space to tonal harmony, as a consequence, the number of scholars interested in offering theoretical reasons for it underwent an increase.[209] We should not be too surprised, therefore, if at that time musicians wrote of science, and mathematicians of music, in search of more certain bases in that moment of crisis. On the other hand, with the fading of Euclidean geometry, substituted in certain sectors by infinitesimal calculus, sure foundations began to disappear even in the mathematical sciences.

The efforts made by Rameau to base tonal harmony on 'nature' received greater attention among the historians of music than among those of the sciences, who were too accustomed to liquidating with ill-concealed sufficiency the non-adepts of their religion. Unfortunately, somewhat intimidated, the former put too much trust in the latter, who are the most conspicuous. Thus it may have come about that

[206] Bailhache 1995a, pp. 13, 21–22 and 32.

[207] See above, Sect. 11.1.

[208] Bailhache 1996, pp. 7 and 17.

[209] Bailhache 1996, p. 5.

Rameau became "regressive", seeing that he continued to carry out investigations into proportions between numbers, in his first treatise. Instead, the French composer was simply among the average of other theoreticians, like Euler himself. While the scholars of harmonic sounds always represented a minority among musicians, not to speak of the natural philosophers who were to transform them into acoustics only in the following century. Thus the idea of calling "acoustic" the model of the second Rameau on the basis of the harmonics would be anachronistic, because it was to become such, partly thanks to him, only with the further evolution of the sciences.[210]

If we wish to put it like this: it was the composer Rameau who contributed with his musical theory to the creation of acoustics as a discipline of physics, which did not exist before him. At most, before him, articles generally regarded the mechanics of *ictus* [strokes]. In the history that we are relating, simplifying as much as possible and undoubtedly too much, here we have found first of all music, and only afterwards acoustics, and not vice versa. Also Thomas Christensen wrote about it as follows: "Yet overtones were not well-understood by scientists in 1722. [...] For the forty-year period in which Rameau struggled to explicate and integrate the phenomenon of harmonic upper partials within his theory coincided almost exactly with the research of numerous continental scientists on the subject. [...] To a surprising degree, this research impinged upon Rameau's own theoretical efforts (of which all the above-mentioned scientists were quite aware). The story of Rameau's theory of music after 1722 is inextricably linked with the progress and polemics of these scientists."[211] At this point, also our reader should have this clear. What would remain to be better understood, carefully avoiding every linear causal historical model, is who influenced who, and why.

Anyway, with the harmony of the French composer, it was becoming impossible to maintain the ancient Pythagoreanism, despite the final variants, hard to die, like those of Tartini, because it was incompatible with the new tonal rules. In his first treatise of 1722 Rameau still tried, amid innumerable contradictions. But when, thanks to the review of the anti-Newtonian Jesuit Cartesian father, Louis Castell (1688–1757), he discovered the higher harmonics of a sound, he understood that with these, he would be able to justify his theory much better.[212] D'Alembert was to be fascinated by them, as we know. Yet the latter did not know how to connect the harmonics of Sauveur and Rameau with his 'smooth' solutions of vibrating strings. However, the event that some relationship must exist between them might have been suspected by him to the detriment of his rivals. With Daniel Bernoulli and Euler, d'Alembert was already involved in a dispute, otherwise he would not have concluded the relative articles criticising those who (like Rameau and Euler) still used ratios for music.

[210] Christensen 1993, pp. 28 and passim.

[211] Christensen 1993, pp. 133–134.

[212] Christensen 1993, pp. 102, 133 and passim.

Our French physicist and mathematician would have liked to save his theory, which contemplated particularly general, even "very irregular" forms for vibrating strings, and not only *sine* curves. While the harmonics of Sauveur and Rameau had to follow some rules. At the time, he came out by distancing mathematics from physics, and unloading the problem on the latter. "We are led to wonder, then, how d'Alembert reconciled Rameau's *corps sonore* [sounding body] with his research on the vibrating string. Surprisingly, the answer is that he did not."[213] The question sounds appropriate to me, too, but the answer remains partial, unfortunately, because it has ignored the final note to the works on vibrating strings, mentioned above.[214] This page the music historian probably did not read.

In the nineteenth century, the next step was to be taken by Joseph Fourier.[215] Daniel Bernoulli made some polemical intermediate contributions, with his empirical approximation to the solution, composed of the succession of *sine* functions. While Euler generalised the solutions found by d'Alembert to curves 'with edges'. The mathematician of the Enlightenment had thus already taken a first step in that direction, detaching himself completely from the neo-Pythagorean tradition, and bringing back music, also as regards its theory, to the environment of auditive sensations. But Mr. Christensen (like Rameau) must have not the *petite difference [little difference]* between numbers and ears very clear. Actually, on one page, he to d'Alembert attributed an inclination towards "ratios", as they were "... expressed more clearly ...". While on the opposite page, we read that a "... progression des quintes ..." ["... progression of fifths ..."], as the definition of a "mode", was still preferred by the mathematician to the triple proportion of Rameau. As if his ears had not been sufficient for d'Alembert to feel the intervals of a fifth.[216]

The distinction between numerical ratios and auditory sensations, which as we know goes back to Aristoxenus, sounded so much the more interesting to maintain, the closer the mercurial, contradictory and changeable Rameau came to the latter with his last writings: the unpublished "Vérités interressantes" ["Interesting truths"]. "Is it not a monumental error to attribute to numbers a virtue that we know was born from sensible relationships, of which those numbers are only signs?"[217] Moving away here from the neo-Pythagoreans and confusing mystical ideas with other pantheistic and post-Leibnizian ideas, was not the renowned composer looking again towards d'Alembert?

If this explicit aspect, linked with the senses of natural philosophy followed by d'Alembert, were not understood better, the interests regarding music would not be

[213] Christensen 1993, p. 156.

[214] See above, Sect. 11.2.

[215] See below, Sect. 12.1.

[216] Christensen 1993, pp. 276–277. For the whole of his essay, Christensen used the terms "acoustical" and "acoustical phenomenon" when he wanted to refer to the phenomena of the ear or to the sense of hearing. This risks misleading the reader, because acoustics as a discipline did not exist yet.

[217] Christensen 1993, p. 297.

understood, either: his pleasure in singing it with his own voice, and in listening to it with his ears. Hence derived his preference for Italian music, and the search for its roots together in the 'sounding body'. Otherwise we could not explain his involvement in writing the *Élémens*. Reducing the text to "propaganda" for the philosophy of the Enlightenment may perhaps have some (little) justification, but this is only a caricature of it. "It was the perfect means to illustrate the merits of his professed empirical-deductive methodology. D'Alembert was simply following a tried-and-true strategy used by all the philosophers – . . . by Voltaire – in which one employs the most innocuous literary genre in order to disseminate covertly some controversial or otherwise opaque idea."[218] To this (pre-revolutionary?) Machiavellianism, I prefer to oppose the sincere love of the mathematician (more or less the prey, at times, of his senses, and at other times of reason) for *La serva padrona* by Pergolesi. Why Mr. Christensen strangely never considered it?

As he dealt above all with music, and probably little, if not at all, with mathematical sciences, this English-(from the United States?)-speaking historian quite rightly did not want to hide to what extent Rameau, d'Alembert and the others were affected by the cultural context. Before the revolution of 1789, the French Enlightenment culture of the eighteenth century influenced all of them in a 1,000 different ways. Though following a similar style of research, I have underlined that Christensen overlooked certain aspects that are remarkable for our history. Was it because he, too, admitted that every observation or affirmation (including scientific ones) is loaded with values? His own, through the adjectives he uses, betray little sympathy for the Enlightenment of d'Alembert and the others.[219]

Surely he prefers Rameau, the practical musician, even if he contradicts himself in the theory, because he knows him better. But why should we impoverish the image of the French mathematician, reducing it, as Christensen has done, to a simple rational mechanist? We do not suffer his limitations in the mathematical sciences, nor do we share his values, although nobody will expect a defence of the Enlightenment cause from me at this point. Equally, the reader is learning about my inevitable prejudices chapter after chapter. Actually they can be understood better, seeing that I am dedicating more space to musicians than Christensen has conceded to scientists. Furthermore, the music historian has a second-hand knowledge of them, through excessively contaminated historical reconstructions, like those of Truesdell. But I will avoid fighting over what Rameau and d'Alembert *really* affirmed, in order to avoid continuing their own *querelle*, shrouded in their relative *truths*, with the precise aim of concealing far more valid aspects.

The curious reader will find other different, or at times similar, images of Rameau, d'Alembert, Diderot, Rousseau and the others, to be compared with my own, scattered over an abundant quantity of literature.[220] As a composer to be listened to with one's ears, Rameau does not arouse great emotions in me today.

[218] Christensen 1993, p. 268.

[219] Christensen 1993, pp. 298–306.

[220] Fubini 1964; 1971; 1986.

Than the *opéra* ballets suitable for the royal court, I find most interesting his pieces for the harpsichord. If he had not been dazzled by the luxury and the vanity of the sitting-rooms and the official theatres, who knows what else he would have produced for us. Although at times repetitive in the rhythmic and harmonic interplay of trills and cadences, in the "Gavotte" of the *Nouvelles suites de Pièces de Clavecin [New Suite of Pieces For Harpsichord]* (1728), he succeeded in moving away from ballet steps in costume to rush into vigorous cavalcades across the keyboard. The agility of the fingers became an imitation in "La Poule" ["The hen"]. Among the *Cinq Pièces [Five pieces]* (1741), the harmonic beauty spots of "L'Agaçante" ["The irritant"] reached an effective expressiveness.

Rameau will forgive me, at this point, but once he would have got very angry, if my (corrupted?) senses are not ravished, but rather made drowsy, by a lyric tragedy like *Castor et Pollux*, written, other things, to celebrate French victories in war. And yet there were also some good instrumental pages, like the "Deuxième et troisième airs" ["Second and third aria", sic!] of "Act I, scene IV" or the grand finale with the apotheosis in Heaven.

> C'est l'ordre du Maître du monde,
> C'est la fête de l'Univers.
> [It is the order of the Lord of the world [God or Louis XV?]. It is the festivity of the universe.][221]

Though often drowned in a rhetorical, melancholy declamation, also *Abaris ou les Boréades* contains interesting instrumental pages, which succeed in expressing drama effectively. Never executed until 1963 and staged only in 1982, in this lyric tragedy which was his last one, the French composer revealed at times an unexpected melodic vein to represent the melancholic sufferings of a love opposed by power and by conventions. This can be heard, for example, in "Act II, scene VI", "Comme un Zéphir qui vole et jamais ne s'engage" ["Like a zephyr which flies and never commits itself"]. If we can believe a testimony, Rameau once said: "Si j'avais trente ans de moins, j'airais en Italie - Pergolèse deviendrait mon modèle... mais quand on a plus de soixante ans, on sent qu'il faut rester ce que l'on est" ["If I were thirty years younger, I would go to Italy - Pergolesi would become my model ... But when one is more than sixty years old, one feels that it is necessary to remain what one is."]. According to Diodorus Siculus (First century B.C.), Abaris, the secret son of Apollo, would have been the tutor of Pythagoras.[222]

Enrico Fubini has quite rightly noted: "..., a continuous gap in the Eighteenth century between musical reality, on the one hand, and musical culture, theoretical reflection, on the other." Much more effective than philosophical hypotheses, written by the various Rousseau, d'Alembert, Diderot or Rameau, the compositions of Pergolesi, Johann Sebastian Bach (1685–1750) and then Wolfgang Amadeus Mozart (1756–1791), changed the musical culture, moving it towards the pleasure

[221] Rameau 2007, p. 102.

[222] Rameau 1990, p. 16 and passim.

of listening and the expression of the passions experienced by the characters. A French abbot of the eighteenth century tried to defend the harmony of nature, seen as a divine order. "Presenting pleasure for pleasure; this is subversion, or, speaking more clearly, it is prostitution." Initially, even the *opéra* of Rameau was defined as "…diabolique…" ["…diabolic…"],[223] but above all it was Italian music that was considered corrupting.

With his extensive studies, having consulted our Euler, Rameau and d'Alembert, the Spanish Jesuit Antonio Eximeno (1729–1808), could criticise them after due consideration. "Fortunatamente col lungo studio della Matematica mi trovava io già munito di alcune riflessioni circa la falsa evidenza di questa scienza, o piuttosto dell'abuso fattosi in questo secolo delle verità ipotetiche della Geometria per dar apparenza di verità dimostrate a certi presupposti parte incerti, parti falsissimi. Per questo le formole algebriche, colle quali il Signor Eulero fa della musica un nuovo trattato di Algebra, non mi spaventarono; anzi questo trattato fu per me il testimonio più autentico de' miei sospetti contra la Matematica, vedendo un Geometra tanto celebre servirsi di quella per fondare una nuova teorica di Musica in pure illusioni della fantasia." ["Fortunately, with my long studies of mathematics, I already possessed certain reflections about the false evidence of this science, or rather about the abuse that has been made in this century of hypothetical truths of Geometry, to give the impression of proven truths to certain presuppositions, some of which are uncertain, and some totally false. For this reason, the algebraic formulas with which Mr. Euler turns music into a new treatise of Algebra, did not scare me. Indeed, this treatise was the most authentic evidence for me of my suspicions against mathematics, seeing such a renowned geometrician make use of it as a basis for a new theory of music in pure illusions of imagination."] Yet he did not express himself too well regarding the French matematician, perhaps because he only knew the *Élémens de la musique*. "Ma alla fine mi avvidi che le ultime proposizioni di quella distruggono le prime, e che tra il fenomeno fisico, che le serve di fondamento, e le regole di armonia non v'interviene che un concorso casuale simile a tant'altri, co' quali s'abbagliano spesse volte i Filosofi, facendo di due cose, che concorrono casualmente insieme, l'una causa dell'altra." ["But in the end, I realised that the last few propositions of it destroy the first ones, and that between the physical phenomenon, which serves as a basis, and the rules of harmony, there is nothing but a casual coincidence similar to many others, by which Philosophers are often blinded, making, of two things that are casually found together, one the cause of the other."] If, like us, he had read the other pages of d'Alembert, the Jesuit would have discovered a side more inclined towards the senses and instinct, like him.[224]

Thus started years of heated disputes, which from time to time shifted the target. In mathematics, the most famous ones were the controversies about infinitesimal

[223] Fubini 1971, pp. 116 and 53.

[224] Eximeno 1774, pp. 298–299. Also Fubini has left d'Alembert's declared Italian and sensual inclinations in the dark; Fubini 1971, pp. 186–187.

calculus, and those about vibrating strings. In music, the followers of Lulli were the first to start, against those of Rameau, and then those of Rameau against Pergolesi, and lastly the supporters of Nicola Piccinni (1728–1800) against the so-called 'reform' of Christoph Willibald Gluck (1714–1787). Their writings gathered around two new literary genres hitherto unknown: musical criticism and the history of music. Among a 1,000 dangers, difficulties and censures, the first liberty that all of them took was that of speaking up, weighing the pros and cons, and doing so openly in public, if necessary without signing their names, for fear of retaliation. Criticism developed. As all this had been cultivated in the context of the Enlightenment, the new attitude could have developed in any environment, from religion to politics, but it was to stop, unfortunately, in the face of what was inspiring the *Encyclopédie*: the sciences. These new *philosophes*, ideologues or whatever else they have been called, substituted the ancient, till then untouchable, faith in God with another faith, equally incontrovertible for them, in reason and scientific progress.

After starting from France, the freedom to criticise, to choose and to change extended beyond the Rhine. Typical of all these healthy polemics, which often presented their advantages for advertising, were the continuous reshuffles in the fronts. These did not seem to be difficult to justify, because the ideology and the criticism could open up large spaces, without too much effort, for the most varied interpretations of the same object. For example, the musician Gluck, who was travelling half-way round Europe, through Germany, Austria, Italy and France, could praise Rousseau without any problems, although he had become the paladin of a reform to return to the refined dramatic nobility of Lulli and Rameau. "The accent of nature is the universal language; Rousseau used it with the greatest success in the simple genre. His *The Village Fortune-teller* is a model that no other author has yet reached."[225] Among other things, this opinion confirms that it was not very different from the other court works.

Seemingly separate from the disputes, because he preferred to play the organ and to prise God, instead of writing pamphlets, we must not forget, however, that in German-speaking countries, J. S. Bach was living a quiet life, creating many compositions, together with his numerous children born from his first and his second wives. At that time, because he was to become famous only half-way through the following century, he could pass for a bombastic, artificial composer who looked to the past: in view of his indulging in counterpoints and fugues, which were presumed enemies of modern harmony and reason.[226] However, as regards melodic invention, the art of harmonic modulations and lastly, effectiveness in representing human passions, Bach not only deserved to be defended, but to receive an emotional admiration. Everybody knows by now that this German composer, destined for well-deserved future glory, had drawn on the best Italian tradition, and had also expressed in his religious *Passions* sentiments that were very earthly.

[225]Fubini 1971, p. 104.

[226]Fubini 1986, pp. 250–254.

In the varied panorama of musical theoreticians with mathematical and scientific claims, did Bach occupy any position? Apart from lyric works for the theatre, he composed all kinds of music in many different styles, re-elaborating them in his own way, and often going further. As regards chromatisms, he could already see so much further, that someone identified him as "the catastrophe of music", and during the twentieth century, in him were sought the limits to that tonality which had just been codified by Rameau. In the end, he even wrote notes to be read rather than be played on earthly instruments, where he revealed his uncommon harmonic and counterpoint abilities. "Vom Himmel hoch" ["From the heavens on high"] was it only the Lutheran God, and sometimes the Catholic God that came down into the pages of Bach, or was there something else, as well?

Towards the end of his life at Leipzig, our German composer took part in the Societät der musicalischen Wissenschaften [Society of musical sciences], which harked back to the philosophy of Leibniz in the person of his leading pupil, Christian Wolff. For this Society, he wrote *Einige canonische Veraenderungen über das Weynacht-Lieder . . . [Some canonical variations on the Christmas song . . .]*. "... precisely because the starting-point had been identified as an element in the church repertoire, the stimulus towards the 'scientific' realisation of the cycle was determined by a spiritual solicitation, albeit of a special kind, ... the astral clarity of the page does not lie so much in the revelation of the guiding formula, which achieves a certain geometrical dimension, as rather in the harmonic vision of musical elements which intertwine in a metaphysical rarefaction of the sound."

On the occasion of a visit in 1747 to King Frederick II of Prussia, to celebrate "... the glory of a monarch whose greatness and proficiency both in the sciences of war and peace, and particularly in music, everyone must admire and venerate", Bach presented another 'scientific' communication in music: *Regis Jussu Cantio Et Reliqua Canonica Arte Resoluta [Song By the Order of the King and Other Passages Solved In Accordance With the Art of the Canon]*. In an acrostic, the word RJCERCAR can thus be read, which had a double meaning. As an Italian word, for the musicians of that country, it could mean a free exploration conducted on the keyboard, but then it also assumed the meaning of composing a rigorous counterpoint in the style of a polyphonic motet, which Bach expanded to six parts.[227]

Then, "..., senza più concedere spazio alle immagini della musica viva, per cercare invece, al crepuscolo, la luce rarefatta della boeziana *musica mundana*, quella delle sfere celesti in movimento, quella che orecchio non è in grado di percepire" ["..., without conceding any space to the images of living music, to seek instead at dusk the rarefied light of the Boethian *mundane music*, that of the heavenly spheres in movement, the kind that the ear is not able to perceive"], Bach wrote *Die Kunst der Fuge [The Art of the Fugue]*. "Ueber dieser Fuge, wo der Nahme B.A.C.H. ... angebracht worden, ist der Verfasser gestorben" ["Over this fugue where the name B. [$B\flat$] A. [A] C. [C] H. [B], the author died."] In the spirit of Leibniz, the composer constructed his universe combining and

[227] Basso 1983, II, pp. 687–688, 702, 705, 710, 713–715.

re-combining 12 notes to obtain orderly designs which could never finish. This was not to happen. It was sufficient to follow the laws of counterpoint and harmony to create organisms that were always new. If *Das wohltemperierte Clavier [The Well-tempered Harpsichord]* could give the illusion that the universe was all enclosed in a finite para-scientific formula, *The Art of the Fugue* refuted a similar claim because it remained suspended.

Nevertheless, there was certainly not a lack of people who may think that the harmony of the spheres had finally been realised with Bach. But, unlike that work, we can fortunately listen to the *Passio secundum Matthaeum [Passion According to Matthew]*, the *Toccata and fugue* in *D* minor, the *Brandenburg Concertos* or the *Concerto nach italienischem Gusto [Concerto In Accordance With the Italian Taste]* with our own ears. Nor did the Lutheran German composer would neglect rhythm, going so far as to make people dance to some fugues of the *Well-tempered Harpsichord*, because his aim was not only to elevate and instruct, but also to amuse.[228]

Even after Bach, new music continued to be composed. Suspended between heaven and earth, the musical science of the devoted German fell into more carnal hands with Wolfgang Amadeus Mozart. But not without the child from Salzburg presenting us with a work that is pertinent to our history of the sciences: *Il sogno di Scipione [Scipio's Dream]*. Metastasio took up Cicero's story.

> FORTUNA: Non lo conosci a tante che ti splendono intorno lucidissime stelle? A quel che ascolti insolito concento delle mobili sfere? A quel che vedi di lucido zaffiro orbe maggior, che le rapisce in giro?
> SCIPIONE: E chi mai tra le sfere, o Dee, produce un concento sì armonico e sonoro?
> COSTANZA: L'istessa ch'è fra loro di moto e di misura proporzionata ineguaglianza. Insieme urtansi nel girar: rende ciascuna suon dall'altre distinto; e si forma di tutti un suon concorde. Varie così le corde son d'una cetra; e pur ne tempra in guisa e l'orecchio, e la man l'acuto e'l grave, che dan, percosse, un'armonia soave. Questo mirabil nodo, questa ragione arcana che i dissimili accorda, proporzion s'appella, ordine e norma universal delle create cose. Questa è quel che nascose, d'alto saper misterioso raggio, entro i numeri suoi di Samo il saggio [Pitagora].
> SCIPIONE: Ma un'armonia sì grande perché non giunge a noi? Perché non l'ode chi vive là nella terrestre sede?
> COSTANZA: Troppo il poter de' vostri sensi eccede.
> [CHANCE: Do you not recognize it, with so many bright stars shining around you? The uncommon harmony of the mobile spheres that you are listening to? What you can see of sparkling sapphire, the greater orb which carries them away on its circuit?
> SCIPIO: And who, ever, among the spheres, O goddesses, produces such a harmonic and resounding concert?
> CONSTANCY: The same proportionate inequality of movement, and measure that exists between them. Together they clash with one another in their turning: each produces a different sound from the others; and a harmonious sound is formed by all of them together. Thus the strings of a lyre are various; and yet in the same way, both the ear and the hand temper the acute and the deep sound so that they produce, when plucked, a sweet harmony. This admirable bond, this arcane reason that tunes dissimilar things, is called ratio: the

[228]Basso 1983, II, pp. 717, 721; Basso 1979, I, pp. 658 and 663.

universal order and norm of the created things. This is what was hidden, the mysterious ray of high knowledge, within his numbers by the Sage from Samos [Pythagoras].
SCIPIO: But why does so great a harmony not arrive as far as us? Why does he who lives there in the terrestrial seat not hear it?
CONSTANCY: Too much the power of your senses exceeds.][229]

In the style of the Italian librettist, the recitative was set to music by a very young Mozart inexpressively. The music of the spheres was no longer an element of interest for him. He took a little more passion in the arias, which distinguished Chance from Constancy. The two goddesses contended the soul of Scipio, who, in the end, entrusted himself diligently to Constancy, because he had to celebrate the bishop Colloredo taking office at Salzburg. But that rascal of a famous musician provided appropriate good warbles rather for the flirtatious Chance. "Lieve sono al par del vento; vario ho il volto, il piè fugace; or m'adiro e in un momento or mi torno a serenar." ["I am as light as the wind; varied is my face, my foot is fleeting; at one moment I get angry, and in a moment I return serene."][230] It will not be necessary to remind many readers that he was to express similar passions in the *Don Giovanni* and in *Così fan tutte*. "Tutti accusan le donne ed io le scuso se mille volte al dì cangiano amore; altri un vizio lo chiaman ed altri un uso, ma a me par necessità del core." ["Everyone accuses women, and I excuse them if they change their love a thousand times a day; others call it a vice, and others a custom, but to me it seems a necessity of the heart."][231] The dissolute rake to be punished substituted Chance, while Scipio became the statue of the commander. The latter deserved to be animated a bit, although deceased, but as in the myth he remained the prisoner of an immobile, inexorable music, which recalled the pages of Rameau, now a thing of the past. Curious that in *Pygmalion*, also the French musician had animated a statue, inspired, perhaps, by an idea proposed by the philosopher Condillac (1715–1780).[232]

Bach would wish to restore the whole universe to us, and thus he needed to exhaust all the tonalities. Mozart, instead, only wanted to communicate himself, with his stories, and he employed few tonalities.[233] But what a lot of beautiful modulations are found here, as well! Let us hear the *Concerto for pianoforte* in *C* major KV 467 (Fig. 11.10).[234] The genius from Salzburg, with a sudden piano solo, passed into C minor, thus flooding our soul with a profound melancholy. Listening to a piece of music like that, all the discussions that had continued for centuries, about mathematical theories of scales and tuning, could be put aside. It had to be recognized if someone was capable of producing similar harmonies on Earth, then every principle and foundation of sound would be compelled to grant them

[229]Mozart 1991, **4**, pp. 1455–1457.
[230]Mozart 1991, **4**, pp. 1430–1454.
[231]Mozart 1991, **8**, pp. 1094–1095.
[232]Christensen 1993, pp. 218–231.
[233]Alfred Einstein 1951, p. 171.
[234]Mozart 1991, **16**, p. 395 bars 109–110 and following.
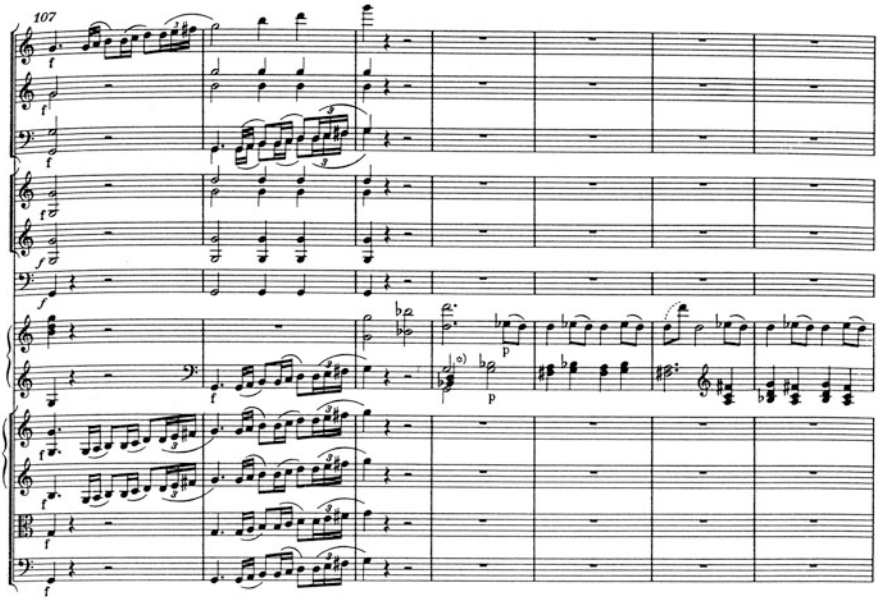
Fig. 11.10 Mozart modulated from C major to C minor in bar 110 (Mozart, *Concerto per pianoforte* in *do maggiore* KV 467, 1991, vol. 16, p. 395)

all necessary freedom. The scientific arguments that claimed to impose limits, or prohibit musicians from obtaining similar results would no longer have any value.

> . . . le mot *Tien* ou *Changti* ne comporte pas précisément la même idée que le mot *Al* donnait en arabe, *Jehova* en phénicien, *Knef* en égyptien, *Zeus* en grec, *Deus* en latin, *Gott* en ancien allemand. Chaque mot entraîne avec lui différents accessoires en chaque langue;
> [. . . the word *Tian* [heaven] or *Shangdi* [emperor on high] does not entail exactly the same idea that the word *Al* communicates in Arabic, or the word *Jehova* in Phoenician, *Knef* in Egyptian, *Zeus* in Greek, *Deus* in Latin, or *Gott* in ancient German. In each language, every word brings with it different additions.]
>
> Voltaire

> Ma in ultimo quelle stolte e superbe domande commossero talmente l'ira del dio, che egli risolse, posta da parte ogni pietà, di punire in perpetuo la specie umana, condannandola per tutte le età future a miseria molto più grave de le passate. Per la qualcosa deliberò non solo di mandare la Verità tra gli uomini a stare, come essi chiedevano, per alquanto di tempo, ma dandole eterno domicilio tra loro, ed esclusi di quaggiù quei vaghi fantasmi che egli vi avea collocati, farla perpetua moderatrice e signora della gente umana.
> [But in the end, those stupid, proud questions aroused to such a point the wrath of the god, that he resolved, leaving aside all pity, to mete out a perpetual punishment to the human race, condemning it, for all future ages, to a misery far more serious than in the past. For this reason, he deliberated not only to send Truth to dwell among them, as they had asked, for some time, but, giving her an eternal dwelling-place among them, excluding from down here all the indistinct ghosts that he had placed in it, he also resolved to make her their perpetual moderator and mistress of the human race.]

TERRA. Senti tu questo suono piacevolissimo che fanno i corpi celesti coi loro moti?
LUNA. A dirti il vero, io non sento nulla.
TERRA. Né pur io sento nulla, fuorché lo strepito del vento che va da' miei poli all'equatore, e dall'equatore ai poli, e non mostra saper niente di musica. Ma Pitagora dice che le sfere celesti fanno un certo suono così dolce ch'è una maraviglia; e che anche tu vi hai la tua parte, e sei l'ottava corda di questa lira universale: ma che io sono assordata dal suono stesso, e però non l'odo.
LUNA. Anch'io senza fallo sono assordata; e, come ho detto, non l'odo: e non so di essere una corda.
[EARTH. Can you hear this very pleasant sound that the heavenly bodies make with their movements?
MOON. To tell the truth, I cannot hear anything.
EARTH. Nor can I hear anything, either, apart from the howling of the wind that goes from my poles to the equator, and from the equator to the poles, and it does not prove to know anything about music. But Pythagoras says that the heavenly spheres make a certain sound so sweet that it is a wonder; and that you, too, have your part in it, and you are the eighth string of this universal lyre: but that I am deafened by the sound itself, and yet I cannot perceive it.
MOON. I, too, am undoubtedly deafened; and as I said, I cannot perceive it: and I did not know that I am a string.]

Giacomo Leopardi

The reason of the strongest is always the best.

Jean de la Fontaine

Il y a ici des philosophes qui, à la vérité, n'ont point atteint jusqu'au faîte de la sagesse orientale: ils n'ont point été ravis jusqu'au trône lumineux; ils n'ont ni entendu les paroles ineffables dont les concerts des Anges retentissent, ni senti les formidables accès d'une fureur divine; mais, laissés à eux-mêmes, privés des saintes merveilles, ils suivent, dans le silence, les traces de la raison humaine.
[Here there are philosophers who, to tell the truth, have not arrived anywhere near Oriental wisdom: they have not been carried off to the radiant throne; they have not heard the ineffable words that echo in the angels' concerts, nor have they experienced the fearful excesses of the divine wrath; but, abandoned to themselves, deprived of holy wonder, they pursue the traces of human reason in silence.]

Montesquieu

Anything is possible: you only need to wait a few thousand billion years. Nothing is necessary: you do something else, if the case so requires. Death does not exist: in the continuous flow that transforms everything.

Anonymous extraterrestrial of the XXI century

Chapter 12
From French to German

> Anche più varia e mutabile si è la condizione così della filosofia come delle altre scienze: se bene al primo aspetto pare il contrario: perché le lettere amene riguardano il bello, che pende in gran parte dalle consuetudini e dalle opinioni; le scienze al vero, ch'è immobile e non patisce cambiamento. Ma come questo vero è celato ai mortali, se non quanto i secoli ne discuoprono poco a poco; però da una parte, sforzandosi gli uomini di conoscerlo, congetturandolo, abbracciando questa o quella apparenza in sua vece, si dividono in molte opinioni e molte sette: onde si genera nelle scienze non piccola varietà.
> [The condition of philosophy and of the other sciences is even more varied and changeable, although at first sight the opposite seems to be true: because pleasant reading regards the beautiful, which largely depends on customs and opinions; the sciences are the result of truth, which is immobile and does not undergo any change. But as this truth is hidden to mortals, except that part of it that the centuries gradually reveal, yet on the other hand, men strive to know it, making conjectures, and embracing, in its place, this or that semblance of it, thus they divide into many opinions and several sects: consequently, a considerable variety is generated in the sciences.]
>
> Giacomo Leopardi
>
> … It happens every two or three years in science that a thing previously considered wrong suddenly overturns all the concepts, or that a humble idea, held in contempt, becomes the queen of a new world of ideas, and these events are not only revolutions, but lead upwards like a heavenly ladder.
>
> Robert Musil

12.1 From Music-Making to Acoustics: Luigi Giuseppe Lagrange e Joseph Jean-Baptiste Fourier

Luigi Giuseppe Lagrange (1736–1813) entered into the controversy about the forms assumed by a string made to vibrate, with some articles about sound. However, we will overlook the details of how much he felt himself, at least at the beginning, closer to Euler than to d'Alembert or to Daniel Bernoulli, as is well known. Among other things, our mathematician from Turin believed in the possibility that mathematics could illuminate physics; indeed, in his famous book about analytical mechanics, he would have desired to reduce the latter to the former.

T.M. Tonietti, *And Yet It Is Heard*, Science Networks. Historical Studies 47,
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I wonder, rather, what positions he would have assumed regarding music. He spoke, referring to his studies, of "... l'application que j'en fais aux cordes de musique, ..." ["... the application that I make of them to the strings of music, ..."]. He considered Tartini's third sound. As regards the nature of consonances and dissonances, he rejected the explanation of Rameau, based on harmonics, preferring that of Euler with ratios. "... la concurrence des vibrations en est toujours le fondement, ... [...] ... on voit évidemment que ce principe même tire son origine de celui de la concurrence des vibrations, principe dès longtemps reconnu pour la source des consonnances et des dissonances, et sur lequel M. Euler a établi sa nouvelle théorie de musique ..." ["... the concord of vibrations is always their foundation, ... [...] ... it can clearly be seen that this same principle [of harmonics] takes its origin from that of the accord among the vibrations, a principle that has long been recognized as the origin of consonances and dissonances, on which Mr. Euler has established his new theory of music ..."]. In the end, however, he did not exclude the role of taste or habit, and he even made reference to d'Alembert.[1]

In his following article, Lagrange concluded as regards Daniel Bernoulli. "... lorsqu'un système quelconque de corps fait des oscillations infiniment petites, le mouvement de chaque corps peut être considéré comme composé de plusieurs mouvements partiels et synchrones chacun à celui d'un pendule simple." ["... when any system of bodies creates infinitely small oscillations, the movement of each body can be considered as composed of many partial, movements, each synchronous with those of a simple pendulum."][2] Although he was inclined towards mathematical formalism, and had filled his formulas with *sines* and *cosines* referring to small isochronous oscillations, our mathematician from Turin, who was to lead his life between Berlin and Paris, did not succeed, however, in seeing the solution of the controversy about vibrating strings when it was right under his nose. Without wishing to make them the effect of a cause, without doubt he was not helped by his prejudices, and his lack of ability to hear harmonic sounds, which he denied could be generated by a single string. "Je suis donc enclin à croire que ces sons peuvent être produits par d'autres corps qui résonnent au bruit du son principal, ..." ["I am therefore inclined to believe that these sounds may be produced by other bodies which resound at the sound of the main body, ..."]. He also wrote this to Euler. "As regards musical notes, I agree perfectly with you, Sir, that the consonant sounds Mr Rameau claims to hear in a single string derive from other vibrating bodies. And I do not see why this phenomenon should be considered as a principle of music, more than the true ratios which are its foundations."[3]

Contrary to d' Alembert, Lagrange stated that the effect of music on him was the following: "Je l'aime parce qu'elle m'isole; j'en écoute les trois premières mesures, à la quatrième je ne distingue plus rien, je me livre à mes réflexions, rien ne m'interrompt, et c'est ainsi que j'ai résolu plus d'un problème difficile."

[1] Lagrange 1759, pp. 45 and 144–147.

[2] Lagrange 1760–1761, p. 316.

[3] Lagrange 1759, p. 147. Quoted in Christensen 1993, p. 155.

["I love it because it isolates me; I listen to the first three bars, and at the fourth, I cannot distinguish anything any more, I dedicate myself to my reflections, nothing interrupts me, and this is how I have solved many a difficult problem."][4] This kind of [in]difference reveals consistency with his idea of mathematical sciences that he made increasingly independent from earthly phenomena. With their transcendence, they could be applied to mechanics a posteriori only. It was significant that drawings were missing in his books. Like Leibniz, he loved new symbolic notations. We are debtors to him for the notation for the derivative of a function $f'(x)$, which tried to avoid infinitesimals like dx.[5]

His failure to understand, and decision to ignore harmonics was to continue to make itself felt, even when Joseph Jean-Baptiste Fourier (1768–1830) took his first example from the vibrating string for his most famous results: developing functions in trigonometric series, defined since then as Fourier's. Actually, Lagrange had still remained powerful through the Revolution in the Parisian scientific institutions, and raised so many, and such, objections against the new theory, that Fourier's first work was never published. "Si l'on applique les principes que nous venons d'établir à la question du mouvement des cordes vibrantes, on résoudra toutes les difficultés que présentait l'analyse employée par Daniel Bernoulli. ... Les inventeurs de l'analyse des équations aux différences partielles regardaient même cette application comme impossible." ["If we apply the principles that we have established to the problem of the movement of vibrating strings, all the difficulties presented by the analysis used by Daniel Bernoulli will be solved. ... The inventors of the analysis of equations with partial derivatives likewise considered this application as impossible."]

Fourier quoted from a passage of d'Alembert and from one of Euler. Then he went on: "... je me suis convainçu, en effet, que le mouvement de la corde sonore est aussi exactement réprésenté dans tous les cas possibles par les développements trigonométriques que par l'intégrale qui contient des fonctions arbitraires." ["... I am convinced that the motion of the vibrating string is as exactly represented in all possible cases by trigonometric developments as by the integration which contains the arbitrary functions."] The idea was repeated: "... on trouve dans l'emploi de ce nouveau théorème la solution complète des diverses objections qui ont été proposées autrefois par Euler et d' Alembert dans la question du mouvement des cordes sonores." ["... using this new theorem, we find the complete solution to the various objections that have been proposed elsewhere by Euler and d'Alembert in the question of the movement of sonorous strings."][6]

Although Fourier's new results dealt, above all, with the *Théorie de la propagation de la chaleur dans les solides [Theory of the propagation of heat in solids]*, we must not overlook the explicit, clearly states relationship also with the movement

[4]Quoted in Bailhache 1995a, p. 33.

[5]Itard 1981. Boyer 1990, pp. 544–545 and 566. Kline 1972, pp. 583–586.

[6]Grattan-Guinness 1970, p. 20; Grattan-Guinness & Ravetz 1972, pp. 250–253, 445–446, 450–453, 457 and passim. Ravetz & Grattan-Guinness 1981. Kline 1972, pp. 672–679. Cf. Bottazzini 1981.

of vibrating strings. Representing the resulting curve in the (infinite) series of *sines* and *cosines*, it was at last possible to formulate the succession of the harmonics by means of mathematics. Now the successive whole multiples of the fundamental frequency had become the variable arguments x, $2x$, $3x$, ... of the series

$$a_1 sin.x + a_2 sin.2x + a_3 sin.3x + \ldots a_i \, sin.ix + \ldots$$

More musicians than natural philosophers or mathematicians, the ears of those who had wanted to hear them had undoubtedly long preceded the efforts to justify them through the theory of equations with partial derivatives and functions. The reader will decide for himself to what extent the history of such an important moment of mathematical sciences can be deformed by taking away music from it. Among other things, this explains why the relative field of research was called, in time, harmonic analysis, although it seemed to take its origin from the phenomena of heat.

We have found in Lagrange attitudes similar to those of Euler: the theory of music would be based on ratios between whole numbers, and *not* on audible harmonics; furthermore, the solutions of the equation that governs the movement of the string, that of d' Alembert, could *not* be developed in trigonometric series, but would be more general. How could this same singular coincidence between attitudes in music and in mathematics be found in two different people? Ivor Grattan-Guinness and Jerome Ravetz have not succeeded in explaining satisfactorily the prejudices of Euler and Lagrange towards trigonometric series, because they have overlooked their aversions against the harmonics of sound. The historians writing in English came closer when they realised that the two famous mathematicians reasoned about sound in terms of "pulse propagation". However, Grattan-Guinness and Ravetz failed to underline that these "pulses" were nothing other than the old theory of music based on *ictus* [strokes]. Thus Euler's relative arguments, with his "... inexplicable dislike of trigonometric series ...", along with those of Lagrange, were not "... confused thinking ...", but rather a way of reasoning that was clear and consistent with their theory of music. The event that this did not coincide with the 'truths' known, with hindsight, to our anachronistic historians, remains a problem for the latter.[7]

Some hand-written notes about music by Fourier are extant, which, to the best of my knowledge, nobody has transcribed yet. Entitled "Théorie des e'changes: Dialectique Psychologie", on sheets 48–60, he considered a new symbolism for musical notes, substituting, instead of the syllables of Guido D'Arezzo, non-alphabetical signs invented by him, also to represent their duration.[8]

[7] Truesdell 1960, p. 299 and passim. Ravetz 1961. Grattan-Guinness 1970, pp. 1–21. Grattan-Guinness & Ravetz 1972, passim.

[8] Bibliothèque Nationale Paris, Manuscrits fond français 22501. Grattan-Guinness & Ravetz 1972, p. 496.

Fourier made reference to the *Regole di contrappunto pratico [Rules of practical counterpoint]* by Nicola Sala (1713–1801), published in Naples in 1794, to the renowned *Gradus ad Parnassum* by Johann Joseph Fux (1660–1741), in an Italian translation, and to the *Storia della musica [History of Music]* by Padre Martini (1706–1784). He also showed interest in the *Rivoluzioni del teatro musicale italiano [Revolutions of the Italian musical theatre]* left to us by Esteban de Arteaga (1747–1799) and in the *Principes de composition de l'école d'Italie [Principles of composition of the Italian School]*, written by Alexandre-Étienne Choron. He also planned to read the *Traité d'Harmonie [Treaty on Harmony]* published by Charles Simon Catel (1773–1830). The short bibliography with the notes for the new notations revealed a work project which was only partly completed in the fair copy of sheets 55r and 55v. On these, our French mathematician wrote the following sentences, among others.

> Lorsque les notes qui se suivent sont reunis dans les chants par les marques qui en designent la valeur, ou les lie en les ecrivant, comme si elles n'etaient qu'un seul caractère ...
> La Mesure se marque par un trait vertical peu étendue |. Les signes de durée s'ecrivent toujours pres de la base des notes. ...
> Lorsque l'intervalle formé par deux notes qui se suivent est plus grand que quarte on ecrit au sommet de la note le signe +. Si l'intervalle est descendant, on met au sommet de la note le signe ^{o}.
> La premiere note d'une mesure porte un Chiffre qui indique la region de l'instrument à la quelle elle appartiens.
> Lorsque dans plusieurs mesures consecutives le Chiffre de la premiere note ne change pas, on omet de l'ecrire.
> Les petites notes s'ecrivent en plus petit caractère et tout barrées ... on met le signe de durée de ces petites notes sur la premiere.
> Les coulés ou piqués s'indiquent par les traits ou points d'usages mis au sommet des notes du chant ...
> Les accords s'ecrivent entre parenthese. On met le signe de durée sur la note du chant qui s'ecrit la premiere.
> Les accords s'ecrivent aussi en placant les notes les unes sur les autres, la durée de l'accord se marque sur la note placée le plus près de la ligne ou on lit *ut mi sol ut*.
> La note d'accompagnement qui dure une partie de la mesure s'ecrit au dessous de la premiere à la quelle elle se joint et cette note d'accompagnement porte le signe de sa durée. Les notes qui composent un accord peuvent avoir les signes d'intervalle descendant ou montants. Pour placer ces signes on part de la note du chant qui est ecrite la premiere. Les notes du chant qui suivent un accord ne se supportent pas aux notes d'accompagnement, mais seulement à la note qui porte l'accord.
> [When the notes that follow each other are united in songs, with the signs that indicate the value, then they are joined together, writing them as if they were a single character ... The bar is indicated with a vertical line which is not very extended |. The duration signs are always written next to the base of the notes. ... When the interval formed by two successive notes is greater than a fourth, the sign + is written at the top of the note. If the interval is descending, the sign ^{o} is placed at the top of the note. The first note of a bar bears a figure which indicates the region of the instrument to which it belongs. When the figure of the first note does not change for several consecutive bars, it is no longer written. The small notes are written in smaller characters, all barred ... the duration sign of these small notes is placed over the first one. The ligatures or the dotted notes are indicated with the marks or the dots in use, placed at the top of the notes of songs. ... The chords are written in brackets. The duration sign is placed over the note of the song that is written first. The chords are written

also by putting the notes one on top of another, the duration of the chord is indicated on the note placed closest to the line where they are read *do mi sol do*. The accompaniment note which lasts for a part of the bar is written under the first note to which it is united, and this accompaniment note bears the sign of its duration. The notes which compose a chord may have the signs of descending or rising intervals. To place these signs, you start from the note of the song written first. The notes of the song that follow a chord are not based on the accompaniment notes, but only on the note bearing the chord.]

Overturning Lagrange's idea, Fourier thought that "the profound study of nature is the most fertile source of mathematical discoveries."[9] In view of his initial uncertainty in the choice between a military career and an ecclesiastical one, for his friendship with Napoleone Bonaparte (1769–1821), for his ability to survive amid the Revolution, the Empire and the Restoration of the monarchy, and his resistance to the obstacles created by his colleagues, this French mathematician-cum-physicist might be a character in a novel like *The Red and the Black* by Stèndhal (Henri Beyle, 1783–1842).

With the important exception of the chemist Antoine-Laurent de Lavoisier (1743–1794), who was condemned to be executed, and a few others, physicists, mathematicians and scientists were generally not maltreated, either by the French Revolution, or by Napoleone Bonaparte, or by the following kings of the Restoration. They changed the institutions where these figures worked, bringing new life. Amid these particularly turbulent, uncertain, difficult situations, in short, Michel Serres has written that the scientific community came into power. The Goddess Reason substituted current religions. The myth of progress was born. Religious truths became relative to cultures, peoples and confessions; scientific truths, transcendent as well, on the contrary, would be universal. Moral, juridical or political laws appeared to be subject to contingent historical changes; scientific laws, instead, would be absolute and invariable. The French Revolution brought movement to what had previously been considered eternal, but by contrast, it made the sciences eternal and unshakable. The roles were changed.

The classic European dualism between being and non-being was enriched with new cases: like the one between the reversible time of mechanics (which had now become "rational" and "analytical" with Euler and Lagrange) and the irreversible time, which flows only in one direction, of the new thermodynamics: the science of heat, of fire and of steam engines. Auguste Comte (1798–1857) founded his positivist philosophy on two components: the systematic classification of knowledge and the historical law of progress.

"... le christianisme caractérise, à ce jours, la seule culture où se développe totalement la totalité des sciences." ["... Christianity characterises, these days, the only culture where the totality of sciences is fully developed."] The Christian mathematician Pascal had in his turn assured "... la passe principale, car il pense ensemble ce temps du salut et le progrès continuel dans les sciences." ["... the main passage, because he thinks simultaneously of this time of salvation and of

[9]Ravetz & Grattan-Guinness 1981, p. 97.

continuous progress in the sciences."] Marie-Jean Nicolas Caritat de Condorcet (1743–1794) prepared an edition of it in 1774, and in the end, before committing suicide in order to avoid the guillotine, left, as his testament, his *Esquisse d'un tableau historique des progrès de l'esprit humain [Sketch of a historical picture of the progress made by the human spirit]*. The Holy Spirit had become the spirit of man; faith in progress towards salvation, whether heavenly or earthly, remained. In order to make it come true, like the old clerics, the new ones, now called scientists and professors, must condemn the heretics. Instead of the saints contemplated for every day, Comte put in his "Calendrier Positiviste" ["Positivist Calendar"] famous historical characters, with scientists at the end of the list. In the 4th week of the 10th month, he included some musicians, from Pergolesi to Rossini, from Mozart to Beethoven, from Lully to Händel, but Rameau and Bach were missing.

> Aucune différence, fonctionnelle ou structurale, ne sépare donc la foi en un dieu transcendante et la croyance qu'il existe un objet scientifique indépendant de nous et pouvant s'exprimer dans et par une vérité universelle pour tous et objective en soi. La transcendance comme telle reste la même dans les deux cas. Et la conséquence sociale demeure stable. [No difference, whether functional or structural, thus separates the faith in a transcendent God and the belief in the existence of a scientific object, independent from us, which can be expressed by means of a truth that is universal for everybody and objective in itself. Transcendence as such remains the same in both cases. And the social consequence remains stable.][10]

From now on, the value, and above all the utility of scientists was to be appreciated, forgiving their political opinions in general, when they were not perfectly in line, with the necessary exceptions. Euler wrote in French with a wholly personal style, whereas Lagrange spoke it with a characteristic Italian accent, the provincial speech of Fourier seemed far from that of Paris, but in spite of all these differences, their faith in the ideal of universal sciences was not in discussion. And yet we have seen that the contrasts between them were neither small nor irrelevant, because they were pertinent to our history. Rather, they were strengthened by the prestige assigned to them by the new context, because capable of unleashing struggles for a power which in certain cases coincided with political power. Famous scientists like Fourier, Lazare Carnot (1753–1823), Gaspar Monge (1746–1818), Pierre-François Arago (1786–1853) or Pierre-Simon de Laplace (1749–1827) became prefects, ministers, presidents and senators.

In connection with religious and scientific transcendence, what happened to Maria Gaetana Agnesi (1718–1799) sounds particularly ironic nowadays. She was a less than rare exception in a scientific community, which was exclusively male (but we shall soon find another one). In the first part of her life, she dedicated herself to mathematical sciences with great success, publishing a book aiming to educate, *Istituzioni analitiche ad uso della gioventù italiana [Analytical institutions for the use of Italian youth]*. In spite of its other merits, as a result of a curious historical case, the most famous part of the book, which was also translated into English,

[10]Serres 1989, pp. 337–361 and passim.

became the study of a bell-shaped curve, called at the time a "versiera". Its prevalent religious inclinations then led the Catholic Italian scholar to give up mathematics and dedicate herself completely to the Church and to works of charity, even though a brilliant academic career lay open before her. But, despite this form of transcendence which she preferred, the curve associated with her name was translated into English as "the witch of Agnesi".[11] It was as if we were still in the previous century, when the scientific interests of a woman seemed to be only diabolical, rather than divine. In the Italian of that period, "versiera" meant "the adversary", that is to say, a "devil of the female sex". What could the devout Maria Gaetana have been the 'adversary' of? Of the treatises on analysis and mechanics written by Euler or Lagrange? Of the prejudices against the female sex? There is no doubt that a sister of hers, who dedicated herself to music and played the harpsichord, was considered more normal.

Moving away from professional musicians, the art of sounds was now cultivated as acoustics by various physicists and mathematicians. The German, from a Hungary stem, Ernst Florentz Friedrich Chladni (1756–1827) caused surfaces to vibrate, showing the formation on them of nodes and antinodes in sand. In the immobile nodes, the sand accumulates, from the vibrating antinodes it goes away. He also constructed curious variants of the harpsichord. The figures that were formed in Chladni's experiments were circles, stars and other symmetrical curves. The sounds generated by the vibrating sheets appeared to be analogous to the harmonics of the strings. The words *Akustik* and *Acoustique* stood out in the titles of his books. Many of the most famous mathematicians and physicists of the age tried to explain the formation of the figures, calculating the oscillating movements of the sheets. His demonstrations of the phenomenon were famous. Even Napoleone Bonaparte was present at one of them.[12]

Among the earliest mathematical models proposed, we recall those of Sophie Germain (1776–1831). However, perhaps in order not to disturb young scholars, and not distract them from their prevalent military interests, the fair sex, albeit scholarly, was not admitted to the austere École politechnique. Therefore Parisian Sophie had to invent the male pseudonym, Mr. Le Blanc, in order to get Lagrange to read her work.[13] In the ardour of renewing everything, the French Revolution had changed the members of many scientific institutions, changing their names, as well: thus the Académie des sciences had become the Institut, but the route towards transcendent (and useful) truths remained the prerogative of the male sex.

The intertwining of the new Parisian scientific institutions with military affairs could be observed in the family of Félix Savart (1791–1841), which included fathers and brothers who had become military engineers. The French physicist studied the vibrations of the violin, using the techniques of Chladni, with the figures in the sand. He ended up by teaching the physics of acoustics at the Collège de France in Paris. And yet his greatest fame was to derive from the laws on the magnetism generated

[11] Agnesi 1748, tome I, pp. 380–382, Fig. 135. Kramer 1981.

[12] Dostrovsky 1981c. Resnick & Halliday 1961, p. 435.

[13] Kramer 1981a.

by electric currents, invented in collaboration with Jean-Baptiste Biot (1774–1862.) His model of a trapezoidal violin was submitted to the judgement of the composer, Luigi Cherubini (1760–1842), who at that time was living in Paris. He succeeded in making visible the formation of nodes and antinodes, also in the columns of air contained in pipes. From the diversities in acoustic behaviour, he deduced properties of the materials used for vibrating bodies.[14]

Subsequently famous for the term "Wronskian", the determinant associated with the partial derivatives of a function, to calculate its development in a series, Józef Maria Hoëné-Wrońsky (1776–1853) also dealt with music in his writings of general philosophy. He began them in the wake of Emmanuel Kant (1724–1804), in order to arrive at the esoteric mysticism of the 'absolute'. His musical idea seems to re-exhume the ancient eternal Pythagoreanism, but basing it, like Rameau, on the harmonics of sound by means of the "harmonic" series of reciprocal whole numbers. However, while the French composer had in the end accepted the equable temperament in order to make music, the Polish philosopher and mathematician with French citizenship, on the contrary, rejected it. Hoëné-Wrońsky sought universal laws of the beautiful, and thus, for him, the pleasure of music did not depend on the differences between peoples in scales or in systems. He calculated a division of the octave into 12 rational parts, with intervals of $\frac{16}{17}$ and $\frac{17}{18}$. These come close to $\sqrt[12]{\frac{1}{2}}$ and thus, as Vincenzio Galilei had proposed, to the equable temperament. Through the binary numbers of Leibniz, the Polish mathematician even arrived as far as the China of the *Yijing*. From the German philosopher and mathematician, he also borrowed the pre-established harmony as well as the principle of sufficient reason.[15]

Among those who had dedicated themselves to acoustics, we recall also Garpar Riche de Proni (1775–1839), who calculated a division of the octave by means of base 2 logarithms.

12.2 Too Much Noise, from Harmony to Harmonics: Bernhard Riemann and Hermann von Helmholtz

Whether they had been aroused or not by the French Revolution, the Napoleonic Wars, or the manufactures transformed into industries thanks to steam power, heat and fire or fires now dominated Europe with their noises. It became increasingly difficult to listen to their harmony. Kepler, Mersenne or Rameau had thought they could reveal the essence of the world through the sounds of music. On Earth, now, there were other thoughts in mind. After losing all universal claims, the relative phenomena were relegated to the particular places of acoustics. Via Fourier

[14]Dostrovsky 1981b.

[15]Dobrzycki 1981, supplements. Orcalli 1996, pp. 128–136 and 143–144.

and through the work of Bernhard Riemann (1826–1866), the music of vibrating strings was to continue, all the same, to make its notes heard, albeit softly, on the mathematical sciences of the nineteenth century.

Besides the inevitable simplifications and some characteristic personal additions, the historical synthesis written by the German mathematician for the controversy about vibrating strings was among the earliest and the best. If nothing else, he recalled that for Daniel Bernoulli, the variables of the trigonometric series were the frequencies of the upper harmonics. "Er erklärte hieraus die physikalische Thatsache, dass eine Saite ausser ihrem Grundtone auch den Grundton einer $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ so langen (übrigens ebenso beschaffenen) Saite geben könne, und hielt seine particuläre Lösung für allgemein, d. h. er glaubte, die Schwingung der Saite würde stets, wenn die ganze Zahl n der Höhe des Tons gemäss bestimmt würde, wenigstens sehr nahe durch die Gleichung ausgedrückt. Die BeoBachtung, das eine Saite ihre verschiedenen Töne gleichzeitig geben könne, führte nun *Bernoulli* zu der Bemerkung, ...". ["[Bernoulli] thus explained the physical fact that besides the fundamental sound that it produces, a string can also give the fundamental sound of a string whose length is equal to $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ of its own length (and whose constitution is otherwise equal); he considered his particular solution to be a general solution, that is to say, he believed that, if the whole number n was determined in accordance with the pitch of the sound, the vibration of the string was always represented, at least very nearly, by this equation. The observation that a string was capable of giving its different sounds simultaneously now led *Bernoulli* to notice ..."].[16]

Although the community of German mathematicians was already steering towards pure and abstract sectors, Riemann also cultivated interests in mathematical physics. Here we shall follow those regarding sound, in order to examine whether any trace of music remained in them. In a note on the propagation of plane waves in the air, we read the following words: "In akustischer Beziehung liefert demnach diese Untersuchung das Resultat, dass in den Fällen, wo die Druckverschiedenheiten nicht als unedlich klein betrachtet werden können, eine Aenderung der Form der Schallwellen, also des Klanges, während der Fortpflanzung eintritt. Eine Prüfung dieses Resultats durch Versuche scheint aber trotz der Fortschritte, welche in der Analyse der Klanges in neuester Zeit durch *Helmholtz* u. A. gemacht worden sind, sehr schwer zu sein; ...". ["As regards acoustics, the present research thus notifies the result that, when the differences in pressure cannot be considered infinitely small, a change is generated in the shape of the sound wave, and consequently in the sound during its propagation. An experimental verification of the result seems to be very difficult, in spite of the progress that has recently been made, by *Helmholtz* [Hermann von, 1821–1894] and others, in the analysis of sound; ..."].[17]

Beyond the range of Helmholtz, the shock waves predicted by the German mathematician were later found and heard all too clearly. More recently, René Thom

[16] Riemann 1990, p. 262; French transl. Riemann 1968, p. 229.

[17] Riemann 1990, p. 210; French transl. Riemann 1968, p. 206.

(1923–2002) reviewed them in his "cusp catastrophe".[18] For the case of appreciable differences in pressure, Helmholtz had already tried to improve the experimental results in acoustics. Here, Riemann made their mathematical theory by means of a non-linear equation with partial derivatives. The renowned mathematician and the equally famous physicist and physiologist were to confront some more times.

In his *Mechanik des Ohres [Mechanics of the ear]*, Riemann tried to explain why the sense of hearing was so precise. This manuscript remained among his last papers and was published posthumously; as few or no historians have examined the article,[19] and perhaps none of those who deal with mathematical sciences *natürlich*, I will give a brief summary of it here. At the beginning, the German mathematician dealt with a question of method. Is it better to start from the anatomy of the ear to study the causes that explain its functioning and performance? A synthetic approach. Or maybe it would be preferable vice versa to start from the results achieved by the ear, and work back to the conditions of its functioning? A different, analytical approach. Granted that the distinction in the method could not be absolute, given the analytical and synthetic presuppositions of the one and the other method of proceeding, Riemann took the second road. In this, he was influenced by the philosopher Johannes Friedrich Herbart (1776–1841), the inventor of experimental psychology.

In actual fact, the analytical procedure would work better in order to study refined organs such as the ear. Thus he searched for the physical properties of the parts composing the organ of hearing, which were able to justify its performance, counting that in this way he could advance even in the knowledge of more general natural laws. "... indem man ... 2) die Anwendung von Analogien (das 'Dichten von Hypothesen') sich zwar nicht, wie Newton will, gänzlich versagt, aber hinterher die Bedingungen, die zur Erklärung der Leistungen des Organs erfüllt sein müssen, heraushebt, ..."[20] ["... 2) not completely refusing, of course, to use analogies (that "composing from hypotheses" [Hypotheses non fingo]), as is preferred by Newton,[21] but underlining, subsequently, the conditions that must be satisfied in order to explain the performance of the organ, ..."].

Having succeeding in overcoming some suspicions as regards finalistic reasonings, our German from Hannover-Göttingen described experiments which illustrated the extreme sensitivity displayed by the ear in perceiving sounds. He was above all interested, not so much in sound in itself, but in the kind that was independent of direction and intensity, which he called "Klang" ["a resounding sound"]. By this term, then, he indicated the spectrum of frequencies that compose a sound. And he then studied by what means it might become possible to transform a sound wave into the refined perception of this "Klang", maintaining intact its

[18] Thom 1980, pp. 71–72. Tonietti 2002a, Chaps. II and III.

[19] Riemann 1990, 370–382.

[20] Riemann 1990, pp. 372–373.

[21] See above, Sect. 12.2.

harmonic characteristics, and also having to amplify the force of the sound dispersed in space, when it derived from a long way away.

With all scrupulousness, Riemann analysed the mechanics of the auditory system. He wanted to do this "... vorher jedoch für die Beschaffenheit der Druckänderung, von welcher der Klang abhängt, einen mathematischen Ausdruck suchen."[22] ["... however, before searching for a mathematical expression to describe the state of changes in pressure, on which the *Klang* depends."] That is to say, in the end he would try to determine what he called the "Klangcurve" ["harmonic curve"]. Here we can again see how much attention he dedicated to the effects caused by oscillating the wave of pressure on the chain formed by the tympanic membrane, by the malleus, the incus, and the stapes, arriving at the liquid in the cochlea. All the contacts between the various parts must be perfect, otherwise we would not have obtained such refined sensations as we experience. For this reason, he also analysed the effects of changes in temperature on the auditory system. The sensitivity of the ear was such as to perceive movements and forces millions of times smaller, he calculated, than the usual ones, not visible even under a microscope.

Riemann rejected the analogy with light, for which we may recall that of Newton,[23] because he did not understand how its effects "... für das genaue Auffassen eines Musikstücks nützen sollte." ["... could serve to understand a piece of music with precision."] Sure enough, his mechanical model intended to serve to understand the conscious hearing of a listener, and therefore he made a distinction "... zwischen dem lauschenden, zum genauen Hören adaptirten Ohr und dem nicht lauschenden Ohr, ... Es bleibt dann unsere Untersuchung für das lauschende, zur genauen Hören absichtlich vorgerichtete Ohr giltig, ..." ["... between the ear that listens, which is suitable for hearing with precision, and the ear that does not listen, ... Then our research on those who listen remains valid for an ear deliberately intent on hearing exactly, ..."].[24] He also thought of the ears of savages and of those who live in the desert, which are virgin with respect to the countless noises of hard-working European cities.

It was his German colleague encountered above who had inspired the first version of his work. "In dem Buche *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* von *Helmholtz*, findet man die Fortschritte zusammengestellt, welche in der so äusserst schwierigen Ermittelung der Thatsachen, die die Wahrnehmung der Töne betreffen, in neuerster Zeit gemacht worden sind und zwar vorzüglich von *Helmholtz* selbst. Da ich den Folgerungen, welche *Helmholtz* aus den Versuchen und BeoBachtungen zieht, entgegen zu treten vielfach genöthigt bin, so glaube ich um so mehr gleich hier aussprechen zu müssen, wie sehr ich die grossen Verdienste seiner Arbeiten über unsern Gegenstand anerkenne. Sie sind aber meiner Ansicht nach nicht in seinen Theorien von den

[22] Riemann 1990, p. 374.

[23] See above, Sect. 10.2.

[24] Riemann 1990, pp. 376 and 380.

Bewegungen des Ohr zu suchen, sondern in der Verbesserung der erfahrungsmässigen Grundlage für die Theorie dieser Bewegungen."[25] ["In the book *The Doctrine of Sonorous Sensations As the Physiological Foundation For the Theory of Music* by *Helmholtz*, we find, united together, the progress that has recently been made in establishing the extremely difficult facts regarding the perception of musical notes, and undoubtedly, the progress made in particular by *Helmholtz* himself. As I am repeatedly forced to dispute the consequences that *Helmholtz* deduces from his research and observations, so much the more do I believe that I should express here that I recognize the great merits of his works in our field. According to me, however, they are not to be sought in his theories of the movements of the ear, but in the improvement of the experimental foundation for the theory of such movements."]

As Helmholtz published the book in 1862, Riemann's manuscript can only have been written a few years later. Unfortunately, Riemann died precociously of tuberculosis in 1866, at Selasca, despite the warm Italian sunshine of Lake Maggiore.[26] In the commemorative speech pronounced on December 1st, 1866 at Göttingen by Ernst Schering (1833–1897), we read that the dying man gave his attention to the present "Mechanics of the ear" in the last few days of his short life.[27] It was written out in fair copy, and only the last few pages were recovered on scattered sheets, when the text had been edited by the anatomist-physiologist quoted in the article. It had not been abandoned, therefore, out of lack of interest, or due to technical difficulties that had arisen, but it was simply interrupted by the author's death.

It will never be possible to know how Riemann would have terminated the fragment. Creativeness cannot be prolonged like an analytical function, because it has too many singularities. However, the indications present in the manuscript indicate to us the direction in which he was moving. What equation could have produced the "Klangcurve"? By means of which equations involving partial derivatives would he have calculated the oscillations in the air pressure that bears on the tympanum, and their propagation in the membrane, the small solid bones and the liquid in the cochlea, arriving at the acoustic nerve? From his model of the great sensitivity of the ear to the frequencies of the "Klang", would he have gone on to propose a theory of music of his own, which would explain, in his way, the consonances and dissonances, unlike Helmholtz, as mentioned above?

Surely, in the mechanical model which gave so much importance to the ear, not the slightest trace is found of the numerical considerations which were still largely present, both in Newton and in Euler, when they had written about music.[28] After all, Riemann's *Naturphilosophie* sounded different from that of his two illustrious predecessors. Considering that on the contrary, the *Doctrine ... for the Theory of Music* by Helmholtz was brilliantly terminated, by changing a few signs (I am only

[25] Riemann 1990, p. 373.

[26] Riemann 1990, pp. 589–590. Freudenthal 1981.

[27] Riemann 1990, pp. 843 and 589.

[28] See above, Sects. 10.2 and 11.1.

joking!) we might obtain something else about Riemann. We shall come back to this after examining it below. As far as I know, this seems to be the only work left by our German mathematician that was *not* followed by innumerable research developments. Another confirmation that the scientific community was abandoning music?

The *Naturphilosophie [Philosophy of nature]* sustained by our prematurely deceased character, while maybe not so empirical as that of Helmholtz, maintained a solid link with the world of phenomena. From their University of Göttingen, this was already underlined by Felix Klein (1849–1925) and subsequently others. To give a single example, I will recall the beginning of the article "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen" ["On hypotheses that serve as a foundation for geometry"], where geometry was derived from "empirischer Gewissheit" ["empirical certainty"] and not from axioms. In the end, Riemann went so far as to imagine that he could modify the formulation of Newton in order to explain new phenomena. After all, his physics was dominated by the idea of the field (of gravity, electromagnetic, ether), rejecting any action at a distance. The prophecy was to come true 60 years later, with Albert Einstein and the theory of General relativity.[29]

Riemann invented a theory for the functions of a complex variable, subjecting them to certain conditions which entailed functions defined as "harmonic". Lord Kelvin of Largs (William Thomson, 1824–1907) had some reason for defining as "harmonic" these functions, which entered into many problems of mathematical physics, such as electrodynamics and the dynamics of fluids. On the one hand, they satisfied a particular case (independent from time) of Fourier's equation for heat (known as that of Laplace), and on the other the best-known example was the (potential) function $\frac{1}{r}$, which recalled the reciprocals of whole numbers contained in the harmonic series.[30]

As we saw above, the reference of Riemann to Helmholtz meant that the mathematician knew in what great depth the latter was working on acoustics. He gave his main treaty the title *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik [The Doctrine of Sonorous Sensations As the Physiological Foundation For the Theory of Music]*.[31]

Once acoustics had been established within the field of physics, Helmholtz wondered what relationship it should have with music. To answer the question, it was necessary to know: "What can musical chords have in common with the ratios of the first six whole numbers?" He was not satisfied by the answers given by those who thought that the soul was unconscious (without quoting Leibniz), but capable of appreciating their ratios, and finding enjoyment in them, because they were simple (he mentioned Euler further on). With his feet firmly planted in research on acoustics, he wanted, on the contrary, to think of psychology and physiology. He

[29] Riemann 1990, pp. 304–305, 318, 539–570 and passim. Riemann 1968, pp. xiii–xxxv, 281 and 297. Freudenthal 1981. Bottazzini 1981, pp. 169–213. Tonietti 1992b and 1994.

[30] Persico 1960, Chaps. II and III. Kline 1972, p. 685.

[31] We can read this in the French edition, approved by the author. Helmholtz 1868.

also made reference to Eduard Hanslick (1825–1904), but he found only "vague, uncertain" answers in aesthetics. As hearing is a sensation, it was necessary to look for it within the field of physiological acoustics. Thus he investigated the relative laws. In this way, he was to discover "the physiological basis of the mysterious law of numerical ratios found by Pythagoras." He confirmed those rules of harmony on which "the best composers" were in agreement. His desire would be to construct the modern scales and tonalities, excluding all arbitrary elements. He did not want to separate the physiological study from the musical consequences. For physicists, he demonstrated his affirmations separately in an appendix, by means of mathematics.[32]

Helmholtz performed experiments on sound, with discs for sirens perforated at 12 and 8 points. These thus already offered at the start discrete pitch of the phenomenon, such as 12:8, that is to say, the fifth, 3:2, of the Pythagorean tradition. Making a disc with 12 holes revolve together with another one with 16, he obviously obtained 4:3, that is to say, the fourth. The advantage of the sirens with discs consisted in being able to measure with precision the frequency of the notes produced, which depended on a closely-controlled speed of rotation. Thus the German physicist believed that it was possible to confirm the law that ". . . consonant intervals lie in ratios between the smallest whole numbers", up to 6.[33] In the historical genealogy, he limited himself to Pythagoras, Galileo Galilei, Newton, Euler and Daniel Bernoulli, for their (presumable?) studies on strings. Thus he obtained the scale of Zarlino. The octaves were distinguished by means of organ pipes with lengths that were multiples of 2; as regards their diameter, he remained silent. The central *la* was fixed at 440 vibrations a second, like today, in accordance with a German convention that was different from the French one.

As Riemann, too, was subsequently to write, Helmholtz noted the greater sensibility of the ear with respect to the eye. With great experimental ability, he succeeded in giving the idea of the shape that the vibrations of bodies assumed with the passing of time. From the prongs of a modern diapason, suitable for tuning the voices of a choir, he obtained an elegant sinusoidal curve (Fig. 12.1). In this way, the *sinx* functions, which had been variously discussed by Taylor, Daniel Bernoulli and Fourier, down to Bernhard Riemann, became really visible. Different musical instruments traced different curves: the timbres were therefore different.

Georg Simon Ohm (1789–1854) had already maintained that only in the case of sound with a sinusoidal form could the single vibration of the fundamental note be heard. The relative movement was like the oscillation of a pendulum. Here, we may observe that while Galileo Galilei had measured the pendulum with music two centuries earlier,[34] now, German physicists and mathematicians measured music with the pendulum. But in general, the sounds of musical instruments were not purely sinusoidal, and thus besides the more intense basic sound, other sounds of

[32]Helmholtz 1868, pp. 2, 6–7 and 21.

[33]Helmholtz 1868, p. 20.

[34]See above, Sect. 8.2.

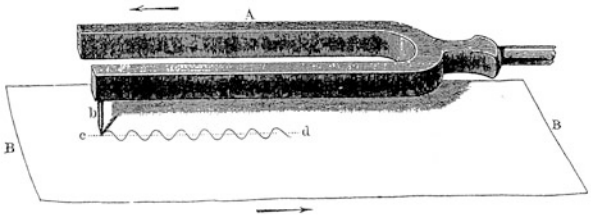

Fig. 12.1 How Helmholtz generated the sinusoidal shape of vibrations (Helmholtz, "Diapason", 1868, Fig. 5, p. 27)

growing pitch could be heard with them, i.e. the famous upper, or partial, harmonics, with a decreasing intensity. The timbre of sounds was thus born from the mixture of harmonics. Helmholtz often compared waves of sound in air with those in water, to explain the former better on the base of the latter. In this case, the eye seemed to go beyond the ear in embracing with a single glance the whole field, where waves in the sea intertwined with one another interfering, while the latter was capable of perceiving only the single point of contact of the tympanum with air in movement. Nevertheless, the ear was capable of distinguishing sounds from one another even if they were confused and mixed.[35]

At this point of the book, Fourier reappeared to demonstrate, with the most rigorous mathematics, that every periodic movement could be analysed in simple components like the pendulum. Although, the ear had been capable of decomposing sound elegantly, distinguishing the harmonics, Helmholtz sought mechanical evidence in the air "... *independent from the ear* ...". We may recall that on the contrary, Riemann started precisely from this organ. Playing with membranes of the kind introduced by Chladni, the German physicist brought out simple harmonic vibrations for their resonance. "... the theoretical considerations, with which mathematicians have been led, first of all, to decompose the compound vibrations, have a real foundation in the nature of things."[36]

By that time, Helmholtz had forgotten that those phenomena had long been part of the real experience of musicians, and was obliged to reconstruct mechanical models of them with various membranes and cavities. He did not trust (his own?) ears, and was forced to use artificial ears, which he called "resonators", but which, though amplifying the sound, had in the end to arrive always at the tympanum of the auditory organ. Though many materials could resound, strings anyway gave harmonics ordered according to the multiples of the fundamental sound, as we know, which not all materials did. All in all, as white light is decomposed into the colours of the rainbow produced by vibrations in the ether, so vibrations in the air are decomposed into harmonics. The German physicist had recovered an analogy dear to Newton.

[35]Helmholtz 1868, pp. 25, 27 and passim.

[36]Helmholtz 1868, pp. 44–45, 47, 58.

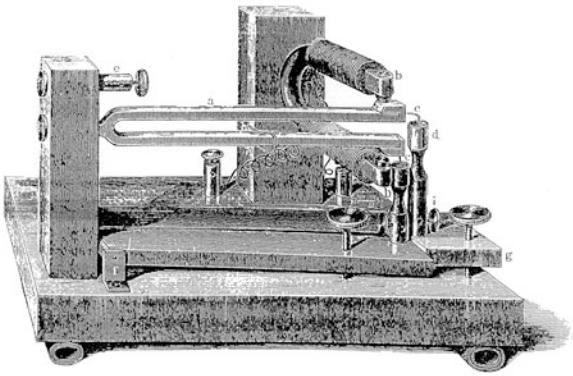

Fig. 12.2 Electromechanical diapason used by Helmholtz ("Diapason", 1868, Fig. 33, p. 155)

And yet Helmholtz gave at the same time details about how to carry out precise harmonic analyses by ear by oneself. With his "resonators", lastly, the German physicist confirmed what had been maintained by Ohm, that the ear was capable of perceiving simple "pendular" sounds (partial, sinusoidal) and decomposing the complex ones into these. Vice versa, by combining together almost sinusoidal sounds, like those produced by closed organ pipes, different timbres could be reconstructed. Thus harmonics were not a curiosity, found by chance by musicians, perhaps to justify the perfect chord, but they really constituted various possible sounds.[37]

The French physicist Jules Lissajous (1822–1880) had invented a "vibration microscope" in order to 'see' the movement of oscillating bodies, even if very rapid. His figures were to be called "figures of Lissajous".[38] By means of these, and "resonators", Helmholtz was able to analyse in detail the harmonics of many musical instruments: strings, wind instruments and the human voice. All things considered, sight was considered more reliable than hearing in scientific matters; or at least it could be controlled to a greater degree. He performed experiments on harmonics, using electromechanical diapasons, for which it was easier to measure the frequency, intensity and phase (Fig. 12.2). In this way, he succeeded in producing artificial harmonics, which he recombined in the desired ratios, to obtain sounds with the characteristics that he sought.

Our German acoustics scholar demonstrated that whereas the eye distinguished different wave forms, produced by changing the phases of harmonics, the ear, on the contrary, perceived the same timbre. Thanks to his studies of medicine and physiology, Helmholtz described how the ear functioned, without any need to have recourse to other scholars of the branch, as Riemann would have done,

[37] Helmholtz 1868, pp. 73–74, 78 and passim.

[38] Dostrovsky 1981d.

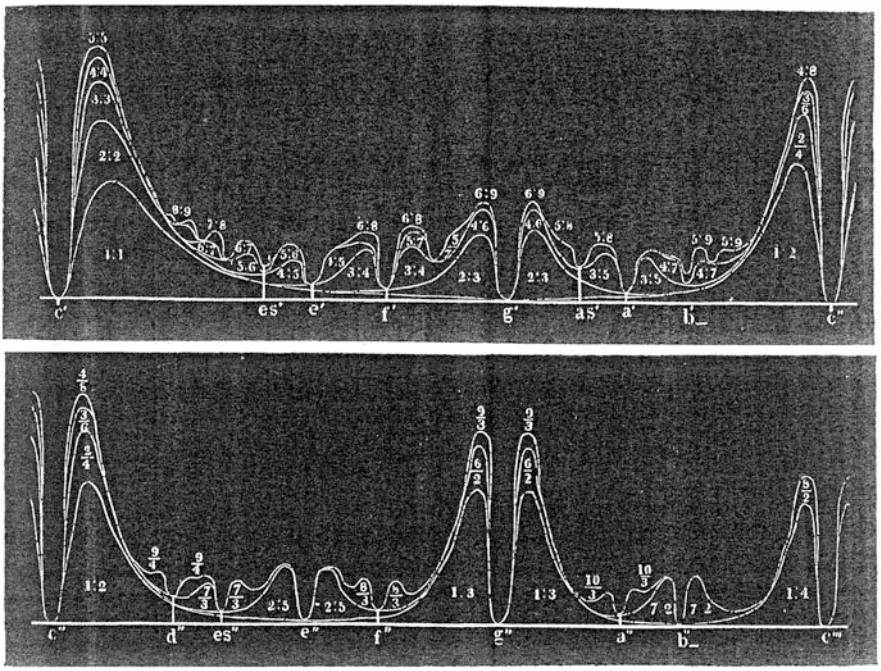

Fig. 12.3 The curves of the beats with which Helmholtz tried to explain dissonances (Helmholtz, "Diapason", 1868, Fig. 52B, p. 247)

on the contrary. By adding and subtracting the sinusoidal waves algebraically, he explained the sounds of Tartini, who, according to him, had been preceded by Georg Sorge (1703–1778), and above all the beats between sounds which were almost equal in frequency. Everything was amplified and made visible by amazing new experimental means.[39]

Basing himself firmly on the beats of harmonics, that is to say, on the periodic variation in intensity of two sounds that are almost equal in frequency, our German physiologist-cum-physicist finally offered his theory for consonances in music. Continuing to call the ratios for intervals of the Ptolemaic and Zarlinian tradition "natural", he calculated a classification of consonances in the ascending order of beats, from unison, the octave, the fifth, the fourth, the major sixth, to the major and minor thirds. The harshness of dissonances was represented by the peaks in a diagram (Fig. 12.3). He concluded proudly: "Almost all theoreticians have admitted, for consonances, series which, in their main characteristics, agreed with one another and with the theory of beats deduced by us." And yet he understood that the

[39]Helmholtz 1868, pp. 154–155, 159–163, 166–188 and passim.

classification could change, if the intervals were not taken separately one by one but linked together in a piece of music.[40]

"A *consonance* is presented when several sounds emitted simultaneously do not exercise this disturbing action [of beats] on one another. From the moment when the numerical ratios of consonances are not observed exactly, beats are generated; that is to say, the same complex sounds or their harmonics or their resulting sounds are reinforced or weakened with one another alternately. These different sounds, instead of co-existing undamaged in the ear, harm one another reciprocally. We call this phenomenon *dissonance*." Helmholtz classified the timbres in accordance with what could best distinguish the consonances from the dissonances. With closed organ pipes and flutes almost devoid of harmonics, "... the harmony loses all its character and ... the listener loses the ability to hear a clear distinction of intervals." While the flute had once been in line with the classical ideal of pure art, now the modern art expressed itself, on the contrary, by means of contrasting effects. In the human voice, the results changed depending on the vowels intoned, because they have different harmonics. Thus, "... it is not indifferent, as regards the musical effect, to sing a piece in one language or in another."[41]

But unfortunately, at least in the French edition, our German did not risk taking up the dispute between Italian and French language which had broken out at the time of d'Alembert and Rousseau.[42] We may also recall that at the time of Helmholtz, European instrumental music had assumed a strongly Germanic character.

Considering how important singing was for modern [tonal] harmony, "... pieces of this kind absolutely need to be sung in accordance with the correct intervals, and, unfortunately, our singers of today rarely arrive at this result, because they are accustomed, from the start, to singing accompanied by instruments tuned in the [equable] temperament, and thus they give inexact consonances. There are only singers who possess a pure musical sensibility, and in this case, find by themselves the true exactness which is no longer taught in schools." The influence of Euler was felt on Helmholtz, when he used the most simple numbers as his criterion. Thus extending an interval by an octave improved the consonance, because this made it more simple, only when the lower number was even, the fifth 2:3 became 2:6, that is 1:3. But the same did not happen when the lower number was odd, because then 3:4 became 3:8, which could not be simplified.[43]

Before they were corrupted by tempered instruments, voices corresponded better to his theory. He found examples in Mozart's *Ave verum corpus* KV. 618 and in the *Stabat mater* by Giovanni Pierluigi da Palestrina (1525/1526–1594). Putting the original Greek at the foot of the page, our German physicist quoted Euclid. "This description of the phenomenon corresponds perfectly to the ancient definition of Euclid: 'Consonance is the association of two sounds, one acute and the other

[40] Helmholtz 1868, pp. 230–239, 247 and passim.

[41] Helmholtz 1868, pp. 261, 263, 265 and passim.

[42] See above, Sect. 11.2.

[43] Helmholtz 1868, pp. 266, 280–281 and passim.

deep. On the contrary, dissonance is the incapacity of two sounds to associate together without producing a harsh impression on the ear'." He admitted that the vibrations of Corti's organ in the ear were only hypotheses, and for this reason, he had based his theory on beats. At one point, when summing up, Helmholtz seemed almost to contradict himself, moving away from the musicians of his time. Here, he now maintained that between consonances and dissonances, there was "... a continuous series of levels, intermediate, ...", whereas musicians made a "... clear separation ...". Then he took up the old dispute about intervals based on the number 7. "The modern musical scale cannot admit within it the sounds determined by the number 7."[44]

Afterwards the physiologist-cum-physicist conceded: "Neither the Greeks nor the great composers of the XVI and XVII centuries were men who would let themselves be enchained by a theory which would contradict their ears." And yet he continued to insist: "As regards the problem posed 2,500 years ago by Pythagoras to the science of causes, concerning the link that unites consonances with the ratios of small whole numbers, its solution is given by the decomposition by the ear of a complex sound into pendular [sinusoidal] vibrations, in accordance with the law of vibration by influence. This was impressed with mathematics by the law of Fourier, who teaches us that every periodic quantity by nature is expressed by a sum [series] of very simple periodic quantities, [*sines* and *cosines*]." "We have seen, effectively, that the condition for having a consonance consists in coinciding two of the deep harmonics in notes produced simultaneously: if this condition is not satisfied, the consonance is destroyed by the beats. Thus the origin of the rational ratios given by Pythagoras is to be found, in the last analysis, in the law of Fourier, which, in a certain sense, may be considered the foundation of harmony."[45]

Helmholtz thus rejected the music of the spheres, but confirmed, with his acoustic model, the ancient Pythagorean ratios in the Ptolemaic-Zarlinian variant. He even quoted a Chinaman, a friend of Confucius, who put his five notes close to the "... five elements [phases] of their natural philosophy (water, fire, wood, metal and earth)." Then he passed on to criticise Athanasius Kircher; even "... such a deeply scientific spirit as Kepler could not free himself completely from this kind of hypothesis; ...". Having thus liquidated the *Harmonices mundi libri quinque* of his famous compatriot,[46] he dwelt at greater leisure on Euler and the *Tentamen*. Resting on psychological considerations, the latter would have gone in the right direction, without, however, completing the course. Helmholtz realised that the hierarchy of Euler coincided with his own, except in the status to be assigned to the major and minor chords. The Swiss scholar, in his opinion, would have not taken into consideration how the resulting sounds influenced [them]." And thus, in his classification, the physicist-cum-physiologist took the liberty of exchanging a

[44] Helmholtz 1868, pp. 291–293 and passim.

[45] Helmholtz 1868, pp. 294–295 and passim.

[46] See above, Sect. 8.3.

major sixth chord with a minor one.[47] Effectively, we know that for music Euler had worked with prime numbers, and not on the physics of harmonics, and beats.[48]

Furthermore, Helmholtz was not convinced about how the Swiss mathematician in exile made music enter into the soul. "The main difficulty in Euler's hypothesis consists in not saying how the soul could arrive at a perception of the numerical ratio of two simultaneous sounds." In other words, let us remember, the great Swiss scholar caused the numerical ratios of intervals to be heard so much the better and with greater pleasure, the more simple they were. When they were made too complex, as they were by the equable temperament, some undefined faculty of ours would simplify them, bringing them back to the closest ratios considered "just". The musician could thus play what he wanted, the ear and the soul would perceive in any case the ratios welcome to Euler. The German physicist wrote here: "... the results of the present study complete what was still missing in Euler's hypothesis." And he underlined the difference between the ability to perceive orderly rational ratios directly, like the Swiss scholar, and investigating "... a physical effect due to these ratios, the continuous or intermittent sensation of auditory nerves."[49] Thus for him, physics substituted psychology.

Through harmonics, Helmholtz took up what had been maintained by Tartini, Rameau and d'Alembert, but he criticised their naturalistic claims. "... when people started to suffer greatly owing to the evils of an artificial social state, it was sufficient to present something as *natural* to demonstrate thereby its goodness and the need for it." All of them had kept silent about beats, and therefore none of them could claim to be complete, as on the contrary, he was, taking advantage of his latest research in physics. After believing that, thanks to acoustics, he had solved the *vexata quaestio* of dissonances in a way he considered "scientific", the German physicist made historical the great mass of remaining musical problems. "... *the system of scales, modes and their harmonic interconnection do not rest on invariable natural laws, but on the contrary, are the consequence of aesthetic principles that vary with the progressive development of humanity* and continue to change."[50]

Starting from Aristotle, Helmholtz made the evolution of music converge in tonality. The Chinese, the Hindus and the Arabs were only mentioned as homophones or complex variants, but he believed that he could isolate one tonic even among them. After the polyphonic period, finally modern harmony arrived. Among its causes, our German scholar included Protestantism. In his opinion, "... the popular melodies of Germanic or Celtic origin betrayed a more accentuated sentiment of the tonality, as we intend it, with respect to the music of the peoples of the South. Also in the sacred Protestant songs of the XVI century, our major scale had already been developed in a very correct way as a harmonic system in

[47] Helmholtz 1868, pp. 295–297 and passim.

[48] See above, Sect. 11.1.

[49] Helmholtz 1868, p. 298 and passim.

[50] Helmholtz 1868, pp. 300, 306 and passim. The criticism of Helmholtz that the others above referred to the "natural" may likewise be extended to himself.

the Ionian mode." In accordance with the decisions taken at the Council of Trent, Giovanni Pierluigi da Palestrina was to simplify polyphony, making it harmonic, not without the influence of the North. Helmholtz quoted his *Stabat mater*, where, in the uncertainty between *A major* and *F major* the tonic *D* emerges. In discussing the birth of the lyric opera at Florence, however, he succeeded in losing the memory of Vincenzio Galilei, until Rameau made a theory of harmony.[51]

Our physicist-cum-physiologist admitted: "... we must not forget that the modern system is the product, not of a blind fatalism, but of a principle freely chosen; we must not forget that, together with it, and before it, other musical systems were generated from other principles, ... it is an aesthetic principle, not subject to natural fatalities." And yet, for him, a progress would have been made with tonality. "If our theory of the modern system is exact, it must be able to explain also the preceding eras, less perfact, in this development." Helmholtz agreed with Hanslick in maintaining that music made the sentiments of the soul vibrate, but without being any more precise. It moved and touched, as Aristotle had already written, because the sounds mimicked the movement. "The musical gamut is, ..., a scale by which we measure the progress in the pitch of sounds, while performing the same task as tempo for the time. [...] As regards the choice of intervals, this is a problem, the solution of which reveals varieties of musical taste, because the number of scales used by different peoples and in different periods is truly considerable."[52]

And yet our scholar of acoustics immediately added that the beats were missing in certain intervals. "Among them, we find some, the octave, the fifth, the fourth, in all the scales known." Modern harmony had anyway developed slowly, however, and did not agree with all the melodies of the different peoples. It could not, as Rameau had imagined, be implied everywhere, and act as an unconscious guide. However, again, the physical and physiological relationships proposed by Helmholtz made him repeat the steps taken by the Frenchman with harmonics. With a certain diffidence, the German had received news about Egyptian scales and Indian quartertones. He compared the Chinese pentatonic scale to that Gaelic one of the Scots and the Irish. Unexpectedly, the division of the octave into 12 semitones, calculated by Prince Zhu Zaiyu surfaced again here.[53] Unfortunately, he took it for granted that everyone must divide the octave, while on the contrary, we know that the theories of the Chinese and the Indians had not considered this. He muddled along to calculate, using fractions, scales divided into five intervals, copying Scottish and Chinese melodies on the stave.[54] The reader will judge by him-/herself how reliable they could be.

Helmholtz recorded the contrast between Aristoxenus and mathematicians, because the former had identified *mi*♯ as *fa*. "But, in practice, this error was quite negligible, and could be ignored without any qualms, above all in homophonous

[51] Helmholtz 1868, pp. 300, 306, 323 and passim.

[52] Helmholtz 1868, pp. 328, 334 and passim.

[53] See above, Sect. 8.1.

[54] Helmholtz 1868, pp. 334, 339 and passim. See Part I, Chaps. 3 and 4.

music." The ratios between the notes of the scale were classified by him by means of the coincidence of harmonics. In this way, the thirds and the sixths, whose fractions were $\frac{5}{4}$ and $\frac{5}{6}$, reappeared, with which he could recreate scale and modes discussed previously. "The numerical values of the direct allies of the tonic are, of course, fixed and unchanging, because they are given directly by the consonant ratios with the tonic, ...".[55]

On this subject, he took the liberty of criticising notes with different ratios introduced by some, including d' Alembert. As had already happened to Rameau, the German was able to re-examine the well-known differences between the scales, inserting and removing *commas* of various kinds, but before encountering the radical alternative to this, he had to pass through the theory of tonality. "... as regards the fineness of sentiment for artistic things, we moderns must consider the Greeks, as models that have never been surpassed. ... they had very special opportunities, through education, to give their ear a greater refinement than ours. From our youth, we have been used to adapting to the falseness of the tempered system, and all the ancient multiplicity of the modes of different expression has been reduced to the distinction between major and minor, which is simple to produce."[56]

The Arabs and Persians attracted the attention of Helmholtz, who interpreted them in accordance with the writings left by Raphael Georg Kiesewetter (1773–1850) in *Die Musik der Araber [The Music of the Arabs]* (1842). The German physicist drew scales from this, adding *commas*, both of the Pythagorean kind and those of Didymus: one octave was divided into 17 intervals. He believed that he could hear preponderantly in them the "natural" intervals obtained by using successions of fifths "judiciously". If Kiesewetter had advanced the hypothesis that Christian missionaries had introduced into Persia the division of the octave into 12 parts, Helmholtz now maintained the opposite, that the reduction of the 17 intervals to 12 had passed to the West together with the lute.[57] In contrast with both hypotheses, we know that Vincenzio Galilei had tempered his own lute by referring to Aristoxenus,[58] and that the Arabs had divided the fourth of the tetrachord in many other ways.[59]

Among the compositions which are believed to sustain the tonality of his period, Helmholtz referred to Händel (1685–1759), Josquin Desprez (1440–1521), Palestrina, Gabrieli (Giovanni 1557–1612), Monteverdi (1567–1643), Johann Sebastian Bach, to the *Requiem*, *Die Zauberflöte [The Magic Flute]* and Mozart's *Don Giovanni*. The major mode would be more important for him than the minor, because the latter "... presents a less perfectly logical construction ..."; anyway, it was appropriate "... for gloomy, restless, mysterious sentiments, to express extraneousness, horror, mystery, mysticism, wild harshness, and everything that

[55]Helmholtz 1868, pp. 356, 365–366 and passim.

[56]Helmholtz 1868, p. 371 and passim.

[57]Helmholtz 1868, pp. 372, 374 and passim.

[58]See Part I, Sect. 6.7.

[59]See Part I, Chap. 5.

is in contrast with artistic beauty." No composers of the nineteenth century were quoted; only Ludwig van Beethoven (1770–1827) deserved a mention for his piano sonata in *E minor* op. 90. His physiological theory of chords rested on the event that "... sounds were already musical, *per sé*, chords of harmonic sounds ... [...] Theoretically it was Rameau, in his *Traité d'Harmonie [Treaty On Harmony]* who first recognized the different value of sounds in a chord, even if he ignored the causes, which were provided by us."[60]

The German acoustic scholar introduced the equable temperament, approximating the exact roots, which he did not write, with the ratios $\frac{3}{2} \times \frac{885}{886}$ for the fifth and $\frac{4}{3} \times \frac{886}{885}$ for the fourth. So far, he had variously annotated *Die Natur der Harmonik und Metrik [The Nature of Harmony and Metrics]* written by Moritz Hauptmann (1792–1868), but here he had to take his distance, at least a little, from another admirer of the Pythagorean system: *Ueber die verschiedenen Bestimmungen der Tonverhältnisse [On the Various Tunings of Ratios for Notes]* by Ernst Naumann (1832–1910). "The main defect of our present tempered system does not lie, therefore, in the fifths; these are only altered, effectively, by an insignificant quantity ... It is rather in the thirds ... [...] The only truly exact system is the one, proposed by Hauptmann, who distinguishes sounds generated by the fifths from those produced by the thirds. As it is important, in a large number of theoretical questions, to be able to make observations about the sounds that form between them intervals proposed by the theory, in order not to be led into error by the imperfection of the temperament, I have attempted to construct an instrument that will make it possible to modulate in all the tonalities, with the natural intervals." For this purpose, therefore, he had a large harmonium adapted, tuning the two keyboards in the "natural" scale, and sometimes taking away the *comma* $\frac{81}{80}$. On this instrument, called "just", Helmholtz compared "natural", "regular" major chords, without beats, with the "tempered" ones, which instead were "contaminated, tremulous, and irregular." As the differences of the former between the major and minor tonalities became more clear-cut, compared with the latter, "... the modulations acquire much more expression than usual, ... the dissonances become more biting ...". The "just" tuning, in his opinion, would bring honour to Palestrina and Gabrieli.[61]

Our German physicist conceded that without the equable temperament, modern instrumental music would not have developed. "But we must not believe that the difference between the tempered scale and the natural one is a mathematical subtlety devoid of any practical value. It has a great effect even on ears that are not very musical ...". After reviewing the milestones of the temperament, under the heading "Inconveniences of the equable temperament", he listed the beats between the relative notes, but he also added that they cannot be easily perceived on quick notes. He was thus forced to admit that there were more advantages than the opposite. It became unpleasant on certain organ timbres. With his experimental "just" and "natural" harmonium, Helmholtz would have listened to the famous violinist Joseph Joachim (1831–1907), a close friend of Robert Schumann (1810–1856)

[60] Helmholtz 1868, Chap. XV pp. 396–397, 406 and passim.

[61] Helmholtz 1868, pp. 411, 415–416, 421 and passim.

and Johannes Brahms (1833–1897), playing "natural" notes. "He played perfectly correct thirds and sixths."[62]

Singers could sing "just" notes, but in this case too for the umpteenth time he repeated that they were ruined by equable-tempered pianos. "The mechanism of the instrument and the consideration for the ease of execution tend to prevail over the natural requirements of the ear. … Mozart and Beethoven are still at the beginning of the period where the equable temperament is starting to dominate. Mozart has still had opportunities to carry out rich studies on pieces to be sung … Beethoven has courageously taken possession of the riches offered in the development of instrumental music; … But he has treated the human voice like a humble servant, … in all this I do not see the need to sacrifice the justness of the voice to the convenience of instrumental music." But, alas!, wishing to modulate between "natural" Ptolemaic-Zarlinian scales, he too had to start again taking away and inserting the usual *commas. Und so weiter* …[63]

In order to avoid excessively visible enharmonic shocks, Helmholtz attenuated the problem in his book, underlining the notes to be diminished in their pitch, and putting a mark over those to be raised. In this way, he dealt with dissonant chords which were necessary for modulations. "Every possible irrational ratio between sounds generating a dissonance, there only exists a finite number of consonances, … […] … nothing totally forbidden exists in music, and there are exceptions to every rule of melodic movement, in the passages of the greatest composers that produce the most effect."[64]

In the end, our German physicist-cum-physiologist confessed his own aesthetic criteria. It was necessary to learn "… to recognize and admire in a work of art the image of a similar order that reigns in the universe, where Law and Reason are everywhere dominant." And yet, at the height of Romanticism, also Helmholtz wrote beatitude descended in us from those parts of the work that were unconscious and far from being able to be fully analysed. "… leaving aside certain hypotheses more or less vaguely formulated, like those of Rameau and d' Alembert for example, the particular cause of the melodic affinity of sounds remains unknown, or at least it has not been presented with the degree of clarity and precision that it requires. I believe that I have arrived at this final result, …". Mixing scientific and aesthetic problems in his essay, our German physicist had made the former more complicated and the latter too simple. An aesthete should, on the contrary, have done the opposite. But on this subject, remaining on his own ground, he declared himself to be a "*dilettante*".[65]

The mathematical appendices did not add much to what had been invented by illustrious predecessors like d'Alembert, Euler or Fourier, except for a modern notation which is familiar to us. From a table of organ pipes (Fig. 12.4), he extracted

[62]Helmholtz 1868, pp. 422, 425–426, 429 and passim. For the different version of musicians, see below, Sect. 12.3.

[63]Helmholtz 1868, pp. 430, 432 and passim.

[64]Helmholtz 1868, pp. 436, 469 and passim.

[65]Helmholtz 1868, pp. 481–482, 486 and passim. In italics and in Italian in the text.

THÉORIE PHYSIOLOGIQUE DE LA MUSIQUE.

NUMÉROS.	HAUTEUR DES SONS.	LONGUEUR DU TUBE.	DIAMÈTRE DU TUBE.	DIAMÈTRE DE L'ORIFICE.
1	$si\flat$	425	138	31,5
2	$si_1\flat$	210	82	23,5
3	fa_2	117	65	16
4	$si_2\flat$	88	55	14,3
5	$ré_3$	58	55	14
6	fa_3	53	44	12,5
7	$la_3\flat$	50	39	11,2
8	$si_3\flat$	40	39	11,5
9	$ré_4$	35	30,5	10,3
10	fa_4	26	26	8,5

Fig. 12.4 How Helmholtz connected notes with the dimensions of pipes (Helmholtz, "Diapason", 1868, p. 494)

some regularities also for the diameter. It was possible to go up from fa_3 to fa_4 not only by shortening the length from 53 to 26, but also by diminishing the diameter from 44 to 26. At least here, he did not leave any comment about a possible regularity, which was not noted because his interest was in experiments about vibrations due to influence.[66]

Helmholtz put formulas to calculate the beats in various cases, and explained how to tune a harmonium with 24 sounds to the octave, so as to be able to play "just" in all the tonalities. Like another English organ, this instrument was not to be adopted by musician, either. Lastly, he insisted in the art of singing "just" notes, and concluded with the three propositions of the last appendix.

> 1) The theoretical intervals, which I have called natural in my book, are effectively so for any ear that has not been contaminated.
> 2) The errors of the tempered scale are in reality perceptible and unpleasant for a just ear.
> 3) Despite the slight difference in the intervals taken in isolation, it is much easier to sing just following the natural scale than following the tempered scale.[67]

Although he defended his cause on every possible occasion, our German physicist did not enjoy the same success among musicians that he had had among scholars of acoustics. The piano and instrumental music had a greater weight than his theory. At this moment, Helmholtz was necessarily still unaware that the initial chord of the *Das Rheingold [The Rhine Gold]* (1869) by Richard Wagner (1813–1883) tuned up "natural" after a short while.

Out of all this, what should Riemann not have found acceptable? Helmholtz started by analysing sound in the terms of a mechanical phenomenon by means of

[66] Helmholtz 1868, pp. 493–497, 507–509 and passim.

[67] Helmholtz 1868, pp. 532–534, 537–538 and passim.

instruments, while the ear came into play later, only later. The mathematician from Göttingen started from here, on the contrary. The latter did not even mention ratios between numbers as a model for sounds, whereas the German physicist, in the wake of Euler, tried to demonstrate their validity through harmonics and beats. Riemann kept the auditory phenomena separate from the luminous ones, and maintained that sonorous movements as small as those induced by very distant sounds could not be seen even with the microscope. Helmholtz made these visible with his experiments, making the ear almost secondary. In reality, every now and then, he relied on his studies on the eye *Handbuch der physiologischen Optik [Manual of physiological optics]*, as if a general theory of sensations existed. The physicist tried to confirm the similarities between the sonorous and the luminous spectra already described by Newton. This English scholar was not criticised at all by him, as on the contrary the mathematician did, sustaining the ether against action at a distance.[68]

Following Kant, Helmholtz defended Newton's idea of space and time, whereas Riemann, who did not refrain from advancing hypotheses, proposed more general space-time theatres, which were also variously curved and modelled by phenomena. After the death of his colleague, the German physicist wrote the article "Ueber die Thatsachen, die der Geometrie zum Grunde liegen" ["On the facts which lie at the foundation of geometry."] In the intentions of Helmholtz, the facts which should have extended the hypotheses of Riemann were the congruences between rigid bodies free to move in space. Trying to explain how refined an attentive, conscious way of listening to music could be, Riemann did not exclude the human "subject" from the auditory sensations. With all his experimental apparatus, Helmholtz went in search of acoustic laws which were as "objective" as possible, relegating the rest to aesthetics. This other German was always hunting for certain "facts": "Die Thatsachen in der Wahrnehmung" ["Facts in perception"]. According to him, every phenomenon should be reduced to matter and to causal forces, which are the only sources of an "objective truth". He believed, like Kant, that the law of causality was a transcendental a priori, the foundation of every possible legality of nature.[69]

Considering that in the long run, Riemann's hypotheses were to have important developments both in mathematics and in physics, I feel the lack of a work by him entitled "On the hypotheses which lie at the basis of music". Consistently with the rest of his work, would we finally have enjoyed a *continuous* theory for the ratios between notes, with a lot of geometry and not many numbers, with a lot of hearing and not many neo-Pythagorean prejudices? Wasn't his mechanical model *continuous*, in all that interlinking in contact between air, water and solid bodies? "…fortwährend mit mehr als mikroskopischer Genauigkeit in einander greifen müssen,…" ["…they must continuously touch one another with a more-than-microscopic precision,…"]. He was extremely worried by any cause that could compromise the continuity, such as sudden, sharp rises and sharp drops in temperature, and thus he dwelt on the corrective mechanisms contemplated in the

[68]Helmholtz 1868, pp. 86–87, 482. Turner 1981, pp. 246–247.

[69]Turner 1981, pp. 243–244.

ear. “Damit der Apparat die kleinsten Druckänderungen der Luft, in stets gleichen Verhältniss vergrössert, dem Labyrinthwasser mittheilen könne, . . .” [“By means of this, the apparatus can communicate the smallest modifications in air pressure to the liquid in the labyrinth, amplified in a ratio that is always the same, . . .”].[70]

A character like Helmholtz had received a very varied *Bildung* [formation], from philosophy to medicine and physiology, from physics to chemistry, whereas for mathematics he was self-taught. It is no surprise that he made relatively valid contributions in different sectors. As had happened also to the less fortunate physiologist, Julius Robert von Mayer (1814–1878), who preceded him in the conservation of energy, his first success in physics derived rather from his research in physiology: “Ueber die Erhaltung der Kraft” [“On the conservation of force [energy]”]. Following the principle that energy was conserved, in the kinetic, mechanical and caloric forms, he obtained important results in mechanics as in electricity, in chemistry as in physiology, in thermodynamics as in hydrodynamics. Helmholtz represented for German sciences what Otto von Bismarck (1815–1898) was achieving for the supremacy of Germany in Europe; what Justus von Liebig (1803–1873) did for chemistry, agriculture and nutrition; he embodied the results obtained by Werner von Siemens (1816–1892) and his family in the electromechanical industry.[71] Without going too far wrong, the intelligent reader can imagine by himself all the relative existing relationships.

This versatile German scientist often started from the inventions made by others, to reorganise them, generalise them, and spread them to the general public; he verified their potential with other clever experiments conducted in the best laboratories in Europe, placed at his disposal by the State and by industrialists. For music and acoustics, which above all concern us here, we have already mentioned the Frenchman Lissajous. From Ludwig Wilhelm August Seebeck (1805–1859), Helmholtz had taken the ideas for the sirens.[72] To our German scholar, the English scientist Thomas Young (1773–1829), who had visited Göttingen for a few years, must have offered a greater source of inspiration. Young had conducted experiments with vibrating strings and with pipes of air, studying the appearance and disappearance of nodes and antinodes.[73] By analogy, he had also developed one of the first wave theories of light, which contradicted that of Newton, and explained the interference. Maybe as a result of his limited ability in mathematics and in the communication of his results, this Englishman did not succeed in convincing the physicists of the period very much; it seems, however, that he was among the first to introduce the term “energy”, while others, like Helmholtz, used the term “force”.[74]

The German physicist probably was more indebted to his compatriot and colleague, Georg Simon Ohm. He, too, had devoted himself to sounds “Ueber

[70]Riemann 1990, pp. 345 and 349.

[71]Turner 1981.

[72]Helmholtz 1868, p. 16.

[73]Helmholtz 1868, p. 71 and passim.

[74]Morse 1981.

die Definition des Tones, nebst daran geknüpfter Theorie der Sirene und ähnlicher tonbildender Vorrichtungen" ["On the definition of notes, together with the theory of sirens connected with it, and similar devices producing sounds"] and "Noch ein Paar Worte über die Definition des Tones" ["A couple more words about the definition of notes"]. Here, Ohm had used Fourier's serial development to explain how the sirens of Seebeck and other instruments produced notes of a certain frequency; he had calculated the quantities of their vibrations by means of the algebra of *sines* and *cosines*, evaluating them at suitable points and phases, and thus how they were reinforced or disappeared, depending on their phases. As a result of the objections advanced by this Seebeck of the sirens, our German physicist had been forced to write the second article, where he, like Helmholtz, followed the "... Thatsachen ... um so wenigstens durch innere Beschauung des Gegenstandes ihr Gewicht zu prüfen, da ich mit dem Ohre in dieser Sache nichts zu thun vermag, weil mir die Natur ein musikalisches Gehör ganz und gar versagt hat." ["... facts ... at least to demonstrate in this way their weight through an internal study of the object, seeing that in this matter, I cannot do anything with my ear, because nature has completely refused to give me a musical ear."] Actually, Ohm would not want to write about music, but only about acoustics. "Jene von Seebeck wahrgenommenen Widersprüche beruhen auf einer Gehörstäuschung, in welcher unser Ohr befangen ist, ... Ich nehme nämlich an, dass unser Ohr unwillkührlich den Haupton für stärker ansicht, als er wirklich ist, und seine Beitöne für schwächer, als sie wirklich sind, ..." ["Those contrasts perceived by Seebeck rest on an auditory deception, of which our ear is the slave, ... To be precise, I assume that our ear involuntarily considers the fundamental note to be louder than it really is, and its harmonics to be softer than they really are ..."].[75]

The conflict between judgements based on the ear and calculations with numbers had become evident, explicit in writings, and it grew deeper rather than being smoothed away in the scientific "truths" brought by the new acoustics. No agreement existed about how to solve it. After being advanced by August Seebeck, the anomalies not explained by the theory, in the pen of Ohm, became "akustische Räthsel" ["acoustic mysteries"] and were translated into other questions. From these problems, in the end, music prepotently emerged. First of all, only the expert, educated ear succeeded in hearing harmonics, which were otherwise ignored. "Worin liegt der Grund, dass in einem Concerte ein falscher Ton dem Ohr so laut sich ankündigt, selbst von einem Instrumente, das ausserdem kaum wahrgenommen wird? Wie lassen sich die Wirkungen der Mixtur an der Orgel begreiflich machen?" ["Where is the reason that in a *concerto*, a false note is announced so clearly to the ear, even by an instrument, which would otherwise not be noticed at all? How are the effects of the *ripieno* at the organ made understandable?"]

Having publicly declared his own musical insensibility, the physicist Georg Simon had to recur to a friend of his, but he did so, minimising the need. "Mehr aus Neugierde als in einer andern Absicht ersuchte ich einen meiner Freunde, der

[75] Ohm 1843; 1844, pp. 7 and 15.

sich ehedem viel mit Musik abgegeben hat, er möge doch einmal an der Violine untersuchen, ob nicht, wenn er einen Ton sammt seiner Octave zugleich anstreiche und dann plötzlich den tieferen Ton weglasse, der übrig bleibende höhere Ton ihm stärker zu werden scheine." ["More out of curiosity than for any other reason, I asked one of my friends, who had previously dedicated himself considerably to music; he therefore intends to verify some time on the violin whether the note that is higher seems to him to become louder, when, together with this note, he plays its octave, and then suddenly abandons the lower note."] Complex results were obtained from the experiment. These depended on how the violin was played. Sometimes the acute note was reinforced, other times the deep note became weaker, but he also reported that the phenomenon disappeared with false notes.

Ohm admitted that he felt ". . . einige Beklemmung in Folge der Kühnheit, womit ich Hypothesen wage in einer Sache, von der ich fast wie ein Blinder von der Farbe rede; . . ." [". . . a certain anxiety as a consequence of the boldness with which I risk hypotheses on a subject of which I speak like a blind man speaking of colours; . . ."]. Consequently, our German physicist submitted the decision to the more solid future research of other scholars who were better equipped to deal with the problem.

After protecting himself in this way, however, he concluded by reasserting the old numerical definition of notes and rejecting Seebeck's solution to the dilemma. "Mir scheint es gerathener, weil die von Seebeck vorgeschlagene Aenderung an der Definition des Tones in ein neues Labyrinth zu führen scheint, lieber gleich die Schuld davon auf unser Ohr zu schieben, das einer Täuschung oder, wenn man lieber will, Verwöhnung in der Art ausgesetzt ist, dass es zu dem tiefsten Tone solche höhere Töne, welche zu ihm das Verhältniss eines Beitones haben, ganz oder theilweise herüberzieth und als zu ihm gehörig ansieth, aber darum unwillkührlich ein falsches Urtheil über die relative Stärke solcher Töne fällt." ["Because modifying the definition of the note, as proposed by Seebeck, appears to lead into another labyrinth, anyway it rather seems better indicated to attribute the blame to our ear, which is exposed to a deception, or, if you prefer, is vitiated in such a way that, wholly or in part, it here attracts to the deeper note such acute notes as stand in the relation of harmonics to it, and it recognizes them as belonging to it, but for this reason in unaware way, a false judgement is attributed to the relative force of such a note."][76]

Apart from coping with an error of calculation committed in his previous article, and pointed out, Ohm wrote that Seebeck had maintained: ". . . dass ein Ton von der Schwingungsmenge *m nicht ausschliesslich* gebildet werde durch eine Bewegung von der Form $\alpha cos 2\pi(mt + \theta)$." [". . . that a note of frequency *m* is *not exclusively* created by a movement of the form $\alpha cos 2\pi(mt + \theta)$."][77] According to the inventor of new sirens, surely also the subsequent terms in the series of Fourier contributed to the fundamental sound. He had compared Ohm's theoretical results with the sounds of his sirens, following a positive experimental method, and he concluded.

[76] Ohm 1844, pp. 15–18.

[77] Seebeck 1843, passim. Ohm 1844, p. 10.

I. Unter den beschränkenden Voraussetzungen, welche Ohm in Betreff der Sirenestöße der Einfachheit wegen angenommen hat, gelangt man zu Resultaten, welche durch die Erfahrung nicht bestatigt werden, und man muß wenigstens von dieser Seite her die Theorie allgemeiner behandeln.
II. Vollzieht man diese Verallgemeinerung, hält aber dabei noch die Annahme fest, daß ein Ton von der Schwingungsmenge m ausschließlich gebildet werde durch eine Bewegung von der Form $acos2\pi(mt + \theta)$, so kann diese Voraussetzung mit mehreren Resultaten der Sirene-Versuche in Einklang gebracht werden, steht jedoch mit einigen anderen Erfahrungen (...), so viel ich habe bemerken können, nicht in Uebereinstimmung.
III) Die Annahme, daß an der Erzeugung jenes Tones auch die übrigen Glieder von der Form $acos2\pi(imt + \theta_i)$, vermöge der ihnen gemeinsamen Periode $\frac{1}{m}$ einen Antheil haben, läßt sich nicht nur mit den ersteren BeoBachtungen ebenfalls in Uebereinstimmung setzen, sondern erklärt auch auf eine einfache Weise die Erfahrungen, welche der in II. versuchten Voraussetzung entgegenstehen. Die Erfahrung spricht also für die in III. gemachte Annahme.
[I.) Under the limitative presuppositions, which Ohm assumed to simplify, considering the siren strokes, we arrive at results which are not confirmed by experience, and from this point of view, the theory needs at least to be treated in a more general manner.
II.) Even after generalisation, all the same, the assumption still remains that a note of the frequency m is exclusively formed by a movement of the form $acos2\pi(mt + \theta)$, and then this presupposition can be brought into syntony with more results of the essay on the siren; however, as far as I have been able to observe, it does not agree with any other experiments.
III.) The hypothesis, to generate that note also the remaining members of the form $acos2\pi(imt + \theta_i)$ may occupy a quota of its common period $\frac{1}{m}$, not only agrees equally with the first observations, but also explains in a simple manner the experiments which contrast the presupposition attempted in II. The experience thus testifies in favour of the hypothesis presented in III.][78]

Invoking also general scientific interests and questions of a method of his, Ohm initially rejected the criticisms of Seebeck, but later he had to concede that his adversary might to some degree be right. "Es könnte daher wohl geschehen, und das Daseyn der Combinationstöne leistet im Grunde Gewähr dafür, dass die folgenden Glieder der Reihe (A.) den im ersten Gliede enthaltenen Ton durch solche unregelmässige Successionen zu verstärken im Stande wären. Desswegen werde ich mich jenes Satzes ... nicht nur nicht bedienen, sonder vielmehr gleich von vorn herein die Möglichkeit, und wenn man will die Wirklichkeit einer solchen Verstärkung einräumen." ["It may very well happen therefore, and thus the existence of notes of combination offers a guarantee as a foundation, that the following members of the [trigonometric] series (A.) were able to intensify the note contained in the first member [the fundamental note] through such an irregular succession. So not only I shall not use this proposition [that only the term of the same frequency contributes to the note] ..., but from now on, I shall concede much more the possibility of this reinforcement, of its reality, if you like."] A bit further on, the famous German physicist found the (all too?) easy solution to all these dilemmas of his: "... stelle ich lieber, keck wie Columbus, das Ei gleich auf die Spitze ..."

[78] Seebeck 1843, pp. 449, 452–453 and 480–481.

[“. . . as cheeky as Columbus, with more pleasure I put the egg down right on its end . . .”].[79] Thus, as usual, the blame would lie with the poor musical ear.

Twenty years later, Helmholtz was to try to substitute the unreliable hearing of scientific researchers with “resonators” and the luminous figures of Lissajous. There is little doubt that he was the one who picked up the final appeal pronounced by Ohm. The latter owed the great fame that he enjoys today to his laws regarding electric currents, but during his lifetime, as he did not come from an academic family, he had to support numerous difficulties, and was to obtain a chair of physics at Munich only 2 years before his death. He developed his theories for electric currents explicitly in analogy with those of Fourier for heat, which he therefore knew well; this is why he could use his series also for sound. And yet, though he was quoted by Helmholtz, Ohm’s articles on the definitions for notes are generally ignored by his historians.[80]

The natural philosophers had first, unsuccessfully, tried to discipline musicians with their whole numbers. Only *obtorto collo* [against their will], the former had been forced to accept the equable temperament, long practised by the latter. Now the physicists who studied acoustics would like to subject the notes used by musicians to those newly-formalised laws taken from the same phenomena that the latter had already heard. Would physics prove to be a more powerful means than ancient mathematical philosophy in the Pythagorean style?

In a scientific community by now more interested in electromagnetism and thermodynamics, while the new chapter of radioactivity was about to open up, the *exploit* of Helmholtz marked the apogee of music as acoustics. Among the leading protagonists of mathematical physics at the end of the nineteenth century, we can find only a few other sporadic examples in the following section.

And in the meantime, what was happening in China, in the Country-at-the-Centre? I would not like the reader to think that we are ignoring it, in favour of Europe; as if scientific research were definitively concentrated in Western countries. In 1857, Alexander Wylie (1815–1887) and Li Shanlan (1811–1882) completed the translation into Chinese of Euclid’s *Elements*, leaving the same title, *Jihe yuanben* as that of Matteo Ricci and his collaborators, who had stopped at the sixth chapter.[81]

Li was a mathematician who did not limit himself to translating and commenting on European works. He wrote the *Ziguxi zhai suanxue [Mathematics from the Study ‘devoted to the imitation of the ancient tradition’]*, where he calculated original formulas of combinatory analysis in the chapter entitled “Duoji bilei” [“Piles of Heaps Summed Analogically”]. He arrived at his fresh rules by means of designs of little circles and cubes variously arranged . The results were later to arrive by-road also in Western books. Among other outcomes, he arranged numbers in the form of a triangle, as if he were generalising binomial coefficients (known as Pascal’s triangle in Europe). “Li Shanlan largely based his mathematical work on models

[79] Ohm 1844, pp. 14–15.

[80] Caneva 1981.

[81] See Part II, Sect. 8.2. Tonietti 2006a.

which did not tend to share the mathematical ideals of the Greeks, but which were characterized, instead, by highly heuristic principles, proofs without words, pattern recognition, algebra, and algorithms, and other components. His writings thus raise many fascinating questions about the differences between Eastern and Western views of rationality, while they provide a rich source of study, not only for historians and mathematicians, but also for cognitive sociologists."[82]

At the end of the nineteenth century, the Protestant missionary Wylie had at his disposal the material necessary to remind us of at least some of the illustrious Chinese physicists and mathematicians of his period. However, his interests almost all regarded the past. If the Chinese imperial men of letters invented something original, in his opinion, this would happen above all with numbers and with algebra. I do not mean only that the necessary geometrical background and the *qi*, on which we dwelt at the beginning, were missing in the missionary's description, but even Li Shanlan, sitting beside him as his translator, would have become almost invisible. "There is one Li Shanlan, ..., who has recently published a small work called *Duishu tanyuan [Discovery of the Source of Logarithms]*, in which he details an entirely new method for their computation, based upon geometrical formulae; which he says in his introduction is 'ten thousand times easier than the methods used by Europeans', and that 'although *they* [Europeans] can just calculate the numbers, yet they are ignorant of the principle.' ... It is well known that there are at present many of a similar turn of mind, in different parts of the empire; and it is surely interesting to find these signs of intellectual activity, striving against the difficulties with which they have to contend, and standing out in bold relief from the mass of mental torpor, so characteristically congenial to the institutions of this truly conservative empire. ... And may we not hope that the same spirit of inquiry, which would lead them to seek after *truth* in science, would also, if rightly directed, teach and prepare them to investigate it in a much higher and more important sphere of knowledge; – even to understand themselves, and the relation they sustain to that great Being, whose name they have preserved, – whose knowledge they have lost."[83]

In China, Christian missionaries have now (2014) been substituted by merchants; the 'torpid' Chinese empire has been transformed into one of the leading world powers. Are there still some other interesting Chinese scholars to be discovered? In the nineteenth century, how was the Orient considered by the Europeans? Among the most representative, Georg Wilhelm Friedrich Hegel (1770–1831) left us a characteristic testimony. He was commenting on *Bhagavad Gita*, which had been presented by the philologist and Sanskrit scholar from Berlin, Wilhelm von Humboldt (1767–1835). As regards the renunciation to pick up the fruits produced by actions, and to pursuit success, as sustained in the Indian poem, the German philosopher expressed doubts. Above all, he could not see in it "any deduction of moral duties". Most times, Hegel judged the Hindu religious rites to be superstitions.

[82]Martzloff 1988, pp. 318–325. Martzloff 1992, pp. 32, 36–37.

[83]Wylie 1897, pp. 193–194.

"The poet has not yet risen above the level of common Indian superstition to arrive at a truly religious or philosophical moral determination."[84]

Although he had to base himself on other people's translations, including, as well as that of Wilhelm Humboldt, also one by August Wilhelm Schlegel (1767–1845) into Latin, our idealistic philosopher did not refrain from discussing significant problems of interpretation about the meaning to be attributed to certain key words. He undoubtedly knew that this was the central point of any attempt to get close to that ancient, faraway culture. But then, when he succeeded in identifying some concept familiar to him, he often fell into judgements of the kind: "But duty is something different from this purely negative indifference to success. The more an *opus operatum* [action] is completed in stupidity and obtuseness, the greater the indifference towards success is."

Even amid all the spirituality that the religions and philosophies of India liberally distributed on all sides, Hegel considered them to be devoid of any true spirit. Because, clearly, also his own was set among the dialectic contradictions of the thesis and the antithesis, but then it should inexorably arrive at the final synthesis. Instead, for him, the typical Indian contradiction between acting by means of warlike actions, as in the *Bhagavad Gita*, and remaining immobile in Brahmanic mediation would not reach a solution. "But this solution is impossible, because the supreme object of Indian conscience, the abstract being Brahm, is himself devoid of determination. Thus this can only be outside unity, only an external, natural determination. In this ruin of the universal and the concrete, the one and the other are devoid of spirit. ... The abandonment of the Indian soul that penetrates into vacuity is rather a way of becoming brutish, which perhaps does not deserve even the name of mysticism, and which cannot lead to the discovery of any truth, because it is without any contents."[85]

Hegel's philosophy was centred around the relationship between the spirit in general and nature, and consequently, around how the absolute spirit entered into a relationship with the finite spirit. "For India, the fundamental determination is that abstract spirituality, the concentration characterised by pure abstraction without any determination or limit, is the absolute power that reigns over the natural element". For this reason, all the rituality described in the sacred Vedic books and in the extremely long Indian poems, seemed to him to be absurd, extravagant superstition. Our German philosopher seemed to admire as "something sublime" the staggering transcendence of the Brahma from the sensible world; the Indians had detached themselves from the empirical diversities, to rise up to the "greatness of thought". "But what characterises them, starting from the prodigious abstraction of this extreme pole, is the event that they did not succeed in opening up a route towards the reconciliation of this extreme with the particular towards the concrete. With them, also the spirit is the unsteady step which, without being able to fasten on to anything,

[84]Hulin 1979, pp. 150, 152.

[85]Hulin 1979, pp. 163, 168, 171.

oscillates from one extreme to the other, the unhappiness of which at last consists of only knowing happiness in the form of the annihilation of personality, which is the same thing as the *nirvana* of the Buddhists."[86]

Besides this modest confusion, the famous Hegel cannot have appreciated the event that the Brahma was so different from his *Gott*, either; on the contrary, the latter had to be as determined and powerful as the God of Newton. "In accordance with his internal determination, Brahma remains the abstract *being*, the universal, the *substance* in itself *devoid of subjectivity*; therefore he is not the concrete, he is not the *spirit* (the moderns, who understand God as the *being* of beings – Wesen der Wesen – no longer determine him as concrete, or as spirit). Possessing such contents, or rather such a lack of contents, this masculine term, in reality, is not an individual subject; ...". As if this were not enough, to exacerbate the divergence between his philosophies with his favourite European religions and the Indian ones, among the defects, Hegel added that the *monotheism* of the only Brahma even became a *pantheism*.[87]

With all their vigorous romanticism fuelled by the infinite and by poetry, the Schlegel brothers had drawn closer with greater love and attention to Indian culture. Also the humanistic and cosmopolitan philologist, Wilhelm von Humboldt constructed universal values for the *Bhagavad Gita*. In parallel with the academic destruction inflicted on this illustrious professor from Berlin, Hegel restored only a caricature of Indian culture, and remained a prisoner of a widespread prejudice: "... that India (like every Oriental historical formation) is an immutable 'monolithic figure', within which no creative originality can be manifested."[88]

To return to mathematical sciences, from the deepest Kerala and Tamil Nadu, the Hindu devotee Srinivasa Ramanujan (1887–1920) was shortly to appear. He was one of the most important theoreticians of numbers who lived in the twentieth century.[89]

Great fame was acquired in the twentieth century by two Chinese scholars, C. N. Yang and Chern Shiingshen, respectively in the field of theoretical physics and geometry. True, they gained it in the West, but in their particular way of thinking, may we not perceive, perhaps, the continuing presence of some traces of the culture cultivated in the Country-at-the-Centre? Continuing the theories of Albert Einstein and Hermann Weyl, the idea was cultivated of submitting to invariance under gauge transformations those equations which rule the fields of physics. It meant interweaving in a circle the behaviour of matter with the geometry of pace-time where phenomena take place. In this, maybe, did they not feel the influence exerted by the Chinese tradition of a unitary universe full of *qi*? "It is indeed remarkable that developments in geometry have been consistently parallel to those in physics."

In his own mother tongue, Chern presented a poem with a vaguely Taoist flavour; its English translation runs like this.

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[86]Hulin 1979, pp. 179, 188.

[87]Hulin 1979, pp. 190–191, and passim.

[88]Hulin 1979, p. 213.

[89]Joseph 2003, pp. 11–13, 289–290.

A Poem

Physics and geometry are one family:
Together and holding hands, they roam to the limits of outer space.
Black hole and monopole exhaust the secret of myths;
Fibre and connection weave to interlace the roseate clouds.
Evolution equations describe solitons;
Dual curvature defines instantons.
Surprisingly, Math has earned its rightful place for man and in the sky;
Fondling flowers with a smile -just wish nothing is said![90]

12.3 Ludwig Boltzmann and Max Planck

Ludwig Boltzmann (1844–1906) partly went back to our history, albeit now in too tenuous a manner: "Zur Geschichte des Problems der Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite" ["For the history about the problem of transporting a plane wave of air with a finite vibration width"]. This Austrian physicist referred to Riemann's article with the same title, accusing him of not knowing the contributions brought by other scholars to the problem. He listed five that had been published in English and French journals, some famous, others obscure. Also Heinrich Weber (1842–1913) was criticised for not adding them when he edited the collection of works. Under the same title, in the following issue, this Weber answered that Boltzmann was wrong: a couple of scholars who were supposed to have been ignored had been quoted in the announcement of 1859, which had preceded the article, while the others either appeared in the relative English literature, or dealt with different arguments. There was only one that Riemann did not know, because he could not possible have known it, as it was printed only after his research.[91]

One might wonder how the famous Boltzmann not so much could ignore Riemann's original solutions to the problem, but instead that he could insist on the presumed unacademic, dishonest behaviour of the German mathematician, who had died more than 10 years before. *Wahrscheinlich*, the physicist from Vienna felt that the atomistic theories, which he loved, did not enjoy the due credit in Germany.[92] Riemann had clearly moved in the opposite direction, thinking of air as a continuous medium, not composed of a statistic group of molecules, as the Austrian preferred to consider gases. In Boltzmann's excessive insistence on trying to find success for his atomistic theories, without too much succeding during his lifetime, some historians

[90]Chern 1990, p. 685–686, 649. Zhang 1993. Roman 1964, pp. 238–239. Tonietti 1966. Tr. Harry Wang.

[91]Boltzmann 1876. Riemann 1990, pp. 188 and 208; Riemann 1968, pp. 177 and 204. Weber 1877, p. 71. Schoeneberg 1981.

[92]Boltzmann 2004. Brush 1981.

have found one of the possible reasons for the depression that led him to commit suicide at Duino, near Trieste.

Anyway, like a good inhabitant of Vienna, the renowned theoretical physicist caused music to emerge from his writings, unfortunately only as a means to divulge his sciences, without really examining its colouristic aspects in detail. He compared the styles followed in theoretical physics to those of music, to dramatise a technical problem in his equations for gases. "...; gerade durch diese Einfachheit, durch diese Unentbehrlichkeit jedes Wortes, jedes Buchstaben, jedes Strichelchens kommt der Mathematiker unter allen Künstlern dem Weltenschöpfer am nächsten; sie begründet eine Erhabenheit, die in keiner Kunst ein Gleiches, – Ähnliches höchstens in der symphonischen Musik hat. Erkannten doch schon die Pythagoräer die Ähnlichkeit der subjectivsten und der objectivsten der Künste. *Ultima se tangunt*. Und wie ausdrucksfähig, wie fein charakterisierend ist dabei die Mathematik. Wie der Musiker bei den ersten Takten Mozart, Beethoven, Schubert erkennt, so würde der Mathematiker nach wenigen Seiten, seinen Cauchy, Gauß, Jacobi, Helmholtz unterscheiden." ["...; with this same simplicity, with every word, every letter, every indispensable hyphen, the mathematician arrives with all artists, as close as possible to the creator of the world; he founds a sublimeness which has no equals in any art: it has a high degree of similarity with symphonic music. And yet the Pythagoreans already recognized the similarity of the most subjective and the most objective in art: 'Extremities touch'. And how expressive, how subtly characterising mathematics is in this! As the musician recognizes Mozart, Beethoven, Schubert from the first bars, so the mathematician distinguishes after a few lines his Cauchy, Gauss, Jacobi, Helmholtz."][93]

Inaugurating a course of lessons at Vienna in 1902, he exalted the technical progress achieved in that period, from the wireless telegraph of Guglielmo Marconi (1874–1937) to the car, he imagined the joy of the pioneers, at the idea of what their successors would achieve. "... for example, what would Mozart experience if he could listen to an excellent execution of the *Ninth Symphony* [Beethoven] or the *Parsifal* [Wagner]?"[94] On the basis of any more or less limited knowledge of music, these comparisons do not seem to be too appropriate. It would be all too easy to deny that in the art of sounds, we witness a truly cumulative historical progress, even if, on the contrary, in the field of sciences, many historians and philosophers continue to deceive themselves on this point. Perhaps those clumsy attempts to construct parallelisms meant Boltzmann felt that music and theoretical physics were both present in him, but also that he no longer understood their relationship, which had become obscure.

In his particularly witty diary of his trip to Berkeley, in American California, among the references to waltzes, to the most famous Viennese composers, to Gustav Mahler (1860–1911), to the Wiener Philarmoniker as well, he narrated that he had performed a *Sonata* by Franz Schubert (1797–1828). "... anfangs freilich war mir

[93] Boltzmann 1905, p. 73.

[94] Boltzmann 2004, p. 183.

die Mechanik etwas fremd, aber wie leicht gewöhnt man sich and das Gute! Schon der zweite Teil des ersten Satzes ging gut und im zweiten Satze, einem *Andante*, vergaß ich mich selbst; nicht ich spielte die Melodie, sondern diese lenkte meine Finger. Ich mußte mich mit Gewalt zurückhalten, nicht auch noch das *Allegro* zu spielen und das war gut, denn dort wäre meine Technik abgefallen." ["At the beginning, actually, the mechanics [of the piano] proved to be a little strange for me, but how easy it is to get used to beautiful things! Already the second part of the first movement went well, and in the second movement, an *Andante*, I forgot myself! It was not me that was playing the melody, but the melody that was guiding my fingers. I had to make an effort to restrain myself from playing the *Allegro* as well, and this was a good thing, because there, my technique would have given way."] He compared that large private sitting-room of his extremely wealthy hostess, where he had played, to the *Bösendorfer Saal* in Vienna, though he had not found the piano that gave it its name, but rather a Steinway. "...; einen Flügel von solcher Klangschönheit hatte mein Ohr vielleicht schon in einem Konzerte gehört, nie mein Finger berührt." ["My ear may perhaps have listened to a grand piano with such a fine sound in some concerts, but my fingers had never touched it."]

As it was thus kept separate from his main profession, it is a pity that he did not write about music with the same dedication with which he was at pains to divulge atomism and scientific progress.[95] He gladly admitted music, above all when he commemorated some deceased colleague.

He wrote a little more copiously about the art of sounds when he maltreated the poor Arthur Schopenhauer (1788–1860), who was accused of being similar to a "... degenerierender Philosophaster ..." ["... degenerate philosophaster..."] because of his idealism. "Ich möchte beipflichten, daß die Musik etwas ganz besonderes vor den anderen Künsten voraus hat. ... Die Musik ist nach Schopenhauer ein Spiegel der Welt, aber nicht ihre Abbildung, oder wie die anderen Künste die Abbildung eines Teils derselben, sondern sie steht der ganzen Welt gleichberechtigt gegenüber, indem die Welt die eine, die Musik eine mit andern Mitteln bewerkstelligte, aber davon unabhängige Manifestation des Weltwillens ist. Daher auch der paradoxe Satz, die Musik könnte auch bestehen bleiben, wenn die Welt nicht wäre. Freilich dann wären keine Geigen, keine schalleitende Luft, kein erregtes Ohr, keine empfindende Seele. ... Das braucht die Musik nicht, welche aus ihren Mitteln direkt das Bild des Weltwillens schafft." ["I would like to admit that music has something very particular beyond the other arts. ... According to Schopenhauer, music is the mirror of the world, but not its copy, or like the other arts, a copy of a part of it, but it stands facing the whole world, with the same rights, seeing that the world is a manifestation, and music is another, executed with other means; and yet it is a manifestation of the will of the world that depends on it. Hence derives the paradoxical proposition that music could continue to exist even if the world did not exist. Obviously, then, there would be no violin, no air to transport

[95] Boltzmann 1905, p. 421. Boltzmann 1993, pp. 18–19, 37–38 and passim. In his youth, he had studied the threshold of intensity for sounds. Rayleigh 1945, pp. 433 and 438.

sound, no ear to stimulate, no sensible soul. … This is no use to music, which directly creates with its means the image of the will of the world."][96]

Consistently with his faith in the material existence of atoms, Boltzmann thus believed that good music could not be made without a piano, better if of the grand kind, and constructed by the Viennese maker, Bösendorfer.

Among the other leading characters who contributed to the sciences of the twentieth century, music assumed an even more marginal role; I would say, practically zero. Henri Poincaré (1854–1912) did not leave any trace of it in his vast work on many subjects of physics, mathematics and philosophy, at least not in his better-known essays. In his numerous writings on the dynamics of heavenly bodies, how can we imagine that this famous French mathematician dealt with the music of the spheres? The only echo, by now hardly audible, remained the "Analyse Harmonique" ["Harmonic analysis"] for the calculation of the tides. Did Poincaré, who would seem to have devoted himself to almost everything, ignore acoustics as well in his writings? Did he do so because he could not hear the song of any open problem, or of anything interesting in it? Was he deaf to notes and harmonics, like Descartes or Ohm, but less daring than them? Only when generalising one of Riemann's problems did our French mathematician and mathematical physicist or philosopher of mathematics calculate the harmonics of a membrane made to vibrate, with the relative "harmonic" functions. The problem had become to obtain solutions of the equation with partial derivatives proposed by Laplace, when one parameter assumed particular values, called "proper values" or "eigenvalues".[97]

As regards the other disciplines, our French mathematician and physicist sang their praises. "Astronomy is useful because it elevates us above ourselves; it is useful because it is great; it is useful because it is beautiful; …". As regards the science and the art of sounds, he seems to have remained silent. He equated the physical law with the differential equation. In a questionable way, he made this go back to Newton, but he ignored that of waves. It goes without saying that as a result, he hardly ever referred to Pythagoras. Ritually, the name of d' Alembert entered into one of his writings about geodesy, but only to demonstrate that in the eighteenth century, the French had surpassed the English. As the champion of German sciences, Helmholtz was quoted several times. Also for his treaty on acoustics and music? Not once.

Poincaré concentrated his numerous talents on the methods of "Analysis situs", that is to say topological analysis, to deal with the still unsolved problem of the three bodies and the stability of the solar system. He had great successes in inventing new mathematical means to solve substantial problems of physics, and therefore from these he derived the faith in the consequent inseparable relationship between the two disciplines. All that led him to consider not so necessary the discussion about the foundations of mathematics, which had been brought up at the end of the nineteenth century. Anyway, he saw the solution of this controversy in the "intuition" of a certain geometrical-Euclidean"continuity" of nature, and in whole numbers that

[96]Boltzmann 1905, pp. 385 and 391.

[97]Dieudonné 1981a.

could not be further analysed by means of axiomatic-formal and set-theoretical systems.[98]

Max Planck (1858–1947), on the other hand, left us an interesting "Die natürliche Stimmung in der modernen Vokalmusik" ["Natural intonation in modern vocal music"]. He entered into the discussion about what practical interest "natürliche" ["natural"] intonation held in the music of that period, considering that the "temperirte" ["equable tempered"] intonation had been measured ". . . mit so glänzendem Erfolge . . ." [". . . with such brilliant success . . ."]. And yet violinists moved away from the temperament in certain cases, such as double strings, and the chorus directors knew that ". . . beim *a cappella*-Gesang in einem *piano* ausgehaltenen Durdreiklang die Terz leicht etwas zu tief genommen wird." [". . . in *a cappella* singing [without instruments] in a major chord held *piano*, the third is taken slightly too low."] String quartets and polyphonic choruses regulated themselves ". . . immer nach der natürliche Stimmung . . ." [". . . always in accordance with the natural tuning . . ."]. Some theoretician would arrive to deny every right to the temperament because it lied to the ear. According to the famous German physicist, this was a point of view that can be judged by itself, considering the effective performance achieved by the tempered way of tuning.[99]

Planck's work on music appears to be an exception, and furthermore, it has generally been passed over in silence by those who were engaged, rather, in exalting his other famous inventions.[100] It therefore here deserves some more analytical attention. This reveals how complex and subjective the problem could be (and is) of choosing the notes to take into consideration in order to make music. The German physicist of the quantum theory thought that a tuning existed for instruments, called variously "natural", "pure" or "just". And yet, with undeniable success, musicians continued to use their equable temperament, with the few sporadic preceding exceptions. Planck brought the question back to the functioning of the ear, the capacities of which analysed in detail. However he did not study its mechanics, as Riemann had started to do, but rather its psychic qualities.

With him, the traditional ratios between whole numbers had disappeared from his article. They only appeared in one case ". . . bei der kleinen Septime, deren Klang im natürlichen Intervall (4 : 7) ganz besonders weich und schön ist." [". . . with the diminished seventh, the sound of which in the natural interval (4 : 7) is particularly sweet and beautiful."] Clearly, by now he took for granted the theory of Helmholtz, who was quoted. Recognizing the difficulty and the complexity of the problem, which had divided opinions and had changed with the passing of different eras, Planck would prefer to deal, above all, with "Thatsachen" ["facts"] that is to say, how the notes used in music were actually tuned. He aimed ". . . irgend

[98]Poincaré 1905–1910, t. III, p. 305. Poincaré 1989, v. I, p. 320, 329; v. II, p. 190. Volterra, Hadamard, Langevin & Boutroux 1914. Boyer 1990, pp. 689–693. Kline 1972, pp. 706–707, 1025–1026, 1170–1179 and passim. Tonietti 1982b; 1983a; 1990.

[99]Planck 1893, p. 418.

[100]Kangro 1981. Heilbron 1986.

ein allgemeines Gesetz in Bezug auf die zur Anwendung kommende Stimmung aufzustellen; . . .” [“. . . to establish a general law of any kind, to be applied to tuning . . .”]. He tried to evaluate “. . . die musikalische Praxis gänzlich regellos . . . zwischen der natürlichen, der temperirten, der pythagoreischen und anderen Stimmungen, . . .” [“. . . musical practice . . . [oscillating] wholly without rules between natural, tempered, Pythagorean and other ways of tuning, . . .”].

Attention was concentrated on voices and intervals as sung in concert halls, judging them directly with the ear. For this reason, the hearing had to learn to recognize also non-“tempered” intervals, which were thus different from those played on the piano. For this purpose, he used a large harmonium, tuned in the “natural” manner, the keyboard of which had been modified by adding innumerable notes which could be combined as desired, in order to make all the comparisons desired, by inserting and taking away *commas*. Commissioned by the Prussian State Government, the instrument had been described in the “Verhandlungen der Physikalischen Gesellschaft zu Berlin (Sitzung von 20. Januar 1893)” [“Discussions of the Berlin Physics Society (meeting of January 20th, 1893)”]. It was while playing this harmonium that Planck had noticed how his ear behaved while listening to the different notes: it willingly allowed itself to be deceived, as Chladni had already written, and as sustained by a famous psychologist of sound. Hearing a note as correct or wrong depended largely on the circumstances, that is, on the instruments played, on the age of the listener, on musical habits and so on.[101]

The ear would have been capable of adapting to the physical sounds perceived. “Aber auch in jedem Augenblick ist die Fähigkeit, sich einem verstimmten Intervall zu akkommodiren, den Umständen und bis zu einem gewissen Grade auch der Willkür unterworfen. Man kann sein Gehör geradezu einstellen auf einen höheren oder geringeren Grad von Akkommodationsvermögen. Das Ohr des Konzertbesuchers ist, bei gespanntester Aufmerksamkeit im Übrigen, akkommodationsfähiger, wenn er als Genießender, wie wenn er als Kritiker erscheint.” [“But at every moment, also the capacity to adapt to a out of pitch interval is subjected to the circumstances and to discretion. It is possible to regulate one’s hearing directly to a higher or lower level of possibility in coming out right. The ear of those who attend concerts is capable of adapting, with the most attentive care furthermore, when it appears to transmit enjoyment, as when it exercises critique.”][102]

Thus musicians would obtain notes that are all the more “pure”, the more they have forbidden their ear to adapt. In all cases, they had to do so less than their listeners. And yet, in spite of these experiences, it would be wrong to conclude that “. . . die Erzielung größtmöglicher Reinheit . . .” [“. . . obtaining the greatest purity possible . . .”] did not have any practical sense. The timbre was influenced, together with “. . . ein gewisses Gefühl der Spannung . . .” [“. . . a certain sense of tension . . .”]. In order to study adequately this phenomenon of the ear adapting to sounds out of tune, and non-“pure” ways of tuning, it should be not necessary,

[101] Planck 1893, pp. 425, 419–421 and passim. Planck 1958, B. I, pp. 435–436.

[102] Planck 1893, p. 423 and passim.

however, to jump in the comparison from one note to another. “Die Änderung des betreffenden Tones …, sondern muß sehr langsam und allmählich vorgenommen werden.” [“But the modification of the notes in question … must take place very slowly, and gradually.”] On the above-mentioned harmonium, it was even necessary to press the keys in a certain special way. Referring to Helmholtz, Planck generated and eliminated “Schwebungen” [“beats”] in order to be able to study the effect on music of the relative “tension”, which would disappear with the “natural” intervals.[103]

The aesthetic comment on the phenomenon made clear to what extent also Planck’s ear had largely remained, after all, a subjective instrument of measurement. “Für mich ist dieser Übergangspunkt geradezu eine Quelle künstlerichen Genusses: es ist, als ob ein gewisser letzter Erdenrest verschwände und zugleich sich ein Schleier lüfte, welcher der Phantasie den Einblick in eine neue, bis in unendliche Fernen reichende Welt eröffnet.” [“For me, this point of passage [towards “natural” intervals] is a direct source of artistic enjoyment: it is as if a certain last earthly remainder disappeared, and at the same time a veil were lifted, opening up to the imagination the vision of a new world that arrives far away, at infinity.”] Unfortunately, our German physicist had to admit that the experiment required “…die konzentrirteste Aufmerksamkeit…” [“…the greatest concentration and attention…”] and thus it could not be performed in public.

In his article, Planck reported which intervals he had tuned, and the effects created on his own ear. He arrived at a rule: “…die Akkomodation um so leichter erfolg um sich so weiter treiben läßt, je weniger konsonant das Intervall ist. … Hier zeigt sich deutlich die Gewöhnung unseres Ohrs an die temperirte Stimmung.” [“…the adaptation is obtained all the more easily, and is cultivated all the more, the less consonant the interval is. … Here, the event that our ear gets used to the tempered tuning is clearly demonstrated.”] He reported the habit of singers to raise the pitch of notes to obtain a more penetrating expression. He appeared to be particularly effective in demonstrating the dependence of the ear on the order of the chords executed: one pure “natural” and another one *comma* higher, due to the persistence of the former in the memory of the auditory organ. The psychic effects could become “…abscheulich … beleidigenden…” [“… horrible … offensive…”]. However they disappeared with the passing of time, or in the middle of other chords.[104]

Planck criticised the habit of singing in choruses *a cappella* with the initial intonation of the pieces. Singers repeatedly hummed the notes assigned by the conductor, thus ruining the beginning of the piece. As to reproduce a note assigned “just”, came into play the “…absolute Tongefühl…” [“…absolute pitch…”], he narrated that he possessed this, by means of an episode from his infancy.

[103] Planck 1893, pp. 423–424, passim.

[104] Planck 1893, pp. 425–426, and passim.

> So lange ich als Kind nur mit einem einzigen Instrument, einem Klavier, vertraut war, besaß ich ein sehr ausgesprochenes absolutes Tongefühl, derart daß ich einst, aufgefordert einen mir sehr wohl im Gedächtniß bekannten Marsch auf einem fremden, etwas tiefer gestimmten Klavier zu spielen, nach den ersten Tönen abbrechen mußte, weil ich sogleich vollständig verwirrt wurde und nicht die Fertigkeit besaß, in der Eile die doppelte Transposition zu machen: einmal im Kopfe in eine tiefere Tonart, und dann wieder auf den Tasten zurück in die frühere Tonart. Heutzutage, wo ich vielerlei Musik in vielerlei Stimmungen gehört habe, ist mein Tongefühl lange nicht mehr so sicher; es gerlingt mir mit Aufwand einiger Mühe ganz gut, den Ton *c* bis zu einem halben Ton höher oder tiefer mir vorzustellen. [For such a long time, as a child, I only trusted one instrument, a piano, and I possessed an absolute sense of the notes [absolute pitch] very remarkable that, once when I was invited to play by memory a march which I knew very well on an unknown piano which had been tuned a bit lower, after the first few notes I was forced to interrupt it; because I was immediately quite confused, and I did not possess the promptness to perform the double transposition rapidly: once to a lower tonality in my head, and then again back to the preceding tonality on the keys. Nowadays, when I have listened to all kinds of music in a variety of tunings, my sense of the notes is no longer so sure; at the cost of a certain effort, I succeed in representing perfectly well to myself the note *do* within a semitone higher or lower.][105]

Although he clearly insisted that he preferred the tuning called "natural", there emerged with force from the German professor's article the determining influence due to the musical context, where he could carry out verifications, and the sense for the memory of the ear. Thus, whether "tempered" or "natural", the ear adapted to the first chord heard, without worrying too much about the *commas* in the difference. If performed first, even the "Pythagorean" tuning would sound better than the "natural" one. Ill-treated like this, the famous Planck's poor ear became confused. "Dabei befindet sich das Ohr in einem eigenthümlichen Zwiespalt: auf der einen Seite die Terz in der vom Ohre gewöhnten und begehrten frischen Höhe, zugleich mit den hämmernde Schwebungen, auf der anderen Seite die Terz in Ruhe, aber doch für das augenblickliche Bedürfniß zu tief." ["Thus the ear finds itself in a particular kind of conflict: on the one hand, the fresh third at the habitual pitch, and the one desired by the ear, together with the hammering beats, on the other, the third in peace, but however too deep for the needs of the moment."] Among the three principal possible tunings, "Pythagorean", "natural" or "tempered", trying again and again passages and comparisons, it was fairly clear that the ear took several liberties. Given that this organ was assigned the central role for the problem, habit easily justified the "tempered" tuning.[106]

And yet Planck still tried other interesting practical musical experiments, which throw light on the theoretical question, both on the behaviour followed by musicians in that period, and lastly, not to be overlooked at all, on the attitudes adopted by Planck himself in his scientific and personal life. Attending concerts and rehearsals, he experienced directly the event that everybody played and sang in the tempered intonation without any ambiguity. But our German physicist followed Helmholtz in

[105] Planck 1893, pp. 428–429 and passim.

[106] Planck 1893, p. 429 and passim.
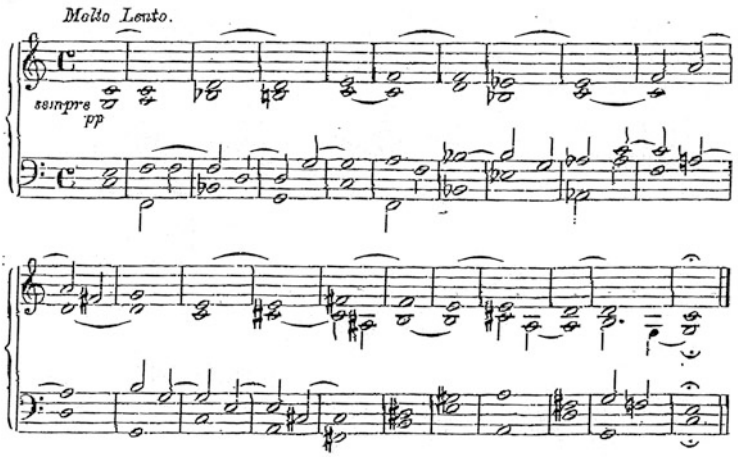
Fig. 12.5 Planck composed music to perform an experiment on "natural" intonation (Planck 1893, p. 435)

considering this system contradictory, because it contained "pure" octaves mixed with "impure" thirds and fifths. And then he wondered how singers solved the dilemma: didn't they pass in practice from the "tempered" to the "natural"?

At Berlin in the winter of 1892/1893, he had listened attentively for the first time to some *Geistliche Gesänge [Spiritual songs]* by Heinrich Schütz (1585–1672). In these, he had noticed that, during some *pianissimo* bars of a chorus of five voices, when he could no longer hear the accompanying piano, the singers' voices tended to drop down by a *comma* towards the "natural" intonation. In order to investigate the phenomenon better, Planck had then written a succession of chords himself in *do* major (Fig. 12.5), which he got a group of singer-friends to sing under his own direction, in circumstances that we shall immediately see below. Thus he discovered that, compared with the initial *do*, the final *do* was sung as many as five syntonic *commas* lower; thus they dropped down by more than half a tempered tone!

What conclusions did Planck draw from a direct experience which he invited other conductors to repeat easily in similar conditions? He was worried above all that the singers should not know the purpose of the experiment, in order not to affect the results. We shall soon see that this condition was consistent with the convictions of the German physicist about the principal epistemological problem of his great epoch. "Wenn dann am Schluß der Chor wieder auf dem Ausgangs-*c* anlangen sollte, so wäre das ein zwingender Beweis dafür, daß für ihn die natürliche Stimmung auch nicht die geringste praktische Bedeutung hat. Dann wäre es, vom Standpunkt der modernen Kunst betrachtet, lediglich müßige Spekulation sich noch einen Augenblick länger mit der Theorie der natürlichen Stimmung zu beschäftigen." ["When, at the end, the choir then had to arrive again at the initial *do*, that would be a convincing demonstration the "natural" intonation did not have the slightest practical meaning for them. Therefore, considered from the point of view

of modern art, it would only be idle speculation to dedicate another moment more to the theory of the "natural" intonation."]

Besides trying several times, the rigorous Planck even performed a counter-experiment. If he had the succession of chords sung backwards, from the last to the first, would he obtain at the end a higher *do*? "Das schließliche Resultat ergab kein Sinken, aber auch nicht das erwartete Steigen der Tonhöhe." ["The final result did not give any lowering, nor, however, did it give the expected rise in the height of the note."] The physicist tried to find some explanation for this, but concluded that further experiments were needed. Anyway: "Diese Fälle beweisen, daß die Differenzen der natürlichen und der temperirten Stimmung die in der praktischen Musik zulässigen Schwankungen der Intervalle unter Umständen merklich überschreiten, und stellen daher jeden Dirigenten vor die Aufgabe, sich darüber ein Urtheil zu bilden." ["These cases demonstrate that, in certain circumstances, the differences between the "tempered" and the "natural" tuning definitely go beyond the oscillations of the interval conceded in practical music, and thus assign every conductor the task of forming a judgement on the subject."]

To the many dilemmas of the conductor executing a piece of music, Planck had added also that of the tuning: but he did not offer us any solution. He left the final word to the composer, and when that was not possible, *force majeure*, he referred the matter to the (subjective?) artistic effect. "Denn die Kunst findet ihre Begründung in sich selbst, und kein theoretisches System der Musik, wäre es noch so logisch begründet und konsequent durchgeführt, ist im Stande, alle Forderungen der zugleich mit dem menschlichen Geiste ewig wechselnden Kunst ein für alle Mal zu fixiren. In dieser Beziehung hat das natürliche System durchaus keinen Vorzug vor dem temperirten, . . ." ["Because art finds its foundation in itself, and no theoretical system of music, even if founded logically, and developed consequently, is in a position to fix once and for all every request of art, which changes eternally at the same time as the human spirit. In this connection, the "natural" system has no advantage at all over the "tempered" one, . . ."].

In the particular case of the choral piece composed by Schütz, however, our German scientist believed that the "natural" intonation was more consonant ". . . mit dem Eindruck der himmlischen Ruhe und Harmonie, . . ." [". . . with the expression of peace and harmony, . . ."] and that relaxing the tension ". . . ein sanftes Hinabgleiten in eine neue Welt von Tönen, hier am besten zusagt." [". . . offers a better promise here of a soft descent into a new world of notes."] But the piece by Schütz was also based ". . . auf dem Gegensatz zwischen der geschilderten selig Ruhe und den folgenden bewegten Kampfesstellen: . . ." [". . . on the contrast between blessed peace depicted and the following animated joining of battle: . . ."]; thus it could not all be sung "naturally". "Wenn der natürlichen Stimmung öfsters nachgesagt wird, sie sei farblos, leer, nichtssagend, . . ." ["If the "natural" intonation is often repeated, it appears to be colourless, empty, insignificant, . . ."].[107]

[107] Planck 1893, pp. 434–439.

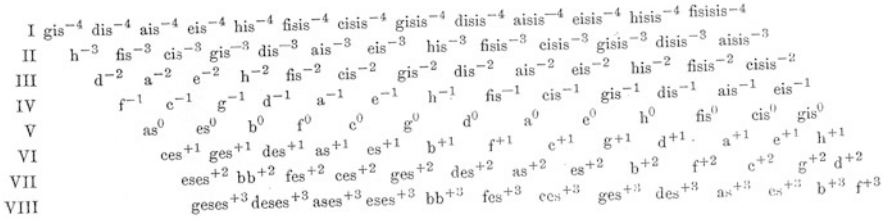
I gis^{-4} dis^{-4} ais^{-4} eis^{-4} his^{-4} fisis^{-4} cisis^{-4} gisis^{-4} disis^{-4} aisis^{-4} eisis^{-4} hisis^{-4} fisisis^{-4}
II h^{-3} fis^{-3} cis^{-3} gis^{-3} dis^{-3} ais^{-3} eis^{-3} his^{-3} fisis^{-3} cisis^{-3} gisis^{-3} disis^{-3} aisis^{-3}
III d^{-2} a^{-2} e^{-2} h^{-2} fis^{-2} cis^{-2} gis^{-2} dis^{-2} ais^{-2} eis^{-2} his^{-2} fisis^{-2} cisis^{-2}
IV f^{-1} c^{-1} g^{-1} d^{-1} a^{-1} e^{-1} h^{-1} fis^{-1} cis^{-1} gis^{-1} dis^{-1} ais^{-1} eis^{-1}
V as^{0} es^{0} b^{0} f^{0} c^{0} g^{0} d^{0} a^{0} e^{0} h^{0} fis^{0} cis^{0} gis^{0}
VI ces^{+1} ges^{+1} des^{+1} as^{+1} es^{+1} b^{+1} f^{+1} c^{+1} g^{+1} d^{+1} a^{+1} e^{+1} h^{+1}
VII eses^{+2} bb^{+2} fes^{+2} ces^{+2} ges^{+2} des^{+2} as^{+2} es^{+2} b^{+2} f^{+2} c^{+2} g^{+2} d^{+2}
VIII geses^{+3} deses^{+3} ases^{+3} eses^{+3} bb^{+3} fes^{+3} ces^{+3} ges^{+3} des^{+3} as^{+3} es^{+3} b^{+3} f^{+3}

Fig. 12.6 How Planck tuned the notes on the experimental harmonium (Planck, "Harmonium", 1958, Band I s. 436)

According to Planck, the main results of his research would be the following. The event that everyone tuned and intoned in the "tempered" manner depended on the ear's adaptation to it. In certain cases, however, it might be better to intone in the "natural" manner, to obtain particular aesthetic effects. The future of the "natural" system would depend on the birth, in future, of a genius who knew how to compose better in that tuning than in the other way: he alone would be capable of solving the dilemmas.[108] Was his main dilemma situated between the pleasure of listening to or making music, and his profession of theoretical physicist, pupil of Helmholtz, who had made the scale of Zarlino "natural" with his harmonics and his beats? Unfortunately, this German physicist would have to face up to many other dilemmas, even more painful ones, in his life and in his profession.[109]

In collecting a large number of his works, Max von Laue (1879–1960) excluded the previous essay on music and included only two short pages, as a report on the meeting of the above-mentioned Society of Physics. They showed the pattern of the notes that could be played on the large experimental harmonium (Fig. 12.6).
The instrument covered four and a half octaves, each divided into 104 note. To facilitate its use, its keys were coloured green, blue, white and red. With these, it was possible to distinguish the syntonic *comma* $\frac{81}{80}$ from the approximations of the Pythagorean *comma* $\frac{74}{73}$ and of the *schisma* $\frac{887}{886}$. Max Planck "... besprach sodann einige beim Studium des Instruments gesammelte Erfahrungen von musikalischer Bedeutung, mit deren weiterer Ausarbeitung er gegenwärtig noch beschäftigt ist, und belegte dieselben durch verschiedene Beispiele." ["... then illustrated some experiments with a musical significance collected while studying the instrument, in the further elaboration of which he is still today involved, and which he documented with different examples."][110]

When our German theoretical physics professor was called to Berlin in 1889, after occupying chairs at Munich and Kiel, he was to find the large coloured harmonium, and he started to play it. From the concise biographical details written

[108]Planck 1893, pp. 439–440.

[109]Heilbron 1986.

[110]Planck 1958, B. I, p. 436.

by him and in his meetings with pupils and colleagues, music emerged every now and then. Max von Laue related: "Er war von Kind an begeisterter Musiker, spielte vortrefflich Klavier und sang gelegentlich bei privaten Geselligkeiten. Deswegen und sicher auch unter dem Einfluß von H. v. Helmholtz, dessen letzte Jahre er in Berlin miterlebte, beschäftigten ihn die physikalisch-physiologischen Grundlagen der Musik. Davon zeugt die Arbeit (. . .) über ein Harmonium in "natürlicher" d. h. in nicht temperierter Stimmung, welche um 1893 erbaut und von Planck immer wieder einmal gespielt wurde - bis es dann den Flammen des zweiten Weltkrieges zum Opfer fiel." ["From his childhood on, he was an enthusiastic musician; he played the piano extremely well, and sang in a group on private occasions. Consequently, sure under the influence of H. v. Helmholtz, with whom he had consorted during his final years at Berlin, he was involved in the physical-physiological foundations of music. This gave rise to his work (. . .) on a large harmonium tuned in the "natural", that is to say, the non-tempered manner, which was constructed in about 1893 and was once played again by Planck, until it fell a victim in the flames of the Second World War."][111]

But this physicist, famous for diffracting X-rays through crystals, must have been less interested in the art of sounds. When introducing the collection dedicated to his master, Planck, he forgot or failed to remember even in a tiny note, that the inventor of the *quanta* had actually published the long, rich article from which we started, even in a journal of musicology. On the contrary, Planck mentioned, in his "Wissenschaftliche Selbstbiografie" ["Scientific autobiography"], undated and published posthumously, his meeting "Zufall" ["by chance"] at Berlin with the harmonium of Carl Eitz. ". . . ich hatte die Aufgabe, an diesem Instrument Studien über die natürliche Stimmung zu machen. Das tat ich denn auch mit großem Interesse, besonders in bezug auf die Frage nach der Rolle, welche die natürliche Stimmung in unserer modernen, von Instrumentalbegleitung freien Vokalmusik spielt. Dabei kam ich zu dem mir einigermaßen unvermuteten Ergebnis, daß unser Ohr die temperierte Stimmung unter allen Umständen der natürlichen Stimmung vorzieht. Sogar in einem harmonischen Durdreiklang klingt die natürliche Terz gegenüber der temperierten Terz matt und ausdruckslos. Ohne Zweifel ist diese Tatsache in letzter Linie auf jahre- und generationenlange Gewöhnung zurückzuführen. Denn vor Joh. Seb. Bach war die temperierte Stimmung überhaupt nicht allgemein bekannt." [". . . I received the task of carrying out, on this instrument, studies regarding the "natural" way of tuning. I subsequently performed them with great interest, referring particularly to the question of the role played by the "natural" method of tuning in our modern vocal music without any instrumental accompaniment. On this I arrived at the result, somewhat unexpected for me, that our ear prefers the "tempered" way of tuning to the "natural" one in all circumstances [just so!]. Even in a chord with the harmony in a major tonality, the "natural" third, compared to the "tempered" one, sounds feeble and without expression. Ultimately, without doubt, this fact is to be connected with the habit [of listening] which has lasted for years and generations.

[111] Planck 1958, B. I, p. V.

Because, above all before J. S. Bach, the "tempered" tuning was not generally known."][112]

To the great disappointment of Helmholtz, and as a result of playing the piano, and conducting domestic choirs, the rigorous Planck became convinced in the end that, "in all circumstances", the "tempered" tuning was the best one to make music. Had that task, which he took upon himself with such a pleasure, been assigned to him by the powerful Helmholtz? The latter had effectively taken a great interest in non-tempered harmoniums, especially a French one with 53 notes per octave.[113] It is therefore highly probable that the whole story of the coloured harmonium started with Helmholtz.

After becoming famous, unfortunately together with others for the fission of uranium and other elements, instead of as a promising tenor, Otto Hahn (1879–1968) relates his musical experiences together with his more mature colleague. "Planck liebte die häusliche Geselligkeit. In den Jahren vor dem ersten Weltkrieg versammelten sich in seinem Hause in Berlin-Grunewald alle 14 Tage eine Anzahl gesangsfreudiger jüngerer Damen und Herren, zu denen auch ich gehörte. Wir bildeten einen vierstimmigen Chor unter Plancks Leitung, sangen Brahms und anderes. Bei einem der Brahmsschen *Zigeunerlieder* durfte ich sogar eine kleine Solostelle singen. Planck redete mir damals zu, regelrecht Gesangsstunden zu nehmen; meine Atemtechnik sei schlect, der Tenor aber könnte gut werden. So begann ich im Juli 1914 mit Gesangsstunden; im August kam der Krieg. Mit dem Singen war es vorbei. Das Gesangskränzchen unter Planck trat nach dem Kriege nicht mehr zusammen, aber die persönlichen Einladungen bei ihm wurden wieder aufgenommen." ["Planck loved small domestic meetings. In the years preceding the First World War, in his house at Berlin Grunewald there was a meeting every fortnight for a certain number of young men and women who were lovers of singing, of whom I was a member. We formed a choir of four voices under the direction of Planck, and we sang Brahms and others. In one of our Brahmsian *Zigeunerlieder [Gypsy songs]* I could even sing a small part as a soloist. Planck once advised me to take regular singing lessons; though my breathing technique was bad, as a tenor I could become good. Thus, in July 1914, I started the singing lessons; in August the war broke out. The singing disappeared. The singing group under Planck's leadership never met again after the war, though his personal invitations were again accepted."]

Thus we have found another crime attributable to the war. Besides Brahms (1833–1897), our German theoretical physicist loved Schubert (1797–1828) and Schumann (1810–1856); by Bach, he preferred the *Passion according to St. Matthew*.[114] The music of Richard Wagner was clearly at the antipodes; we may say that he had classic romantic tastes.

[112]Planck 1958, B. III, pp. 383–384.

[113]Serravezza 1996, pp. 79–80.

[114]Planck 1958, B. III, p. 421. Heilbron 1986, p. 34.

In the abundant secondary literature dedicated to Planck by physics historians, there is little to be found about what interests us here. His music has been mentioned as a "hobby". We read of him that he had only conducted experiments as a student, subsequently becoming one of the first theoretical physicist, whereas here we have discovered that he had undoubtedly experimented with sound on the organ. Thus this little that was written about him, on closer inspection, appears to be unreliable. Indeed, to Helmholtz is even attributed a preference for the "tempered" scale, while Planck was said to prefer the "natural" one, together with other imprecisions.[115]

Though often with debatable results, every attention was reserved to Planck's most famous inventions. Among these stands out the calculation, by means of discrete elements, of energy exchanges of the "bloß Hohlraum" ["black body"]: elements since then known as Planck's *quanta*, with the famous relative constant h. A new epoch began with a controversial epistemological revolution, during which a conspicuous part of the scientific community renounced the principle of causality expressed by differential equations, preferring laws of nature which in some cases were probabilistic: however Planck remained faithful to a sort of Newtonian-Kantian causal vision. He liked more to make a clear-cut separation between the world of "nature", seen as unitary, and the human one: he did not admit that man could influence in any way the universal, eternal laws of the former. Thus he never wanted to accept, as other inventors of the newborn quantum mechanics subsequently came to sustain, that by making experiments, the scholar could influence their results. We found the same concern above in his experiments with the family choir to verify the differences between "natural" or "tempered" singing.

This German physicist exchanged letters also with the humanist from Alsace, Albert Schweitzer (1875–1965): he was an organist, a scholar of Bach and a missionary doctor in Africa. The subjects included the most suitable philosophies to give a meaning to life. In view of their positive attitude, Schweitzer preferred the wise Chinese to the Indians. Planck commented that every philosophy must explain the meaning of life, without which no morals would be possible.[116]

Expressed by the *Da-sein* of German, where "being", "essence" and "existence" were united, the laws of "nature" invented by Planck in his opinion remained oscillating in the dualism between truth and error, which were above all transcendent, for him, a committed attender at his Lutheran parish church. But under the blows of Ernst Mach (1838–1916) their eternal truth had been transformed into a more modest ability to foresee the determined evolution of phenomena. Unfortunately the new quantum mechanics, which owed so much to him, was trying to subtract this last certainty from him, and was to rely on chance and on "Wahrscheinlichkeit" ["probability"] in the study of atomic phenomena. Physicists would have to be content to describe the results of experiments, and could not claim to assign to them the true essence of "nature".

[115] Kangro 1981, p. 7.

[116] Forman 2002. Heilbron 1986, p. 186.

Published at Leipzig in 1883, reprinted and translated many times, with one of the most famous, but also most hotly-discussed histories of mechanics, Mach provided good evidence of how the role of music in the sciences had been overlooked. To qualify his history as based on facts, he made reference to "... absurd fables, suitable to arouse only our diffidence, like the one about Pythagoras and the hammers in the smithy, which produced a harmonic interval proportional to their weight." Unfortunately, together with the bathwater, in this way they threw away the baby as well. In a page of Vitruvius Marcus Pollio, he appreciated the event that sound had been described as forming circular rings in the air, like the ripples in a pond, but Mach stopped here. Even when he dealt with Kepler, Mersenne or d' Alembert, he did not pay the slightest attention to their pages on harmony. The omission appears to be all the more significant, seeing that he had devoted himself to the study, both of the theory of Helmholtz, and to the problems of harmony.[117] In spite of all his acoustics, perhaps he could not reduce music appropriately to material sensations?

Famous as the co-founder of logical positivism, the Viennese philosopher had faith in sciences free from any metaphysics, derived only from sensible experiences, from mathematics and from the relative logic. The thesis obliged him to ignore everything that could throw doubt on it. As evident as music, yet this time inevitable, he had to find a way of justifying the religious and theological convictions of his best champions. Sometimes our man from Vienna even went so far as to deny the written text: "... In fact, Newton never mixed theology with questions of physics." We know, instead, that the English scientist had written about God even at the end of the *Principia*.[118] Then, according to him, scientists would gradually learn to keep their religious convictions separate from physics. Although it was written otherwise in his book, Mach and Planck could not find any agreement, either in philosophy or on the existence (or not) of atoms. On the one hand, we have met a theoretical physicist imbued with metaphysics, religion and music, the creator of a new theory, and on the other hand, a neo-positivist philosopher who simplified history at will for his own purposes. The reader may choose for himself which he prefers.[119]

Every now and then, Planck played his piano in a trio, or with the violinist Joseph Joachim. The latter and Brahms had once gone together to visit Helmholtz, who undoubtedly wanted to convert them to the "natural" tuning. Who knows whether the violinist told the theoretical physicist how the meeting really went: not according to the preceding version of Helmholtz.

> Einmal waren Joachim und ich bei Helmholtz, der uns seine Entdeckungen und die reinen Harmonien auf den von ihm erfundenen Instrumenten vorführte. Er behauptete, die Septime müsse etwas höher, die Terz tiefer klingen als gewöhnlich. Joachim, der ja ein sehr höflicher Mann ist, wollte erst einen ganz eigentümlichen Eindruck von der Intervallen empfangen haben und tat so, als ob er sie gerade so höre wie Helmholtz. Da sagte ich ihm, die Sache

[117] Serravezza 1996, pp. 65–66 and 206.

[118] See Sect. 10.2.

[119] Mach 1977, pp. 36–37, 451, 492 and passim.

> sei doch zu ernst, als daß auch hier die Höflichkeit entscheiden könne; ich hörte immer das Gegenteil von dem, was Helmholtz behauptete. Da gab er denn zu, daß es eigentlich auch bei ihm der Fall sei. Helmholtz hatte mehrere Klaviaturen. Ich machte ihn darauf aufmerksam, daß die Töne der zweiten viel schärfer klängen als die der andern; er mußte es zugeben. Er selbst ist eben in musikalischen Dingen ein entsetzlicher Dilettant. [Once Joachim and I [Brahms] were at the house of Helmholtz, who presented to us his discoveries and the 'pure' harmonies on instruments invented by him. He maintained that the seventh should sound a bit higher, and the third lower than usual. Joachim, who is undoubtedly a very polite person, must have received a highly particular impression from the intervals, and made semblance as if he could hear them in the same way as Helmholtz. Then I told him, however, that the question was too serious for the decision to be taken on the basis of politeness; I could always hear the opposite of what Helmholtz said. He [Joachim]then admitted that it was exactly the same for him. Helmholtz had several keyboards and I pointed out that the notes of the second one sounded much more acute than the others; he had to admit it. In musical matters, he really is a terrible *dilettante*.]

In his youth, Planck devoted himself a good deal to music. He composed *Lieder* and short pieces, and even an operetta: *Die Liebe im Walde [Love in the Wood]*. He sang female parts several times in other lyrical representations. At the beginning he was unsure whether to dedicate himself to music, but a "... großer Musiker ..." ["... greater musician ..."] discouraged him [perhaps Brahms?].[120]

Historians of physics have not generally ignored Planck's friendship with Albert Einstein (1879–1955) and the role he assumed in defence of the theory of relativity, even in difficult circumstances, or the painful events that he went through in Germany during the two World Wars.[121] We know that Planck sometimes accompanied Einstein's violin on the piano; on many scientific topics, the two physicists were often in agreement, though in politics they were not on the same sides.[122] In the following section, we shall deal with this other theoretical physicist, who was also born in Germany, but who would not have been at all happy to hear himself defined as a German. In his own, different way, he too assumed a role in our history.

> Far from all dogmatism, may we be left the free opportunity to investigate in every direction, both the atomistic one and the anti-atomistic one!
> The first human utensil was the club. Even the orang-utan uses it, for a purpose to which, still today, when we consider ourselves so far superior to this animal, a large part of the human inventive spirit and technical subtlety is dedicated. What name can I give to this purpose? Pacifists call it murder; soldiers call it putting the high value of life at stake for the most noble possessions of humanity, for honour, for freedom and for the homeland.
>
> Ludwig Boltzmann

> Les lois, considérées comme existant en dehors de l'esprit qui les crée ou qui les observe, sont-elles immuables en soi? La question est insoluble.

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[120] Kalbeck 1976, B. I, p. 279. Cf. Serravezza 1996, p. 80. Grassmann 1973, pp. 92–94, 96 and passim.

[121] Kangro 1981. Heilbron 1986, pp. 54, 58, 66 and passim. Forman 2002. Giovannini & Tonietti 1968. Tonietti 1976. Tonietti 1980.

[122] Grassmann 1973, p. 96. Heilbron 1986, p. 34. Tonietti 2005.

[Are laws, considered as existing outside the mind that creates them or observes them, unchanging in themselves? The question is insoluble.]

Henri Poincaré

. . . it is possible to describe, or reproduce conceptually in thought only that which is uniform and regular, seeing that description presupposes the use of nouns that are comprehensible only if they refer to constant elements.
Chinese writing offers an example of writing in ideograms, the meaning of which can be understood by various peoples, even though they pronounce them in a phonetically different manner. A more simple system of signs might make its use universal. . . . *reading* would coincide with *understanding*. Our children often read things that they do not understand; the Chinese, on the contrary, can only read what they understand.
In general, with every observation and every opinion, both the environment and the observer enter into the [scientific] expression . . . Worrying about a physics that will be valid in every age, for all peoples, including Martians, seems extremely premature to me, almost comic, even, while many physical matters of every day press upon us.

Ernst Mach

. . . philosophical systems follow one another, and the latest one is not necessarily the best one . . ., there is only one single science, and this is binding for all humanity . . . it marches on, although it will never reach, and can never reach its ideal goal.
If natural sciences could not predict, they would be nothing more than an amusement for scholars.
Besides the many horrible aspects [of war], there also exist many that are unexpectedly large and beautiful, every good and noble thing rises up high. What a glorious age we are living in! It is a great feeling to be able to call oneself a German.
. . . [we should have] faith, at least faith in a certain reality external to us.
In the end, the highest tribunal stands in the conscience and in the conviction of everybody -. . .- and before all science first of all there is faith. For me, it is the faith in a perfect legality for everything that happens.
It is the firm, continuous, never-relaxing struggle against scepticism and dogmatism, against unbelief and superstition that religion and science conduct together.

Max Planck

. . . military power and science are the twin towers of Germany's greatness.

Adolf von Harnack

[*Die Lehre von den Tonempfindungen* by Helmholtz] is a monument of truth based on an enormous error.

Jules Combarieu

. . . if only we knew what kind of dance of atoms of carbon, hydrogen, nitrogen, oxygen, phosphorus and other elements corresponds to the pleasure of listening to music.

Emile Du Bois-Reymond

What a contrast between the unrefined sonorous sensations of a savage, whose maximum musical enjoyment is the rhythmic repetition of a sound, or at most of the sound of the drum or a penny whistle, and the musical intelligence of a cultured civilised man, whose ear delights in the classical harmonies of an opera by Mozart or a symphony by Beethoven! And what great contrast between the civilised man and the super-civilised advocates of Wagner's music of the future, who only find the true purpose of their aesthetic enjoyment of sound in complicated disharmonies! These musical men of the future look at the sonorous sensations of us, ordinary civilised men, with the same commiseration that we reserve for the unrefined music of savages, for the monotonous sonorities of a tam-tam or a strident reed-pipe.

Ernst Haeckel

> A distinction must be made between the reliability of astrology and that of astrologists. The hidden sciences are not those that nobody wants to teach, or it is not legitimate to teach, but those that it is not possible to teach. A similar knowledge cannot be learnt, either: it is decisively innate.
>
> Arnold Schönberg.

12.4 Arnold Schönberg and Albert Einstein

Before describing other characters of our story, I will briefly say something about the attitude, not followed here, which is the opposite of ours. In environments which, to cut a long story short, we might call positivist, where it is therefore claimed that the solution to every problem could be found in the development of the sciences, attempts were consequently made to make music scientific, as well: "Musikwissenschaft" ["Science for music"]. It, among other things, provided the title for the journal where Planck's article was published.

What were the viewpoints of this different style regarding the questions that we have dealt with so far? Helmholtz's book and other writings lent themselves perfectly for comments and for use; consequently, they were used with success. Leading theoretical musicologists like Hugo Riemann (1849–1919) and Carl Stumpf (1848–1936) had to come to terms with him. The latter of these illustrated their weaknesses and explained consonances rather as a "Verschmelzung" ["fusion"] of notes which could only be judged by the ear: "solvitur audiendo" ["it is solved by listening"]. Thus, listening to Siamese music, a scale could even be found, in which the octave was divided into seven levels, which were perceived equal, as the 12 of the equable temperament. The event that they could be perceived as equal, though they were intervals that were different in pitch, was explained by studies conducted by physiologists and psychophysicists like Ernst Heinrich Weber (1795–1878) and Gustav Theodor Fechner (1801–1887). While the stimuli of sound and of light grow in a geometrical progression, the corresponding sensations experienced by the ears and the eyes follow an arithmetic progression: we find here again the inventions of the Pythagoreans, united by logarithms. Even in this Siamese scale some pleasant intervals could be heard, although they were in contrast with Western theories for consonances. The physical-mathematical psycho-physiology of listening thus did not exclude at all the equable temperament in favour of intervals considered by some to be "natural".[123]

In the conscience, furthermore, feeling time flowing from note to note did not coincide at all with the time of a mechanical clock, as was also maintained by the

[123] Serravezza 1996, pp. 56–57, 315 and passim. In contrast with Helmholtz, Stumpf maintained that in consonances, sounds fused together, whereas in dissonances, they did not. He confirmed his theory of "fusion" by studying the judgement of listeners directly. For him, basically, the phenomenon of music manifested that psychological unity which could not be broken down into mechanical elements without losing its main characteristics. Green & Butler 2004.

philosopher Edmund Husserl (1859–1938).[124] Stumpf wrote: "... [the] aesthetologists of music ... cannot decide to admit the possibility and the existence of a non-harmonic music, not based on triads. When Riemann affirms that 'a harmonic concept of the relationships between sounds corresponding to ours' has 'always been immanent to every experience of listening to music', he expresses an opinion that is widespread in most of literature. And yet today, this formulation can no longer be sustained in the presence of ethnomusicological research rigorously founded on phonographic recordings." Hugo Riemann reacted: "... neither acoustics, nor the physiology or psychology of sound, but only a 'theory of sonorous representations' can give us the key to the deepest essence of music."[125]

And yet, for this Hugo Riemann, musical subjects, whether active as composers or passive as listeners, would be bound to even stricter and more absolute laws and codes than the others based on physiology and conscience. For the famous German musicologist, the tonal triad of the chord with thirds superimposed would remain the basis of every possible harmony, above and beyond space, time, cultures and historical epochs. "... I have now been working for twenty-five years with all my energies constructing the theory of the natural legality of harmonic successions, ..." and again "... the history of musical theory ... shows the gradual discovery of all the laws nowadays legitimately in force." "It is totally wrong to doubt the universal validity of the foundations of our musical system basing ourselves on the study of the musical practice of races far from the European culture."[126]

Hugo Riemann had gone so far as to think of music independent from the ear and from instruments to play it. He had substituted, in place of the astral numbers of the Pythagoreans, equally transcendent, ideal musical notes, which existed only in the mind: in *his* mind? In the initial period of his studies, referring to Helmholtz, he had even thought that he could hear the lower harmonics of a note, because they were indispensable to sustain his "dualist" theory of major and minor chords. Our abilities to represent notes to ourselves would cancel the differences in *commas* between the different intonations, leading us "spontaneously" back to "natural" ratios. Thus the "pure" way of tuning would be the "true" one, but in practice it would be not necessary at all, thanks to this ability. We know that this hypothesis had already been advanced by Euler.[127] With his highly successful *Vom Musikalisch-Schönen [On the Musical Beautiful]*, also Eduard Hanslick was to comment on Helmholtz, to indicate its limits, with respect to his own "formalistic" conception: music expressed only music, and nothing else, disputing polemically above all with the expressive contents claimed by the Wagnerians.[128]

Subsequently, in the twentieth century, our history acquired the dramatic tones of the period in which the Great War took place, lasting for more than 30 years, 1914–

[124] See below.

[125] Serravezza 1996, pp. 74, 76.

[126] Serravezza 1996, pp. 90, 96 and passim.

[127] Serravezza 1996, pp. 83–85 and passim. See above, Sect. 11.1.

[128] Serravezza 1996, p. 110 and passim.

1945. It changed many things, including the mathematical sciences and music, together with the conflicts we have been seen to mark their relationship. It is significant that the most representative composer and the most famous theoretical physicist should have known each other and written to each other: on January 1st 1925 actually, Arnold Schönberg wrote a letter to Albert Einstein.[129]

After initial compositions, where the influence was felt, above all, of Wagner, together with a hint of Brahms, this musician started to abandon the traditional, more widespread "tonal" patterns for other styles. First, he invented music called "atonal" and in the end, the one called the "way of composing with twelve notes" or "serial composition" (currently and popularly better known as "dodecaphony"). While he was still at Vienna, where he was born, we may easily imagine the hostile environment of the public, the conservatoire and critics, who were not favourable to him, varying between Hugo Riemann and Hanslick. However, this first problem grew, with the addition of others, equally serious, which emerged clearly from the same letter.

> Der Wunsch, mich dort verschwinden zu lassen, meine Existenz zu negieren, ist so gross, wie die Zahl derjenigen, die ihr Brot bestenfalls in Tränen ässen, wenn eine von der Tonalität entbundene Musik sich als lebensfähig erwiese. Einmal diese Nabelschnur durchschnitten, die heute weniger die Musik, als die Musiker verbindet (und nährt, wie nicht verschwiegen sein soll), und die deutsche Musikwissenschaft, die sich bis heute ein 'Mittel-Ding' zwischen Musik und Wissenschaft dünkt, ist nur mehr Wissenschaft, was sie gewiss kaum war. Darum die hitzige Besorgtheit. Während ich aber nun im Ausland mindenstens als der führende deutsche Musiker gelte, ist man unbegreiflicherweise in Deutschland gerne bereit auf die Vorherrschaft in der Musik zu verzichten, wenn man nur damit verhindert, dass sie an meinen Name anknüpft. Darin, in dem Hass gegen mich, sind Juden und die zuständigen Hakenkreuzler eines Sinnes. [The desire to make me disappear, to deny my existence, is equally as strong as the number of those who, in the best of cases, season their bread with tears when a music freed from tonality shows that it is capable of living. Capable of living, once we have cut this umbilical cord [with tonality], which today binds less the music than the musicist (and which nourishes him, as we must not hide); capable of living, once the German *Musikwissenschaft*, which till today has considered itself something intermediate between music and science, has become only science, which, without doubt, it scarcely was. Hence derives the most acute concern. But while now, at least abroad, I am considered to be the leading German musician, in Germany, incomprehensibly, they are ready and willing to renounce their leadership in music, if they can thus prevent it being linked with my name. In this hatred against me, Jews and *Hakenkreuz* are characterised by the same idea.]

If the self-defined science for music claimed to dictate the rules of composition, then Schönberg denied it the status of science, and claimed the right to move out of the "tonal" pattern. Of Jewish descent, like many others, he had tried to assimilate himself into the context where he lived. Above all with the defeat of 1918, he felt less Viennese, and more German, also because his music was part of that cultural tradition from Bach down to Beethoven and Wagner. But in the face of the first episodes of anti-Semitism, after he personally had experienced it in 1921, it became increasingly difficult to pass off either as a good "pure" German, or as a faithful Jew,

[129] Tonietti 1997; Tonietti 2004.

which he had never been. The double link had been broken; the balance between life and art no longer appeared possible.

It was curious that our musician could be hated both by Jews and by Nazis wearing the swastika. The former rejected him because he felt that he was a member of the great German culture and had even been converted to Lutheranism. The latter attacked him because they considered him to be co-responsible for the defeat, and irremediably extraneous, in spite of all his efforts. Furthermore, Schönberg had just refused to sign an appeal which would have confined him to the ghetto of Jewish composers. Nor could he adhere to the newborn Zionist movement, which tried to transport Jews back to Palestine. In the end, the purpose of the letter became the request to meet Einstein in order to explain his position, and maybe to propose his solution to all these problems. His prophetic impulses were to find expression in the drama *Der biblische Weg [The Biblical Way]*, which contained his proposal about how to set up a Jewish state, and in the work *Moses und Aron [Moses and Aaron]*.

Anyway, Schönberg tried to explain that truly Jewish artistic music did not exist, even though perhaps the Jewish spirit had contributed to Western music. "Die Kunst der Niederländer einerseits erinnert in vielem an das was man von Talmud und Kabbala weiss und anderseits haben wir in der zum Teil von Juden durchsetzten Zigeunermusik das Gegenstück zu dieser auf wissenschaftlichen und okkulten Erkenntnissen beruhenden Hirn-Kunst. Davon abgesehen (denn es ist eine Hypothese für die Beweise nicht leicht zu erbringen sein dürften) giebt es nur deutschte, italienische, französische, etc. Musik, die von Juden geschrieben ist und daher gewiss jüdische Züge besitzt." ["The art of the Flemish, on the one hand, recalls in many aspects what is known of the Talmud and the Kabbala, and on the other, we have, in gypsy music, partly spread by Jews, the opposite of this cerebral art, which rests on scientific and hidden knowledge. Apart from this, (because it is a hypothesis, the demonstration of which might not be easy to present), there is only a German, Italian, French, etc. music, which is written by Jews, and thus possesses certain Jewish characteristics."][130]

In our history, it should be underlined that, in reconstructing a genealogy for is own musical style, this "German" composer also appealed to the sciences, or rather, to those that he considered to be such. Thus he called this tradition of his: "cerebral art that rests on scientific and hidden knowledge". It is thus clear why Einstein did not reply to him, either. In this period, they had opposite ideas on almost all the important problems. However, their relations were to change, partly, only during the exile of both of them in the United States of America. Schönberg had tried to take part in the Great War, albeit without too much success; Einstein was notoriously a self-declared pacifist. Schönberg considered himself to be a good German; Einstein had wisely kept his Swiss passport. Both stayed outside practising Jewish families. But the former revealed a religious transcendence typical of heretical mysticism; he was nostalgic about the heroic Biblical times when the Jewish people fought for their "religious and national independence". The latter helped lay Zionism of

[130]Tonietti 1997, pp. 1–2. Tonietti 2004, pp. 1329–1330.

the socialist kind set up in the *kibbutz*, which aspired to a pacific co-existence with the Arabs; he once declared that he was close to the philosopher Baruch Spinoza (1632–1677), the heretical Jew, famous for positing a God who was immanent in the world.

And music? Or sciences? Did these divide them or bring them together? We already know that the theoretical physicist loved to play the violin: he knew, and also consorted with some musicians, too. Schönberg also arrived in Berlin in 1926 to hold a composition class for the Academy of Arts, which had remained vacant as a result of the death of Ferruccio Busoni (1866–1924). May he have held a meeting sooner or later with the theoretical physicist, in spite of the first unsuccessful attempt? Hans Heinz Stuckenschmidt (1901–1988), who frequented the entourage of Schönberg in that period, sustained, when he was 85 years old, that there had been direct personal contact. Unfortunately, the biographer of our musician did not remember the circumstances any longer. The famous violinist Fritz Kreisler (1875–1962) and the equally famous pianist Arthur Schnabel (1882–1951) were two of their common acquaintances. It would appear to be likely, therefore, that the meeting really took place. The favourite musicians of Einstein seem to have been Antonio Vivaldi (1677–1741), J. S. Bach, Mozart and Schubert. He did not like Beethoven very much, because he considered him too dramatic. He listened to Wagner with a sense of disgust. For him, the music of Richard Strauss (1864–1949) expressed only external effects. Contemporary composers aroused little interest in him. However he had a great consideration for the (Jewish?) composer, Ernest Bloch (1880–1959).

The only musical point of contact between Einstein and Schönberg appeared to be J. S. Bach, perhaps because this musician revealed several aspects: monodic and polyphonic, melodic and harmonic, for the violin and for the keyboard, continuous and discrete, Mediterranean and Nordic, passionate and cerebral. Partly joking, our artist considered Bach to be "the first musician with twelve notes", because fugue n^o 24 in $B\flat$ minor of the first book from *The well-tempered harpsichord* began with all 12 notes. The idea of music that Einstein had did not seem to agree very much with that of Schönberg . Probably the physicist would not have accepted the distinction between gypsy music (spread by the Jews) and the music of the brain (as hidden science), or perhaps he preferred the spontaneous rhythmic liveliness of the gypsies. Dialogue between the two did not seem to be easy on this level, either.

During their exile in the United States, they undoubtedly met at least a couple of times. The University of Princeton, where Einstein was working at the Institute for Advanced Studies, invited our musician to hold a lecture on 12-note composition. The secretary of the physicist, Helene Dukas, remembered that on that March 6th, 1934, among those who listened to him, there were also Einstein and his wife. She also remembered the contemptuous comment expressed by Einstein at the end of the talk: that "the idea of the twelve notes was *'verrückt'* [crazy]". As this corresponded to the musical tastes of Einstein we can take it as a faithful account, failing proof to the contrary. Furthermore, on April 1st of the same year, the pianist Leopold Godowsky (1870–1938) organised a concert in favour of Jewish and Zionist associations, to support the settlement of German Jewish children in Palestine. It was

held under the auspices of Albert Einstein as a "tribute of music to science". A photo is extant where the physicist appears arm in arm with Schönberg and Godowsky.

In spite of their irreconcilable differences, their common exile in the United States and the Jewish cause which became increasingly dramatic, brought Schönberg closer to Einstein. The last letters exchanged between the two are interesting, because music and sciences necessarily emerged from them. Thus, on August 10th, 1938, the musician asked the theoretical physicist to help a friend of his, the philosopher and violinist Oscar Adler (1875–1955). As a result of the *Anschluss*, that is to say, the forced annexation of Austria to the Germany of Adolf Hitler (1889–1945), he had to escape from Vienna.

Schönberg described him in the following terms: "Er ist der Hauptsache nach Philosoph und er hat als solcher eine Reihe Bücher geschrieben über verschiedene Gegenstände: Aesthetik, Musiktheorie, etc. welche meines Wissens Manuskript geblieben sind. Veröffentlicht hat er bis jetzt drei Bände (der 4te ist im Erscheinen) eines siebenbändigen Werkes: 'Einführung in die Astrologie als Geheimwissenschaft'. ... Und ich kenne ein Buch von ihm über musikalische Theorien, das allein ausreicht, meine Eindrücke (und hier bin ich kompetent - mehr als auf meinem beruflichen Gebiet) zu bestätigen. Auf alle Fälle weiss ich dass das Buch einen wissenschaftlichen Wert - zumindest - dadurch haben muss, dass es eine Gesamtdarstellung in wissenschaftlicher Form gibt, die, wie Dr. Adler sagt, 'seines Wissens noch nicht unternommen wurde'. Es fasst also sicherlich Unbekanntes, Wenig-Bekanntes aus verschollenen Quellen zusammen mit Einigem-Neuen." ["Considering the most important thing, he is a philosopher, and as such, he has written a series of books on various subjects: aesthetics, musical theory, etc., which, to the best of my knowledge, have remained in manuscript form. So far he has published three volumes (the fourth is above to come out) of a work composed of seven volumes: *Introduction To Astrology As a Hidden Science*. ... I know a book of his about musical theories which is enough, by itself, to form my impressions (and I am more competent in this than in my professional fields). In any case, I know that the book must have -at least- a scientific value, because it provides a general representation in a scientific form, which, as Dr. Adler says, 'has not been undertaken yet to his knowledge'. It thus undoubtedly contains things that are unknown, things that are little known from missing sources, together with some things that are new."][131]

For him, he went so far as to request a chair of "Astrology", "History of astrology", "Scientific bases of astrology", or a cycle of lectures in universities. Einstein replied immediately on August 28th. "Frau Goldner, die Schwiegermutter von Eugen Ormandy, hat mir das Werk von Herrn Adler gesandt und ich habe in demselben mit grossem Interesse gelesen. Es ist sehr gut geschrieben und gewiss wertvoll für diejenigen welche sich für den Gedankenkreis der Astrologie interessieren. Für mich ist es natürlich ganz unmöglich, mich für die in dem Werk vertretenen Ansichten einzusetzten, da ich diese für ganz abwegig halte. Ich glaube

[131] Tonietti 1997, p. 12. Tonietti 2004, pp. 1351–1352.

sogar, dass das Werk, gerade weil es so gut geschrieben ist, für unreife Geister eine wirkliche Gefahr bedeutet." ["Frau Goldner, mother-in-law of Eugene Ormandy [1899–1985], sent me the work of Mr. Adler, and I read it with great interest. It is very well written, and undoubtedly of great value for those who are interested in the various ideas about astrology. For me, of course, it is quite impossible to commit myself as regards the opinions sustained in the work, because I consider them totally misleading. Likewise, I believe that precisely because the work is so well written, it represents a real danger for ingenuous spirits."][132]

Oscar Adler was the brother of the better-known philosopher, sociologist and Marxist politician, Max Adler (1873–1937). By profession, he was a dermatologist, but he was attracted by philosophical and esoteric studies, which he cultivated for all his life. He played the violin so excellently that he became professionally involved in musical activities. He also loved teaching. A contemporary of Schönberg , he had made a significant contribution to the formation of his basic culture. They had played together in a small youth orchestra, and our musician had learnt how to play the cello from him, but at the beginning with the fingering of the violin. Schönberg wrote that he was a debtor to him for his introduction to musical theory, and also for his interest in poetry and philosophy. After the Great War, Oscar Adler took part in the Verein für musikalische Privataufführungen [Union for private musical performances] founded by Schönberg at Vienna, playing his instrument on occasions.

Few people could have expected a different answer from Einstein. In any case, this time Einstein did not reject the request completely, and declared that he was ready to write a letter to help Adler. Only he did not want to get involved in astrology. Those people had the right to live, like everybody else, but earning a living by teaching at University appeared to be a completely different matter. In the twentieth century, a physicist, who had won the Nobel prize, could not have anything to do with astrology. The common attitude consisted in equating astrology to plain superstition. And yet, although this appeared at first sight to be the opinion of Einstein , some details deserve attention. We have to notice that he looked at Adler's book, *Introduction To Astrology As a Hidden Science*, and wrote that he had "read it with great interest". He judged it to be harmful for immature people only *afterwards*. We may wonder whether any other professor of physics (with the possible exception of Wolfgang Pauli)[133] would have behaved in the same way. It is likely that today, a similar book would not even get as far as the desk of a scientist.

It was not the first time, however, that Einstein allowed himself to be contaminated by debatable, unorthodox ideas. At Pasadena, in about 1930, when he met the writer Upton Sinclair (1878–1968), he was involved in telepathy experiments. Our renowned physicist even wrote a short preface for Sinclair's book, *Mental Radio*, where this psychic research was reported. The conclusion of the writer left no room for doubt: "TELEPATHY HAPPENS!". "If a man of the honesty, of the social

[132]Tonietti 1997, p. 13. Tonietti 2004, p. 1352.

[133]See below.

consciousness, and the critical power [of one] like Upton Sinclair writes a book on telepathy based on his own experiments, then such a book deserves the most serious interest of professional psychologists and of laymen interested in science". Perhaps these pages reveal a little of that curiosity for unusual phenomena and something of that taste for non-conformist attitudes for which our theoretical physicist was famous, in his life and in his science.

However, to use an expression of Schönberg, not all hidden sciences are equal. Perhaps Einstein could make a distinction between the single happening, of a simple, limited 'psychic consistence' and the grand synthesis of an ancient wisdom, religious and remote. We also have to think that the ideas of Einstein were closer to a manifest science, within reach of everybody, prepared to learn its symbols, rather than to an esoteric knowledge reserved to the elect of some religion.

The point of view expressed by Oscar Adler, on the contrary, appeared to be of this second kind. His books offered an esoteric image of science, with many links with music. Was this Schönberg's idea of science? Could it have been that of Einstein ? Adler wished to re-introduce the original wisdom that the ancients once knew and through which it was possible to grasp the sense of an incommensurable, unrepresentable universe. Only *one* science existed, the broken fragments of which had given rise to the single subsequent disciplines: coloured rays of a primordial light. He felt himself to be "only one note in the great symphony of the world". "Astrology was the theory of that indivisible cosmic connection of all the events". The exact sciences, those that Adler knew, had taken the opposite route. Hidden science for him meant not only a science to be maintained secret and to be revealed to the elect; it also meant, above all, a knowledge that budded in the mystery of the human soul.

Einstein, the passionate defender of causality, the constant, irreducible antagonist of every physical law that depended on the subject, could not appreciate these ideas. However, when Adler was elaborating his idea, the laws of physics underwent a crisis in the link between cause and effect. The new quantum mechanics, according to the orthodox interpretation of Göttingen-Copenhagen, opened up the way towards an indissoluble marriage between the subjective experience of the observer and the phenomena of the nature observed, by means of the principles of indeterminacy and complementarity.

When they were trying to classify those singular non-continuous spectra of atoms with luminous lines, which seemed to elude the classic laws, through formulas that were still mysterious and whole numbers, the physicists spoke of "Zahlenmysterium" ["Numerical mystery"] and "Atomistik" ["atom/mystique"]. In his *Atombau und Spektrallinien [Construction Of the Atom and Lines Of the Spectrum]*, Arnold Sommerfeld (1868–1951) wrote about the ". . . mysterious organ on which nature plays the music of the spectra", referring to atomic spheres. In one article, he repeated: ". . . a certain inclination exists on the part of modern physics towards a numerical mysticism of Pythagorean origin. The most successful researchers in the field of the theoretical analysis of spectra - . . . - were passionate supporters of a mysticism of numbers . . . If only Kepler had been able to feel the experience of the modern-day quantum theory!" In 1922, Richard von Mises (1883–

1953) went so far as to affirm that atomic physics had again taken up "the question of old alchemists"; "numerical harmonies, even numerical mysteries, play a role which recalls no less the ideas of the Pythagoreans than certain ideas of followers of the cabbala".[134]

Also Oscar Adler thought that mathematics played a central role in his consistent cosmos: it constructed the bridge between interiority and the external, between the microcosm and the macrocosm. Pentagons, regular polyhedra, whole numbers, everything revealed the hidden symbolic links between the earth and the skies. The earth and the whole universe became a unitary organism, the life of which was revealed in well-ordered numerical ratios. How was it possible to have a direct experience of that cosmic harmony? "Music is the immediate intimate experience of numbers", he wrote, paraphrasing Leibniz. It communicated the intimate solidarity of the whole universe; in this sense the ancients spoke of the music of the spheres.

Our scientific astrologist often quoted Wolfgang Goethe (1749–1832), but then he also appealed to Kepler's *Harmonices Mundi Libri Quinque*. And so on. As symbolic elements of cosmic knowledge, here numbers emerged, there music, here philosophy, there poetry. As the physiologist Emil Du Bois-Reymond had already done at the end of the nineteenth century, Oscar Adler placed his *Ignoramus-Ignorabimus* as a stronghold against mechanicism. In the Fifth lesson, he played with the numbers 7 and 12, connecting them with the notes of the musical scale and these with the planets. He again met with Kepler's *Harmonices mundi* and concluded in the apotheosis with the angels of Goethe's *Faust*, who alluded to the music of the world, cosmic numbers and other enigmas.

However, Adler seemed to be lacking the main characteristics of the *Harmonices*, and flattened the ideas of the famous astronomer into the habitual numerical divisions of the Pythagoreans. On the contrary, we know that Kepler made use of geometry.[135] By means of an original geometrical model, the astronomer had considered the "hard" major third as "masculine" and the "soft" minor third as "feminine". Kepler's theory seemed to confirm here that the surmounting of the major and minor tonalities, achieved by Schönberg by his compositions with 12 notes, also represented the surmounting of "masculine" and "feminine". Angelic beings were the protagonists of a story, *Seraphita* written by Honoré de Balzac (1799–1850), which inspired the composer for his *Jakobsleiter [Jacob's Ladder]*. Here we find a series of 12 notes assigned to the character of Der Auserwählte [The one who is chosen] probably the first row included by him in a score[136] (Fig. 12.7).

Einstein must have felt the lack of a clear criterion of demarcation between science and astrology particularly dangerous, as between the natural sciences and psychology. In this case the laws of physics would remain contaminated by the subject, opening up the way to that quantum mechanics he loved least. The presence of Kepler in the history of science appeared to be a problem for him, and not only

[134] Forman 2002, pp. 66–67.

[135] See above, Sect. 8.3.

[136] Schönberg 1917/1977, pp. 98–99, bars 360–363.
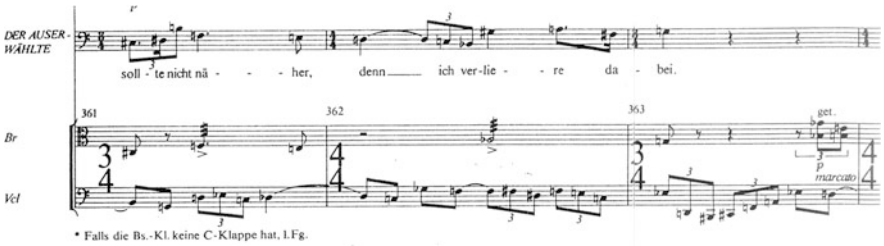
Fig. 12.7 Already in *Jacob's Ladder* Schönberg assigned a series to "The one who is chosen" (Schönberg 1977, bars 361–363, pp. 98–99)

for him. Neither could he cancel him from the evolution of his disciplne, nor could he purify him easily from his astrological sins. Einstein preferred Isaac Newton, in other words, the official amended, self-censured Newton of the *Mathematical Principles Of Natural Philosophy*, that is to say, the Newton without 'scholia' and without comments on *Revelation*.

When he commemorated Kepler in 1930, Einstein presented him as a person who observed the sky "serenely" and discovered the famous three laws. In the preface to a book on the *Life and Letters* of Kepler, the physicist wrote of is need of ". . . freeing himself to a large extent from the spiritual tradition in which he was born. . . . from an animistic, teleologically oriented manner of thinking in scientific research." If he had not succeeded in doing so, the successful astronomical results would not have come. Yet, reading those letters, even Einstein had to admit that astrology, ". . . the inner enemy, conquered and rendered innocuous, was not yet completely dead."[137] However, where had Kepler placed the Third Law, the most important one, because by making the planetary system united, he fixed the common cause of movement in the Sun, and gave a decisive impulse to Newton's gravity? We find this precisely in the *Harmonices mundi*, that is to say, in the book which described musical harmony in Book III, and which in Book IV dealt with the "metaphysical, psychological and astrological" influence of the heavenly bodies on the earth.[138]

Thus, unlike Einstein , I believe that Schönberg must have seen in Kepler his real model of a scientist. Besides, living in his period, Kepler had supported an existence rich in ups and downs, and troubles: from religious controversies to the 30 Years' War, to the trial against his mother for witchcraft. He had cast horoscopes for Wallenstein, and had not disdained to calculate them for himself. He appeared to be such an interesting character as to attract the attention of many people, leaving traces in literary, philosophical and musical works. One of his pupils and leading followers, Alban Berg (1885–1935), received *Harmonices mundi* as a Christmas present in 1920, and then wrote to Schönberg about it. With Oscar Adler, our composer was to remain in contact. On March 3rd, 1951, a few months before leaving an even more

[137] Baumgardt 1951, p. 13.

[138] See above, Chap. 8.3.

mutilated Earth, he thanked him for sending him *The Testament of Astrology*, which he considered "a truly admirable work, both for the style and for the ideas expressed there". In exchange, he sent him "psalms, prayers and other conversations with and about God". Thus we see that the common feelings between Schönberg and Oscar Adler lasted for their whole lifetime.

When Schönberg wrote to Einstein about cerebral music, which was not so different from "scientific and hidden knowledge", he must have been thinking of Oscar Adler and Johannes Kepler. Einstein did not believe that music influenced his own research work. For him however, both derived from the same inspiration and they complemented each other. Science used the language of logic, art that of intuition; the common element was a passionate devotion for what transcends the will and personal interests.[139] This may have represented a (weak) echo of that destiny which led also Schönberg to accomplish his mission to compose a transcendent, prophetic music.

Talking about contacts with the scientific world, our composer also knew a mathematician, Emanuel Lasker (1868–1941), who had made a non-negligible contribution to modern algebra, travelling around the German universities of Berlin, Göttingen and Erlangen. It was likely, however, that Schönberg (like everybody) saw in him above all the genius who was capable of holding the title of world chess champion for 25 years.

Einstein succeeded in arousing in Schönberg images, not only of science and Hebrewism, but even of the Cabbala. This term was used only very rarely by our composer-prophet, although his mysticism showed a certain relationship also with that ancient Biblical tradition. He included an allusion to the Cabbala in his *Harmonielehre [Manual of harmony]*. The Cabbala returned at the beginning of *Kol Nidre [All the Vows]* op. 39. Composed in 1938, commissioned by the Rabbi of Los Angeles, Jacob Sanderling, the text opened with the words: "Cabbala narrates a legend". Here, Schönberg , who had very probably chosen the words personally, presented the creation of light as a blaze: "God shattered that light into atoms. Myriads of sparks are hidden in our world, but not all of us perceive them".

Jewish orthodoxy did not accept this *Kol Nidre* by Schönberg. Various disputes followed. In this connection, together with a letter that was not sent, the composer underlined certain points. Among these, we read:

> II) If we were to use only the elements of the Biblical cantillation, then why not also exclude all modern musical instruments, and use only those of the Biblical period? What about the Parisian intonation? What about the well-tempered tonality?
> III) Music has its own laws and develops in accordance with them. If, in spite of this, a national flavour is desirable and possible – then let us leave it to them.
> But why keep people far away from progress in music, when they are allowed to benefit from the products of every technical, scientific, legal, social, medical and philosophical progress?
> IV) It is curious that the Jews are always the last to accept my conquests, both in Israel and in the rest of the world. They perform everything: Debussy [Claude, 1862–1918],

[139] Einstein 1980.

> Ravel [Maurice, 1875–1937], Hindemith [Paul,1895–1963], Strawinsky [Igor, 1882–1971], Schostakovitch [Dimitri, 1906–1975], Bartók [Béla, 1881–1945], etc., - but not me! In spite of all my contributions, they are my fiercest enemies!

The inventor of serial music, the composer who revolutionised most deeply the music of the twentieth century, had met with the physicist who changed the evolution of the sciences (even if not in the sense that he would have preferred). Under many aspects, they appeared to be so different that it is easy to understand why their relationship remained difficult and unilateral. The physics of Einstein was a science of space and time, whereas, on the contrary, the music of Schönberg was conceived as a mystic enterprise to leave space and time.

If our composer had known the other physicists of the new quantum mechanics, just those with whom Planck and Einstein had entered into a heated dispute, he might perhaps have found more acceptance and more success. Might the Viennese physicist Wolfgang Pauli (1900–1958) have felt a certain sympathy for his ideas? At least, he seemed to be the most appropriate scholar to discuss them: in a book written together with Carl Gustav Jung (1875–1961), *The Interpretation of Nature and the Psyche*, he wrote an essay on "The Influence of Archetypal Ideas on the Scientific Theories of Kepler". In presenting him and discussing long passages of his *Harmonices mundi*, Pauli made him an ancestor of modern physicists who reconstructed the unity and the symbolic totality of the cosmos, thanks to the principles of complementarity and indeterminacy of quantum mechanics. Thanks above all to his own particular and innovative mathematical formalism, the physics of the atom lost the possibility of representing microscopic objects in space and in time, and was released from the chains of causes and effects.[140] The prohibition here of depicting the electron was similar to that for the God of *Jacob's Ladder* and of *Moses and Aaron*, as conceived by Schönberg.

Our musician-prophet and the physicist of relativity thus proved to be almost irreconcilable in their professional attitudes, as also in politics and religion. Nevertheless, in the depths of their beings, both Schönberg and Einstein seemed to share a small fragment of common faith. In an article on Kepler, in the end, even Einstein had to express his admiration "... for the mysterious harmony of the nature in which we were born".[141] Each with his own means, both had tried to grasp a spark of it. It did not seem to be their fault if, in this world of dust, harmony, as always, appeared to be distant.[142]

What positions did Schönberg adopt on the relatively scientific questions of tuning musical instruments? He dealt with this in his *Harmonielehre [Manual of Harmony]*. In a book, our serial composer had read: "... he quotes Prof. Stumpf and sustains that it is plausible, and admissible, 'that also the ratios 4 : 7, 7 : 8, 5 : 7, etc. may gradually be raised up to the rank of consonances'. ... Oh, if I had imagined that a scholar with the credentials of Stumpf defended my same idea! I

[140] Jung & Pauli 1955.

[141] Einstein 1934, p. 179. Einstein 1991, p. 52.

[142] Tonietti 2004, Chap. 58.

am not acquainted with all these sources; the only one that I have at my disposal is reason. Thus the work proceeds very slowly, but it moves forwards. I was right, then, when I instinctively opposed the 'return to nature', surprised that a Debussy thought he could find nature by following the ways of art in the course already followed. . . . Onwards, towards nature! If I had a *slogan*, it might be this one; but I think that something even higher still than nature exists."

Schönberg, who was self-taught, sometimes agreed with musicologists, when he found his own ideas, and sometimes did not. ". . . those who are in search of theoretical truths *basing them on aesthetic judgements given, without controlling whether these aesthetic ideas and these judgements are exact*, reveal with their theories only one aspiration, the aspiration to obtain laboriously proofs of these judgements; they are devoid of any value. . . . I have dealt with this problem and I did not give an answer that was a pipe-dream, like [Hugo] Riemann, for example, who was proud that his theory succeeded in formulating rules with greater rigour and precision, without realising that for this very reason, it will be surpassed so much the more rapidly."[143]

When discussing dissonances, he wrote: ". . . our ear has not only been educated by natural conditions, but also by the conditions created by a system which in the meantime has become a second nature. Today, we cannot, or we can only gradually subtract ourselves from the force of this culture and this cultural product; and the reflection about nature may have a gnosiological value without necessarily producing direct artistic results." The passage between those dissonances and the consonances was gradual for him. Considering seventh chords, he expressed himself on the tonal order: ". . . which claims to be natural, and on the contrary, is disowned by the healthy ear of the natural, uncultured person: precisely because it is not a natural order, but artificial, because it is a product of civilisation."

Schönberg saw a dualism between major and minor modes in parallel with the masculine and feminine genders. He criticised those who [like Hugo Riemann] had rendered them absolute. ". . . it is heresy that these two modes are the only true natural foundation in music, the definitive one that is destined to last, and that they fully satisfy the will of nature. For me, things stand differently: *we have drawn close to the will of nature*, it is true, but we are still far from it; the angels, who are our higher nature, do not have any sex, and their spirit does not know any caprices."[144]

He retained that the function of harmonic sounds had been repudiated ". . . or at least put in doubt by science. . . . both the theory of harmonics and the interpretation that has been given of them are wrong." Yet our German-Jewish composer had not found anything better than introduce the 12-note scale by means of (natural?) harmonics. Thus he fluctuated between the sensible ear and his theoretical brain, between the artistic requirements of musicians and historical evolution. Some problems thus remained for him to solve. "If we can avoid these problems for now, we owe it almost exclusively to a compromise between the natural intervals

[143] Schönberg 1922/1963, pp. 494–495, 81–82, 512–513, and passim.

[144] Schönberg 1922/1963, pp. 59, 82, 116, 119, and passim.

and our inability to make use of them, that is to say, to that compromise which is called the 'tempered system', and represents an indefinite armistice. However, this reduction of natural ratios to more manageable ratio will not be able to resist evolution with the passing of time; and when the need is felt, the ear will have to deal with these problems. Then also our scale will resolve itself into a higher order, just as the Gregorian modes resolved themselves into the major and minor modes: . . . Perhaps this new subdivision of the octave will be non-tempered and will have little in common with ours."

Not only did the transpositions and the modulations need intervals that were all equal, but now also the style of composing with 12 notes required them; these could only be combined freely on this condition which was guaranteed by the equable temperament. Schönberg must have felt this crucial point to be so delicate that he put a heavy-handed note at the foot of the page, subject to modifications in subsequent editions. He composed it just during the First World War, while he was moving increasingly further away from the tonal system, to invent his method which placed all 12 notes on the same level, without worrying any more about tonics, dominants or consonances. "A music like that of today, for which the true nature of sound remains unknown, will appear to be imperfect to our descendants, as a music not differentiated even within the range of the octave might appear to be to us, . . ."

Some years later, he added. "Things will change, therefore, even though not so fast as some think, and the change will take place in a different manner, not as a result of external reasons, but due to internal causes, not, obviously, by imitation of given models, and not as a technical conquest, either: because it is not so much a material question as a spiritual question, and it will be the spirit that will lead it to completion. It seems that the recent fashion of contrasting European civilisations with those of the most ancient Oriental and exotic peoples intends to penetrate also into music. Now, however great the conquests of these peoples may be, they constitute the perfecting of a lower evolutionary stage, or the degradation of a stage higher than ours."[145]

Someone proposed a tempered system divided into 53 equal parts. With polyphony, which he loved and like using in mirror forms, 12° were sufficient for Schönberg in the scale, and he did not feel any need for quarters or thirds of a tone. ". . . much of that which is considered to be aesthetic, . . ., is not always founded in the substance of things, . . . it is the imperfection of our senses that obliges us to make those compromises through which we obtain order, seeing that it is not the object that requires order, but the subject; . . . thus the laws, which they would like to pass for laws of nature, derive from the desire to treat material in an exact manner from the artisan point of view; . . .". The composer commented on the discovery that sound could be analysed in its harmonics: "Having to follow the will and the ingeniousness of theoreticians, musicians should have stopped at this point. This they did not do. They had discovered the scale, but they did not know how to find its fundamental: . . . they helped themselves with *mathematics* and with the resources

[145] Schönberg 1922/1963, pp. 22–23, 29–30, 31–32, and passim.

of *combination*, and a wonderful system was born: wonderful if proportional to our intellectual faculties, but infantile with respect to nature, which uses a superior mathematics. . . . The tempered system was an expedient, a genial expedient because the situation was serious, and the need for help was great." Historical evolution had substituted "natural" sounds with "artificial, tempered" ones, with the result that ". . . harmonics become 'non-musical', seeing that if a chorus member sings a major third exactly, he is out of tune from the musical point of view."[146]

Troubled by such problems, and claiming artistic freedom in the face of theoreticians and naturalists, in his *Manual of Harmony* the serial composer invoked ". . . yet another element which it is not the case to examine here. [. . .] in reality, art is composed of a fourth factor which is always present in these masters, even though they, with respect to their material, have considered to a lesser degree the final end of nature." It would be sufficient to examine what kind of music Schönberg composed to understand that this "fourth factor" was a divinity that transcended nature and man. "At its highest level, art occupies itself only with reproducing interior nature. [. . .] if we want to found a true theory of sounds, we should always go back to the subject, that is to say, to the ear." He thus advanced doubts about the value of experiments, and preferred to try to give his theoretical ideas form and coherence. For this reason, he referred to alchemists, who, as regards the possibility of transforming elements, had had clearer sight than the experimental chemists of the nineteenth century, who considered them to be indivisible and unchangeable.[147]

Subsequently, in a letter of 1934, Schönberg stated: "I have always requested the tempered tuning from players of stringed instruments with whom I worked . . . (. . .). Certainly, one of the difficulties for my music is that an ear that is used to classical music will wonder, when it hears a *do*♯, whether it might be a *re*♭. While in reality, it is nothing other than the measurement of half-way between *re* and *do*, without any relationship with harmonic problems. . . . Because being musical means having an ear in the musical sense, not in the natural sense. A musical ear must have assimilated the tempered scale. And a singer who produces natural pitches is not musical, just as someone who acts 'in accordance with nature' in the road may be immoral". Lastly, the composer presented a highly significant comparison with language. "Just as phonetic writing is bound to be superior to syllabic writing or pictograms (. . .), a music that is not limited tonally is naturally destined to have a greater development through the wealth of its combinations, ideas and forms of notes."[148]

His *Manual of Harmony* was directed at self-taught pupils like himself; it suggested technical solutions and artifices. "The latter should be, if possible, a secret science, to which only those who are able to find it by themselves have access. [. . .] You will ask me why I write a treaty on harmony if I want technique to become a secret science. [. . .] In his instinct, in his unconscious lies a treasure

[146] Schönberg 1922/1963, pp. 37, 396–397, and passim.

[147] Schönberg 1922/1963, pp. 20-21-22, and passim. Tonietti 2004.

[148] Tonietti 2004, p. 733.

of ancient wisdom that he will cause to take effect without wanting to; . . .".[149] Schönberg presented this "secret science" not only to musicians, but also to scientists like Einstein. It guided his compositions *Jacob's Ladder*, *Serenade* op. 24, *Fünf Klavierstücke [Five pieces for the piano]* op. 23, *Moses and Aaron*, *Variationen für Orchester [Variations for orchestra]* op. 31. I have little doubt that for him, the method of composing with 12 notes was the core of his "secret science". Initially, he even concealed it, like the series in *Jacob's Ladder*. Then, afraid of the competition, above all of Joseph Matthias Hauer (1883–1959) or his pupils, he brought it to the light, and laid claim to it as his own.[150]

Still today, unfortunately, our composer who invented serial music (vulgarly, called dodecaphony) is either loved or hated (deeply!) for the wrong reasons. He is often misunderstood as a scientific, or cerebral composer; above all, he would be a theoretician, a champion of revolution in music, controlled (too much?) in his sentimental expressions. Schönberg would incarnate the model of rationality and progress after the crisis that music went through following Wagner. We can see here how deeply, in reality, he believed in a knowledge which was scientific, it is true, but archaic, rather for a few elect, and inspired by a transcendent divinity.

Arriving at the end of this second part, we may say that the idea of science offered by Schönberg was not wholly absent from our history, and that, on the contrary, it was manifested precisely in connection with music. But, during the twentieth century, the scientific community (not to say, almost all the schools of philosophy for the sciences) loved to present itself as a model of rationality independent both from music and from religious beliefs. In order to offer this "neutral", sterilised image of themselves, physicists and mathematicians had to lose the memory of their past, and cancel as "heretics" those who maintained other ideas on the subject.

For music, the idea of rationality was to be proposed by the well-known sociologist, Max Weber (1864–1920), in the same period as the above-mentioned *Manual of Harmony*. He rationalised the scales of notes with the numbers of the harmonics, although at times he partly criticised Helmholtz, who remained, however, his main referent. This German sociologist followed the idea of his physicist colleague that the custom of hearing the equable temperament on the piano had ruined our ear. Also for him, the "pure" or "harmonic" tuning would be that of Zarlino and Ptolemy, and the divisions of the octave into equal parts would be conclusively extraneous to (tonal) harmony. Referring to the composers intent on dissolving the old tonalities, this Weber indicated, as the inevitable foundation, the "tonal *ratio*", based on ratios between whole numbers. Thus, as at the time of Pythagoras, the "irrational" numbers of roots, necessary for the temperament, produce irrationalities also in music. Stumpf and other ethnomusicological studies on the Chinese, Indians, Arabs and Siamese gave him the possibility of making comparisons, as a sociologist, with

[149] Schönberg 1922/1963, pp. 518, 521–522 and passim.

[150] Tonietti 2004.

other cultures less – according to him – developed that is to say, non-rationalised. These second-and third-hand observations are not always reliable.[151]

Also between this disenchanted German professor and our musician-prophet, the distance was not short. And yet in those (these?) dark, destiny-laden times, not even a Max Weber could avoid quoting a "true" prophet from the *Bible*, Isaiah, to distinguish himself from the "false" prophets. He did so in order to put a good seal on a lecture of his to students on "Science as a vocation".[152] Among the "false" prophets, Schönberg had with certainty placed Oswald Spengler (1880–1936) for his images on the decadence of his great Germany. But this time, we can do without this hypocritical, superficial character, seeing that we have already had to suffer him elsewhere.[153] As he boasted of credits also in music and mathematical sciences, in his approximate philosophical style typical of an unprepared fifth-hand disseminator, it perhaps might be instructive to make a comparison with the present historical essay. But I hope that my couple of dozen readers have better to do than waste their time on an old-time success, highly appreciated by Fascists and Nazis, which reached 60 editions and 200,000 copies sold, and was read by everybody, including Schönberg, Einstein and Thomas Mann (1875–1955).

If not a cure for insomnia, music gave Einstein a certain pleasure, though it can hardly be said that he had original scientific ideas on the subject. With few exceptions, this was the place of music in the sciences of the twentieth century. The mathematician Hermann Weyl (1885–1955) did not seem to be an exception. In philosophy, he followed, above all, Edmund Husserl, who he had frequented at Göttingen.[154] The philosopher of phenomenology moved against the background of sound, above all when he considered the time of psychological experience in the conscience, but he only left us scanty general indications to form continuous successions of notes. In 1933, having chosen exile in the United States, our German mathematician spoke on the radio once during a concert of the New York Philharmonic Symphony. "But the stimulus to this mathematical development had originally come from a ground where music and physics meet, the acoustics of vibrating bodies. Studies undertaken to understand musical harmony have thus finally enabled us to understand the richest harmony in the visible world - that of the spectral lines emitted by radiating atoms."

Unlike Planck, Weyl had not received any musical education. "Hermann war in einer völlig a-musikalischen Welt aufgewachsen, hatte nie gelernt, ein Instrument zu spielen, und von Musikgeschichte und Formlehre und Harmonie keine Ahnung. Aber nun, in Zürich, in dieser der Musik so zugewandten Stadt, entwickelte er augenscheinlich ein so überwältigendes Bedürfnis nach Musik, daß er sich zum Kauf einer Pianola entschloß, eines Instruments also, das es ihm ermöglichte, sich quasi als Interpret großer Werke zu betätigen. . . . Später, . . ., kam dann auch noch

[151]Max Weber 1980, pp. 67–68 and passim. Cf. above, Part I.

[152]Tonietti 2004, pp. 714–721.

[153]Tonietti 2004, Chap. 49.

[154]Tonietti 1988.

ein Grammophon in's Haus, und nun ertönte auch des öfteren Gesang bei uns, Schubert- Wolf- und Strauss- Lieder, Caruso mit Volkstümlich-Italienischem, das Sextett aus der *Lucia* etwa, aber auch einiges an Kammermusik, wie z. B. die zwei großen Klaviertrios von Schubert oder Dvoraks Dumkytrio. ... Doch die Pianola ließ sich nicht durch das Grammophon übertrumpfen, sie stellte größere Forderungen an den Musiker in Hermann und hat ihm manche Jahre viel Freude und musikalische Befriedigung gegeben." ["Hermann [Weyl] grew up in a totally a-musical world; he had never learnt to play an instrument, and did not have the slightest idea of musical history, the theory of form, or harmony. But now, in Zurich, in this city so devoted to music, he rapidly developed such an impelling need for music that he decided to purchase a [mechanical] pianola, an instrument which made it possible for him to behave almost like a performer of great works. ... Later, ..., also a gramophone arrived in the house, and now it was more common to hear singing in our house, Lieder by Schubert, Wolf, Strauss, Caruso with Italian folk language, the sextet from *Lucia [of Lammermoor]*, but also some chamber music, such as, for example, the two great *Trios* by Schubert with pianoforte, or the *Trio Dumky* by Dvorak. ... However, the pianola did not let the gramophone surpass it; it presented increasing requests to the musician in Hermann, and for more than a year, it gave him great joy and musical satisfaction."]

Although he frequently tended not to restrain his pen, in dealing with philosophy and sciences in general, our German mathematician very rarely presented, in his writings, the music of the spheres that harked back to the Pythagoreans or to Kepler.[155] As regards Helmholtz, Weyl very often discussed his idea of space, to which, however, he preferred that of Bernhard Riemann, although he never considered the essay of music. He dealt with symmetry in general and in artistic ornaments, but as regards music, he had recourse to the study of his Swiss colleague, Andreas Speiser (1885–1970): "Formfragen in der Musik" "[Questions of form in music]".[156]

The Swiss mathematician divided up into parts, like a, a, b or $a, a, b; a, a, b; c, c, d$ famous pieces like the *Sonata* for pianoforte op. 28, the "Pastoral" by Beethoven. "Man sieht schon an dieser trockenen Aufzählung, wie die Symmetrien die Komposition durchdringen. Man müsste das ergänzen durch die Harmonik, doch diese bildet ein gebiet für sich, das ich beiseite lasse. ... Die ausserordentlicher Genauigkeit, welche der Künstler und der Zuhörer hierbei in Funktion setzen, ist einer der deutlichsten Beweise für die mathematische Struktur des menschlichen Geistes." ["We can already see in these arid calculations that symmetries permeate composition. This should be completed with the analysis of harmony, but this forms a field by itself which I leave aside. ... The extraordinary exactness which the artist and the listener put to work here, is one of the clearest demonstrations of the mathematical structure of the human spirit."][157]

[155] Weyl 1932, p. 22.

[156] Weyl 1962, p. 59. Speiser 1932, pp. 23–36.

[157] Speiser 1932, pp. 30–31 and passim.

With the algebraist Speiser, who studied symmetries by means of group theory, now mathematics (a posteriori) served to give a form to compositions. The process was now inverted, with respect to the problems followed so far, which had prompted natural philosophers to find original mathematical solutions. "Das Wesentliche am musikalischen Kunstwerk gehört dem Seelischen, nicht dem Sinnlischen an und besteht daher aus Relationen." ["In the musical work of art, the essence belongs to the side of soul, not to that of the senses, and thus it consists of relationships."] "In früheren Zeiten ist der arithmetische Character der Musik unbestritten gewesen, weil jedermann bei den Zahlen unmittelbar an Töne dachte. Heute denkt man unter dem Einfluss der analytischen Geometrie an räumliche, visuelle Dinge, nicht akustische. Trotzdem ist es überraschend, dass die Verwandtschaft der beiden Gebiete noch im letzten Jahrhundert gefühlt wurde." ["In remote times, the arithmetic character of music was not challenged, because with the numbers, everyone thought of the notes. Today, under the influence of analytical geometry, people think or spatial or visual things, and not acoustics. Nevertheless, it is surprising how the affinity of both the domains has been felt in the last centuries."][158]

In order to sustain that these were not empty abstractions, Speiser even based himself on Schopenhauer and Nietzsche, who had written about music as an expression of the will, and of the possible numerical and geometrical forms present in the experiment itself. This mathematical way of thinking was to prove fruitful also with the works of Mozart, Wagner and Beethoven. Throwing dice at random in a *harmonic game*, Haydn and Mozart had composed some pretty minuets. "Aehnliche wie es für die algebraische Gleichung eine Metaphysik, die Gruppe, gibt, deren Kenntnis das Innerste der Gleichung enthült, so gibt es auch für das Kunstwerk eine Metaphysik, nämlich eine Symmetriegehalt, dessen Kenntnis gestattet, beliebig viele schöne Stücke zu componieren, und die Auffindung solcher Konfigurationen ist die wahre künstlerische Leistung." ["Similarly to the way in which the group [of permutations] provides a metaphysics for the algebraic equation, the knowledge of which contains the most intimate part of the equation, so precisely a quantity of symmetry gives also to the work of art a metaphysics, the knowledge of which makes it possible to compose at will as many beautiful pieces as are desired, and the discovery of these configurations is the true artistic result."][159]

On the other side of the Atlantic Ocean, a similar power of the mathematical sciences, which was claimed to be universal, was put forward by the mathematician from the United States, George David Birkhoff (1884–1944). In some publications and articles which culminated in his book *Aesthetic Measure*, he gave his formula to evaluate by means of numbers the work of art $M = \frac{O}{C}$. The aesthetic measure is the ratio between the ability to introduce order O into the material, and its degree of complexity C. Out of courtesy, I omit some common places which sound funny on the lips of an "American" like him, e.g., "But only in Europe since the Renaissance

[158] Speiser 1932, pp. 32, 33 and passim.

[159] Speiser 1932, p. 36.

has it [music] broken the bounds of homophonic song and thereby acquired an almost transcendental expressive power."

Having recognized "the decisive superiority of the equably tempered scale", Birkhoff attributed whole numbers to chords and melodies, in accordance with whether they sounded better "ordered" to him. Thus he could calculate the factor O. Of course, he was in agreement with the current manuals of harmony. For the denominator C, what could there be more objective and quantitative than counting the number of notes directly? All this seems arbitrary to me, conventional and fundamentally useless. Lastly, he applied the procedure to a passage taken from the "Fourth movement" of the *Ninth symphony* of Beethoven. "Thus the grand total for this Beethoven chorale is 288 or an average of 4.8 elements of order for each note. In other words the aesthetic measure M for this Beethoven chorale is 4.8 which turns out to be very high in comparison with that of most other melodies." Aesthetically satisfied by the (surprising?) result, the renowned mathematician in search of objective certainties then turned his arithmetic power towards the field of ethics. But someone else was to deal with that.[160] At the time of Picasso and Strawinsky, his idea of beauty as order sounds like an anachronistic nostalgia for Baroque mathematical harmonies.[161]

Written sometimes by mathematicians and physicists, at other times by historians and philosophers, from now on, it becomes possible to read in scientific literature, between the amateur and the scholarly, essays on the relations between music and the mathematical sciences. They could be grouped together under the main categories of "Rest for the scientist", or "Mathematical truth in acoustics", or "The mysterious harmony of the cosmos" and "How much musicians should learn from mathematicians". To bring Part II to a rapid conclusion, I shall only mention briefly all these different cases. The list of those who variously dabbled in music, together with some of their main scientific occupations, could be lengthened with Pierre Louis Maupertuis (1698–1759), William John Herschel (1792–1871), Carl Gustav Jacobi (1804–1851), Hermann Grassmann (1809–1877), János Bolyai (1802–1860), Augustus De Morgan (1806–1871), Gaspar Monge (1746–1818), Walther Nernst (1864–1941).

The Departments of Mathematics and Physics are undoubtedly not without people who take pleasure in listening to and playing music. Instead, Sofja Kowaleswskaia (1850–1891) would have liked to be able to sing, but she did not possess any talent. Niels Abel (1802–1829) once sought the relationship between the number of times that the keys of a piano were pressed by the fingers during a concerto. It would be equally easy to find other cases of mathematicians and musicians who are at a great distance from each other.[162]

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[160] Birkhoff 1950, vol. III, pp. 288–306, 320–333, 382–535, 755–777, pp. 435, 446, 524 and passim. Birkhoff 1933.

[161] Knobloch 1995.

[162] Archibald 1924. Hiebert 1981, pp. 434 and 449.

Federigo Enriques (1871–1946), an exponent of the Italian school of algebraic geometry, completely ignored music in his historical, philosophical and popularising works. In them, I have succeeded in finding only the few, inevitable lines about the ratios of the Pythagorean musical intervals, together with the mysticism of numbers. While Tannery and Helmholtz came out in all kinds of contexts, but never with music. In his contrast with Benedetto Croce, was the Italian mathematician afraid perhaps that he would infect his rationalism by coming too dangerously close to aestheticism? It is interesting, however, that in his last posthumous book about algebraic surfaces, "hidden harmonies" emerged also for him. ". . . si soleva dire che, mentre le curve algebriche (già composte in una teoria armonica) sono create da Dio, le superficie invece sono opera del Demonio. Ora si palesa invece che piacque a Dio di creare per le superficie un ordine di armonie più riposte ove rifulge una meravigliosa bellezza, e ch'Ei volle -diciamo col Poeta-

del creator suo spirito
più vasta orma stampar.

. . . quell'ordine meraviglioso degli enti matematici, che il pensiero trova innanzi a sé e quasi raccoglie, al pari delle specie viventi, dalla Natura Madre; . . . c'è realmente in questo mondo di enti una divina armonia, . . .". ["... it used to be said that while algebraic curves (already composed in a harmonic theory) are created by God, surfaces, instead, are the work of the devil. Now, on the contrary, it is clear that God chose to create for surfaces an order of more hidden harmonies, where a wonderful beauty shines forth, which He wanted in them - let us say, with the Poet [Alessandro Manzoni (1785–1873), "Cinque maggio"] -

of his Creator Spirit
to imprint a vaster trace

. . . that wonderful order of mathematical beings, which thought finds before itself, and almost picks up, together with the living species, from Mother Nature; . . . there really is a divine harmony in this world of beings, . . ."]

This harmony no longer regarded music; it was only a metaphor. The comparison was taken up by the editor of the edition, Guido Castelnuovo (1865–1952). He noticed that this geometry was now practised with new mathematical techniques. Under the impulse of David Hilbert, actually, algebra had become formalist, and demanded to cut every tie with "Mother Nature".[163] "Oggi più che il terreno da esplorare interessa la via [l'algebra] che vi conduce, e questa via ora vien seminata di ostacoli artificiali, ora si libra tra le nuvole. Che questa tendenza non sia una manifestazione di breve durata si è indotti a credere dal paragone col fenomeno analogo che si verifica nelle arti plastiche e nella musica. Anche qui la fantasia è bandita quale sopravvivenza dell'epoca romantica, anche qui il contenuto dell'opera d'arte interessa meno della tecnica o dei mezzi d'espressione che l'artista impiega. Vari critici illustri pensano che ciò rappresenti una decadenza dell'arte e ne traggono

[163] Tonietti 1992a; 1992b; 1994.

foschi presagi sull'avvenire della nostra civiltà." ["Today, more than the ground to be explored, what is of interest is the way [algebra] that leads there, and at times this way is scattered with artificial obstacles, at times it rises up among the clouds. We are led to believe that this trend is not a short-lasting manifestation from a comparison with the analogous phenomenon that we see in the plastic arts and in music. Here too, imagination is banished as a fossil of the romantic age, here too, the contents of a work of art are less interesting than the technique or the means of expression that the artist uses. Various illustrious critics think that this represents a decline in art, and foresee dark omens for the future of our civilisation."][164]

The literature of those who propose acoustics as a scientific system to found music on sure, objective bases appears to be extensive. This has not been my point of view, and we shall not deal with it here, even to confute it. Our history shows something different. Starting from the second half of the twentieth century, there has been a continuing increase in the number of those who use the computer, both for acoustics and for music.[165]

The historians or philosophers of the mathematical sciences who have at least realised the non-negligible significance of music in their disciplines are rare. Sometimes, they have enriched the models of cosmic harmony proposed through the ages with some interesting musical elements. Eberhard Knobloch has underlined the role of music for the first combinatorial art, and has compared the positions of Athanasius Kircher and Robert Fludd with those of Kepler, Mersenne, Leibniz and Euler. In the face of two very similar manuscripts by Leibniz and Euler, our German scholar concluded: "La structure semblable de la musique et de l'arithmétique a entraîné la même méthode de recherche chez les deux éminents savants." ["The similar structure of music and arithmetic gave rise to the same research method in the case of both the eminent scholars [Leibniz and Euler]."][166]

In criticising the equable temperament, Patrice Bailhache followed Helmholtz, above all, rather than the ears of musicians (with some reservations and corrections here and there), preferring to seek justifications for musical scales and consonances in the principles of acoustics. "... l'objet n'est pas de produire de la musique, mais d'en comprendre la nature par des principes scientifiques si tant est que la chose soit possible. [...] ... la fonction de dissonance que l'on obtient ressemble à celle de Helmholtz, mais les 'vallées' sont beaucoup moins accentuées. Cela signifie que les consonances devraient se manifester bien moins nettement que ne le prévoyait Helmholtz. Et pourtant, notre expérience musicale, lorsqu'elle a été convenablement *éduquée*, nous atteste que Helmholtz ne se trompait pas." ["... The aim is not that of producing music, but understanding its nature, by means of scientific principles, as far as possible. [...] ... the function of dissonance that is obtained is similar to that of Helmholtz [Fig. 12.3], but the 'valleys' are much less accentuated. This means

[164]Enriques & De Santillana 1932, p. 86. Enriques 1949, pp. 464 and vii. Enriques & Castelnuovo 1996, p. xxx.

[165]O'Keefe 1972.

[166]Knobloch 1992; 1995.

that consonances should manifest themselves much less distinctly than Helmholtz expected. And yet, our musical experience, if appropriately *educated*, confirms to us that Helmholtz was not wrong."]

The patient, benevolent reader will judge for himself to what extent the history narrated so far is in agreement with the metaphysical conclusion of Bailhache. "... il a 'globalement' progrès scientifique; je pense justement que l'histoire des théories de la musique s'accorde avec cette idée générale. De toute évidence, l'explication physico-physiologique des sensations sonores approche la'réalité' de bien plus près qu'une simple mise en relation des phénomènes avec la théorie Euclidienne des proportions. Du reste, comme cela arrive fréquemment en science physique, la théorie récente ne contredit pas entièrement la théorie ancienne." ["... scientific progress 'globally' exists; I think, quite rightly, that the history of theories of music is in agreement with this general idea. Clearly, the physical-physiological explanation of sonorous sensations comes closer to reality than a simple relationship of the phenomena with Euclid's theory of ratios. Moreover, as frequently happens with physical science, the recent theory does not completely contradict ancient thinking."][167]

But *à la rigueur* of logic, once the decisive role of the ear has been admitted in the *education* of listening, the presumed 'scientific progress' dissolves into the social context that organises the relative musical culture. The Spartans, or Plato, had their own, which was not that of Aristoxenus; the Germans of the nineteenth century, with Brahms, Joachim and Planck preferred the equable temperament to the scale of Helmholtz. Thus our French scholar should resign himself. And we have not had to pull either Chinese, or Indians or Arabs into the history.

The metamorphoses to which J. S. Bach or Anton Webern (1883–1945) had subjected their polyphonic themes can become geometrical transformations. The continuous curves of sounds have been "sampled", that is to say, made suitably discrete, and analysed in computers.[168] Observing sciences from too far away, some would thus like to make music "rational": music as a science. In this, a difficult balance is sought with historical evolution, maintaining a distance from other modern theoreticians of music in the name of a resurrected "Pythagoreanism" *à la* Pickering Walker.[169] However, whether Pythagorean or not, mystical or not, numbers continue to exert their secret fascination, and have followers everywhere, including the most scientific departments.[170] Not even the empiricism of modern acoustics is completely immune from it. Here we can find practically everything: those who confuse the scale of Aristoxenus with that of Zarlino, or those who attribute the greatest discoveries to Helmholtz or to Lord Rayleigh (John William Strutt, 1842–1919), forgetting too many previous musicians.[171]

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[167] Bailhache 1989; 1996, pp. 1, 27 and passim.

[168] Simonini 1995.

[169] Asinari 1997. Gozza 1996.

[170] Pekonen 1993. Klotz 1995. Bizzi 198?. Scimemi 1990.

[171] Anfilov 1966, p. 128. Cingolani 1995. Lindsay 1972. Lindsay 1981.

Since the times of Iannis Xenakis (1923–2001),[172] never again will there be a lack of musicians attracted irresistibly, for various reasons, by the mathematical sciences. However, sometimes what is taken from the scientific panorama is, above all, the easy common places that are best publicised.[173] Thus, in the great age of the computer, the amusements of Douglas R. Hofstadter are a good example of how to apply digital thought successfully to every aspect of the universe. But these have little to do with music or with history: above all, nothing to do with our history. However, this American computer scientist is a clear indication that it is possible to remain prisoners of dualist categories engaged in discriminating truth from error by means of transcendent logical and mathematical laws. Attempts are made even to trace the music of Johann Sebastian Bach back to these, reducing it to a recursive game. Hofstadter defended so-called Artificial Intelligence, publicising it with an appealing artistic and philosophical digression. Details are deferred to the forthcoming reincarnation.[174]

Before jumping on to our boats and sailing away to faraway seas, let us turn back for a moment to see what we have left behind us in our long history. Many others have related the evolution of the sciences as the clash between truth and error, as the development of universal rationality, as the conquests of the experimental method, as the rigorous application of logic, to bring to light the hidden mysteries of nature, as the defeat of superstition and of religion, thanks to the light of reason. Here, instead, the patient reader has had ample possibility to verify that the mathematical sciences underwent a completely different history.

Bringing music into the foreground has made it possible for us to understand better the weight of the differences between scientific cultures, the development of Greek Pythagoreanism within a European secular orthodoxy, some critical cases like Kepler's Third Law, the geometry of Descartes or the birth of logarithms. In general, we have witnessed the various phases of a conflict between those who would have liked to reduce the problems of scales and of consonances to numbers and the others, more used to verifying the notes produced by their instruments with their own ears. Natural philosophers, first, and then mathematicians and physicists, tried to dictate their rules to musicians, who, on the contrary, played and composed, following their own artistic choices. While the former would have liked to impose their idea of order, consistent with their political, religious and social values, the latter often succeeded, following the impulses of the context, in taking many of the desired liberties.

Despite all those secular resistances, Pythagorean and Zarlinian ways of tuning were to change in time into the equable temperament, as our sensibilities in listening to dissonances underwent considerable changes. All this demonstrates how far, in the end, luckily for us, musicians preferred to develop the variety of a myriad of different expressions, instead of bowing down to laws and rules, which regularly

[172] Xenakis 1963. Napolitano & Tonietti 1972.

[173] Domínguez & Pecci 199?.

[174] Hofstadter 1980. Cf. Nolan 2004.

proved to be arbitrary means of control, rather than being suitable to procure sensible joy to listeners.

> My Lutheranism is in agreement with you, because it sees in theology and in music territories that are close together and very similar, and furthermore, music has always seemed to me personally a magical union of theology and amusing mathematics.
>
> ThomasMann

> Bach, Mozart and some ancient Italian and English composers are my favourites. I appreciate Beethoven much less. On the contrary, I like Schubert. [. . .] I admire the creative ability of Wagner, but I consider his lack of architectonic structure a sign of decadence. Furthermore, I find his musical personality so indescribably offensive that for the most part, I listen to him with a sense of disgust. I believe that [Richard] Strauss possesses great talent, but he lacks an interior truth, and is only interested in exterior effects.
> Music does not *influence* research, but both derive from the same source of inspiration and complement each other in the sense of relief that they procure for us.
>
> Albert Einstein

> No eternal laws exist for us, but only indications which are valid until they are surpassed and eliminated, wholly or in part, by new conditions.
> The man "of character" is (modifying an expression of Karl Kraus [1874–1936]) the one whose arteriosclerosis derives from the conception he has of the universe.
> . . . beauty gives itself to the artist, who did not desire it, because he aspired only to truthfulness: only to that. . . . In order to be something real, beauty should be, for us, something that does *not* let itself be exposed like an arithmetic problem; . . . Perhaps also truthfulness can be expressed mathematically, and perhaps better still than beauty, where the formula, in the end, is not sufficient; but in order to understand, what is needed is the sense of beauty. . . . here, there is room also for error, which, like truth, has its part in truthfulness. And error deserves the place of honour, because it is thanks to it if the movement never stops, if the fraction does not become equal to one, if truthfulness never becomes truth: because knowing the truth would be difficult to suffer.
> Only the ear can guide us, a sensibility to sound, the creative impulse, imagination and never mathematics, the pure combination, aesthetics.
>
> Arnold Schönberg

> *Philosophy as a science*, as a serious science, rigorous, indeed, apodictic, - the *dream is over*. . . .. The conception of the world is thus an individual operation, a kind of personal religious faith, which is different from traditional religion, from revealed religion, because it lays no claim to an unconditional truth, binding for all men, a truth that is to be transmitted to everybody -just as a scientific truth of the absolute is not possible, so it is impossible to demonstrate the general validity, for all men, of a truth that falls into the order of the conceptions of the world.
>
> Edmund Husserl

> "Mathematizing" may well be a creative activity of man, like language or music, of primary originality, whose historical decision defies complete objective rationalization.
>
> Hermann Weyl

> Arithmetica est exercitium musicum occultum nescientis se sonos comparare animi.
> [Arithmetic is a hidden musical exercise of the soul, who does not know that it compares sounds in itself.]
>
> T. W. Preyer

> May not Music be described as the Mathematic [sic!] of Sense, Mathematic as the Music of reason? the soul of each the same! Thus the musician *feels* Mathematic, the mathematician *thinks* Music, - Music the dream, Mathematic the working life -, each to receive its

consummation from the other when the human intelligence, elevated to the perfect type, shall shine forth glorified in some future Mozart-Dirichlet, or Beethoven-Gauss - a union already not indistinctly foreshadowed in the genius and labours of a Helmholtz.

James Sylvester

Gran verità, ma bisogna ponderarla bene. La ragione è nemica di ogni grandezza: la ragione è nemica della natura: la natura è grande, la ragione è piccola.
[A great truth, but it needs to be pondered well. Reason is the enemy of every greatness: reason is the enemy of nature: nature is great, reason is small.]

Giacomo Leopardi

Without music, life would be an error. The German thinks that God Himself sings songs.
'Reason' in language: ah, what a cheating old hag! I am afraid we shall not get ride of God, because we still believe in grammar . . .

Friedrich Nietzsche

Part III
It Is Not Even Heard

It can’t be all in one language.

Ezra Pound.

Chapter 13
In the Language of the Venusians

a dernière chose qu'on trouve en faisant un ouvrage est de savoir celle qu'il faut mettre la première.
[The last thing you discover when writing a book is what to put first.]

Blaise Pascal

13.1 Black Languages

In Africa, the populations living along the upper Niger used to call the number 20,000 *captif*, which means "slave" in French. This indicated his price as that number of shells.[1] I would like I could write these last few chapters in a language in which it is impossible even to express concepts of the kind. This has never existed on our poor tormented Earth, and I doubt if it will exist in the near future. Maybe it is spoken on Mars, or on Venus.

So far, we have been able to tell our history, following the traces that have remained in books. But not all cultures have left written documents. And yet, on the basis of the objects of use that have survived the passing of time, even if more difficult and sometimes obscure to interpret, also in these cases we can try to work back to the relative scientific ideas. In such cases, we find engraved bones used to count the days of the calendar, or symmetrical designs as decorations, or games with pebbles.[2]

Has the idea ever occurred to anyone of listening to African music with the ears attached to the head of a Leibniz? Is it not true that in oral cultures, melodies and rhythms are, also in this case, archaic traces of ethnic groups present in the black continent, capable of understanding a context suitable for living there? Without much hope, I am trying to find something suitable in the secondary literature. At

[1] Zaslavsky 1994, p. 88.

[2] Gerdes 1990; Gerdes 1997; Gerdes 1999. Lumpkin 1997. Ascher 1991.

T.M. Tonietti, *And Yet It Is Heard*, Science Networks. Historical Studies 47,
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the moment, books abounds in writers who have rediscovered the beauty of Africa by means of their scientific studies and some problems in teaching mathematics to the many ethnic groups who have emigrated to Western cities.

"La musique dans les sociétés de tradition orale peut également contribuer à la mise en évidence de représentations mathématiques" ["In societies with an oral tradition, music can contribute all the same to the demonstration of mathematical representations."] maintains Marc Chemillier. In the way in which an African tribe plays a five-stringed harp, he has tried to demonstrate a recursive "canon-like" structure and a "central symmetry", in the structuralist tradition going back to the famous Lévi-Strauss. Certain pygmies beat on tree trunks, generating asymmetrical rhythms which Chemillier has analysed and classified, using the combinatorial methods of information technology.[3]

We must ask ourselves, however, how much of all this came out of the head of a French computer scientist, ending up superimposed on the local culture, with too few traces really present in the acts accomplished by African natives. "La question qui apparaît en filigrane de cette analyse est celle de l'universalité de la pensée rationnelle, …" ["The question that emerges between the lines of this analysis is that of the universality of rational thought, …"].[4] From Descartes to Claude Lévi-Strauss, by way of the Illuminists, and down to Chemillier, it has generally been maintained that, in all cases, a no-more-clearly defined "universal rationality" would exist. But it seems to be a kind of "araba fenice" [unknown rarity]: "cosa sia ciascun lo dice / dove sia nessun lo sa" ["…what it is, everyone is ready to explain, but where it is, nobody knows."] Often "rationality" is sought in the mathematical sciences, and at that point, it is necessary to demonstrate its universality. What better proof can there be than the event that even the thought found in cultures that are not European and not written exhibit it?

Far away from a world like that, with other (historical) styles of research, here we continue to show that, whether rational or not, every culture is interesting because it is different, also in its mathematical and scientific expressions. *Ratio* and *logos* are European Graeco-Latin values, and furthermore they are devoid of eternal qualities. But weren't anthropologists supposed to be specialised, above all, in showing us cultural diversities while maintaining their validity intact? Instead, with his information technology, is it not true that Chemillier flattens them out, creating an umpteenth Eurocentric model?

Paulus Gerdes has studied the geometry "hidden" in various forms in indigenous cultures of sub-Sahara Africa. In particular, he has provided examples, with some evidence, in favour of the idea that the fundamental property of right-angled triangles (known as the theorem of Pythagoras in the West) was known and present in various forms in Africa. (Fig. 13.1)

[3]Chemillier 2002. Chemillier 2004, p. 276. Chemillier 2007a. Chemillier 2007b. Chemillier 2008.
[4]Chemillier 2008, p. 31.

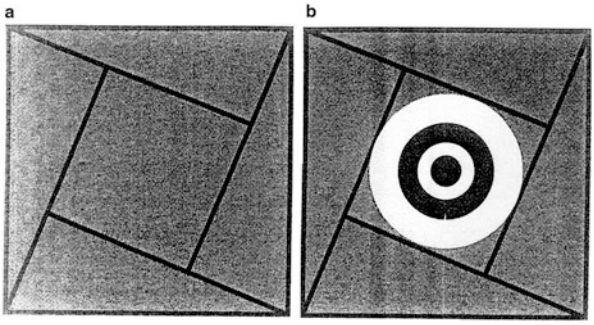

Fig. 13.1 African drawing which can be traced back to the fundamental property of right-angled triangles ("African square" in Gerdes 1999, p. 66)

The drawing was produced by the Kuba tribe of Congo/Zaire and is called "mwoong", which means "defence of the elephants".[5]

13.2 Stones, Pieces of String and Songs

In Central America, the Maya culture left us traces of its non-alphabetic form of writing, sculpting stones and decorating ceramics with its so-called *glyphs* [engravings]. In this way, they expressed their far-from-banal astronomic knowledge: they calculated ritual temporal cycles based on the number 20; they foresaw eclipses; they included right angles in their squared-off temples.[6]

In South America, the Incas preserved the memory of their empires, recording accounts and events by means of coloured pieces of string, variously intertwined and knotted: *quipus*. In North America, the oral traditions of the Navajo Indians related the interweaving of space and time in complex geometrical shapes.[7]

Music at last re-emerges with the Hopi Indians, although its transcription in books has been performed using Western notes. Dancing and singing often reappeared in the telling of legends and descriptions of social events. Plants grew tall thanks to their singing; the sun and the moon were created in sung rituals; hunting dogs were won over by dancing and singing; even the stalks of corn did so. In other accounts of legends, music often underlined the events. There were clans who were called "pipes", "horns" or "flutes"; rites were celebrated, known as the "Ceremony of the flute", and a certain village is indicated as "Lenyanovi" ["Place of the flute"].[8]

[5]Gerdes 1999, pp. 65–66 and passim.

[6]Lounsbury 1980. Closs 1994.

[7]Ascher & Ascher 1994.

[8]Courlander 1971, pp. 24, 51, 61, 98, 127, 137, . . . 235 and passim.

Any possibility thus remains that ethnomusicologists and ethnomathematicians may study some archaeological findings of such ancient musical instruments. From the holes made in them, it would not be difficult to work back to the scale of their notes, with the relative ratios. Wouldn't this be a good way of beginning to investigate their mathematical culture?[9]

> The first female singer begins: like a proud bird, she rapidly rises up to the heights of passion. The powerful high note falls and rises again, gliding like a bird, while the others fly around the star like faithful satellites. Then the men, with a single barbaric cry, conclude in harmony on the tonic. These are the songs of Tahiti, . . .
>
> Paul Gauguin, *Noanoa [Fragrant]*

13.3 Dancing, Singing and Navigating

Traced on the beaches of Oceania, the designs of the Malekula reveal particularly decorative complex geometries.[10] Across the Pacific Ocean, the Ma'ohi cultures organised their religious rituals in precisely delimited spaces called *Marae*.[11] Whether they were magical ceremonies or warlike activities, public festivals or secret meetings, the Ma'ohi culture almost always organised them with plenty of songs, dances and music. Though deformed and censured by Western prejudices of various kinds, we can still find, in the accounts of missionaries, the traces of these ancient traditions.

"Faa-oto-raa" ["to cause to play", or also "to cause to cry"] or "upaupa" ["dancing"] are all periphrases that indicate the essential stimulus to produce these expressive movements: music. To obtain the sounds, the main traditional instruments were of three kinds. First of all, they beat *pahu* [drums]. To arrive at her wedding, a princess, Hutu-hiva, is said to have been transported by the waves from one island to another, closed inside a drum. All the characters had their own *pahu*, certain events even took their name from it. Sacrifices on the *marae* were accompanied by that sound. During navigation in their pirogues, they emitted signals with their drums, or by blowing into large *pu* [shells]; the rhythm of the drums corresponded to that of the paddle strokes. Festivals, songs, ceremonies, dances, rituals were all performed to the sound of innumerable drums constructed in various forms and dimensions: created by hollowing out tree trunks, and covering them with shark skin.[12]

For our arguments, the most interesting instrument appears to be the flute. "Le *vivo*, flûte de bambou a environ 30 cm. de long, une de ses extrémités fermée par le

[9] See below, Sect. 13.3.

[10] Ascher & Ascher 1994.

[11] Aria 2007. I am indebted for a large part of this section to the help of Matteo Aria, who was my guide on a journey to the Society Islands in French Polynesia.

[12] Henry 2004 , pp. 107, 135, 130, 163–164 and passim. *Collections* 2001, pp. 84–87.

noeud du bambou porte un petit trou par lequel on souffle avec la narine. La flûte a cinq trous sur le dessus pour les doigts ed un en-dessous pour le pouce. Cet instrument donne un son doux et plaintif." ["The *vivo*, a bamboo flute, is about 30 cm long; one of its extremities, closed by a knot in the bamboo, has a small hole into which you blow with one nostril. The flute has five holes on the upper side for the fingers, and one underneath for the thumb. This instrument produces a sweet, plaintive sound."][13]

As the distances between the holes generate the ratios of the intervals, *ceteris paribus*, we may try to work back from them to the notes that could be played on the *vivo*. The three *vivos* present in the Tahiti Museum offer us the following ratios listed below. The real dimensions are bigger, but of course, these would change the number listed only proportionately, and would therefore not have any influence on the relative heights of the sounds produced. Anyway, the height in millimetres of the three *vivos* considered is (a) mm 355 (b) mm 425 and (c) mm 615.

On the basis of a proportionate reduction with respect to the figure in the catalogue, the following numbers are obtained. The holes are counted starting from the one closest to the part blocked by the bamboo knot, where the player blew with his nostril.

(a) Total length 120, external diameter 11:
distance of the holes from the closed extremity 1st 4–7; 2nd 23; 3rd 74
(b) Total length 144, external diameter 10:
1st 3–7; 2nd 19–20; 3rd 85; 4th 114; 5th 120
(c) Total length 213 external diameter 15:
1st 2–4; 2nd (lateral) 48–49; 3rd 107–108; 4th 204–205[14]

On the basis of these extant musical instruments, can we thus now imagine any criteria in making the holes at different distances to obtain the desired melodies? As they could easily find bamboo canes of different lengths and various diameters in nature, what could it mean if they generally chose larger diameters for the longer ones? For the longer *vivos*, did they cut only bigger canes with the relatively greater distance between the knots? Or had they realised that the notes harmonised better in this case which was spontaneously present in their forests, where the diameter increased without doubt together with the length?

Played simultaneously with all the holes stopped up, the ratio created between *vivo* (a) and *vivo* (b) is

$$120 : 144 = 8 \times 15 : 8 \times 18 = 15 : 18 = 5 : 6$$

between (a) and (c)

$$120 : 213 = 8 \times 5 \times 3 : 71 \times 3 = 8 \times 5 : 71$$

[13] Henry 2004 , p. 283; *Collections* 2001, pp. 86–89.

[14] *Collections* 2001, pp. 88–89.

and between (b) and (c)

$$144 : 213 = 16 \times 9 : 71 \times 3 = 16 \times 3 : 71.$$

Some of the ratios that can be obtained from the distances between the holes of the single *vivos* are as follows, where we have adjusted the numbers a little when the holes were wide.

(a) Between the 1st and the 2nd holes 6:23;
Between the 1st and the 3rd 6:74;
Between the 2nd and the 3rd 23:72.
(b) Between the 1st and the 2nd holes 5:20 = 1:4 ;
Between the 1st and the 3rd 5:85;
Between the 1st and the 4th 5:114;
Between the 1st and the 5th 5:120 = 1:24, equivalent to 1:3, cutting the octaves;
Between the 2nd and the 3rd 20:85;
...
Between the 3rd and the 4th 85:114;
...
Between the 4th and the 5th 114:120;
(c) Between the 1st and the 2nd holes 1:24, equivalent to 1:3, cutting the octaves;
Between the 1st and the 3rd 1:54;
...
Between the 2nd and the 3rd 48:108;
Between the 2nd and the 4th 48:204;
Between the 3rd and the 4th 108:204.

In this case, my indulgence in a certain rhetoric of numbers, clearly reminiscent of Leibniz and Euler, does not stem from the claim to reconstruct an ancient presumably authentic scale of Polynesian notes. More than anything else, I would like to succeed in pointing out to the benevolent reader a possible course followed in the choice of the intervals. Hoping that these *vivos* preceded the arrival of coloniser missionaries, they should be played in their reality, to obtain the intonations used by the natives of the islands, following their ears. Though it would not be difficult to get them with a certain approximation, we should not expect to find our usual intervals, e.g. of the octave, the fifth and the fourth, but others different. The *vivos* that are preserved in Paris would likewise lead us to other notes.[15]

And yet, whatever they were, an artisan experience had fixed them in the distances between the holes made in the bamboo canes. Can anyone stop me from imagining that once a satisfactory *vivo* had been made, the artisan might make some more, following the same criteria? And would he not have had to measure the distance between the holes in some way? And could he not then have realised

[15]In Paris, the *Musée du quai Branly* exhibits several *vivos*, though I have not yet had access to them in order to study them.

Fig. 13.2 *Marae* of Taputapuatea at Raiatea ("Taputapuatea", photo of the author)

that it was necessary to change the distances appropriately on different canes, in order to produce the desired sounds? And so on.

In this process, the results of which have remained in the *vivos* that have survived colonisation from the West, we are forced to recognise the birth of criteria for musical intervals. As these had been translated into distances between holes, here we are in the presence of something scientific and mathematical. It does not seem to be so important to me whether this is called "natural mathematics", or "implicit mathematics" or "unwritten ethnic mathematics". Whatever name is given to it, we must avoid projecting on to it some conviction of our own about universal mathematics or Pythagorean-Platonic philosophies.

The Ma'hoi carried out their religious ceremonies on their *maraes*. Marked off by layers of stones, these reveal a certain geometry. I describe below the shape of the *maraes* visited and measured with Matteo Aria on the island of Moorea, in the archipelago of the Society Islands. In general, the *marae* also includes an *ofai'tià*, a larger stone set upright. (Fig. 13.2)

The Muura *marae* includes an *ofai'tià* and various squared stones. Also the Titiroa one has its squared stones, with an *ofai'tià* set upright. Its sides measure, in cm,

$$3525 - 3780 - 1565 - 1460$$

The sides of the Ahu-o-mahine [Altar of the daughter] *marae* are

$$2280 - 2110 - 880 - 960$$

That of Afareaito

$$3140 - 2950 - 1550 - 1260$$

Fig. 13.3 Squared steps of a *marae* at Raiatea ("Squared steps of a *marae*", photo of the author)

At Raiatea, one can climb up squared steps on to the *ahu* [altar] of a *marae*. (Fig. 13.3)

The *ahus* at Taputapuatea [vast space with *tabù*, sacred, prohibited] are made of squared blocks of coral.[16] Squared stones can also be found in various *maraes*. In the ma'hoi language, *tuea* means 'setsquare'. Are these all good evidence that the inhabitants of the islands knew how to construct relatively precise right angles? How did they do that? Is it legitimate to imagine some relative geometrical property, seeing that they succeeded, and the results have been preserved?

The shape of the *maraes* visited with Matteo Aria is not rectangular, because the two long sides are too different. They are reminiscent rather of slightly crooked trapezia. Ancient accounts of the most famous *maraes* described them as "...en forme de parallélogramme pyramidale ..." ["...in the shape of a pyramidal parallelogram ..."]. "Sous la direction d'un artisan appelé *tahu'a-marae*, ils taillaient les pierres irrégulières en blocs et en dalles avec instruments de pierre dure; les pierres rondes étaient laissées telles quelles et servaient à boucher les interstices. [...] Si le *marae* royal était construit dans une plaine ou sur un plateau, il prenait une forme de parallélogramme. S'il était contruit sur le sommet d'une colline, il prenait une forme carrée, contituée par un mur élevé." ["Under the guidance of a craftsman called a *tahu'a-marae* [artisan of the *marae*], they cut irregular stones into blocks and slabs, using hard stone instruments; the round stones were left as they were, and were used to fill the interstices. [...] If the royal *marae* was built on a plain or on a plateau, it took the shape of a parallelogram. If it was built on a hilltop, it took the square shape of a high wall."][17]

[16] Aria 2007, pp. 173–176.

[17] Henry 2004 , pp. 126, 138–139.

May there have been some connection with the Indian rituals delimited by the *vedi*?[18] Or did each of them invent similar sacred spaces?

At Tahiti, in the nineteenth century, the Maha'i-atea [prolonged satisfaction] *marae* still existed: this was a large pyramid with a rectangular base, built on ten flights of steps. "Les marches sont formées de rangées régulières de pierres de corail taillées en carré ayant environ quarante-cinq centimètres de hauteur, ensuite de galets bleuâtre presque ronds d'une matière très dure utilisés tels quels et de quinze centimètres, ... il leur a fallu d'immenses efforts et un temps très long pour apporter toutes ces pierres, ainsi que pour tailler les dalles de corail avec les instruments dont ils disposaient à l'époque." ["The steps are made of regular rows of coral stones cut into squares about 45 cm high, and then almost round bluish pebbles of a very hard material, used in their natural state, measuring 15 cm. ... it took them enormous efforts and a very long time to transport all those stones there, and also to cut the slabs of coral with the instruments available at that time."][19] To the best of my knowledge, the (implicit?) geometry of these *maraes* has not been studied yet. The problem of gaining a good knowledge of them is further complicated by the event that archaeologists have at times been careless in their restoration work, as regards the original forms not clearly surveied.

The largest and most sacred *marae* was the ocean. Certain stakes planted in the ritual space represented the stars in the sky, indicating each of them precisely by its name.[20] The ma'hoi culture crosses the whole of the Pacific Ocean from the Hawaii to New Zealand, which is inhabited by the Maoris. There is no doubt that their pirogues were capable of surmounting enormous distances over the open sea, arriving at their destinations in order to take part in the religious ceremonies at Raiatea on the most important *marae* of Taputapuatea, or maybe to make war on one another. How did they succeed in sailing from one island to another?

In the eighteenth century, Captain James Cook (1728–1779) had been guided on a tour of the archipelagos by Tupaia, the religious chief of Raiatea. "Etant un bon navigateur, il avait dessiné grossièrement une carte indiquant la position des diverses îles, les courants et les vents. Il leur indiqua aussi les noms de beaucoup des îles de la Polynésie. ... Actuellement à Raiatea, les discendants de Tupaia sont encore de bons navigateurs et pilotes." ["As he was a good seaman, he had roughly made a map indicating the position of various islands, the currents and the winds. He also told them the names of many islands of Polynesia. ... At present [in the nineteenth century] the descendants of Tupaia at Raiatea are still good sailors and pilots."][21] However, Cook is said not to have understood Tupaia's schematic map, because he read it wrongly.[22]

[18] See above, Part I, Sect. 3.1.

[19] Henry 2004 , p. 146.

[20] Henry 2004 , pp. 141, 151 and 370–371.

[21] Henry 2004 , p. 29. *Oxford Companion* 1979, pp. 199–203. Lewis 1994, pp. 9–10 and 342–348.

[22] Matteo Aria, private communication. Lewis 1994, pp. 342–345 and 398.

On his first voyage to Tahiti, the English sea captain's aim was to observe the passage of Venus in front of the face of the Sun, and to search for any continent that might exist to the south of Australia or New Zealand. For this reason, he had to calculate various longitudes using the methods available. But he still did not take with him the precision chronometers invented for this purpose by John Harrison.[23] Instead, he measured them, in a more complicated way, by referring to the time of Greenwich, measuring the angles between the Moon and the fixed stars, which change rapidly in time. It was only on his subsequent voyage, ordered by the Royal Navy, that Cook's ship had a chronometer on board.

Thus there had been a direct comparison between the art of navigating followed in the ma'hoi culture, and the Western method based on the compass, nautical maps, the clock, the sextant and astronomic observations. Also the original inhabitants of the islands in the Pacific knew how to navigate so well that they were able to provide precious information for the colonisers.

Starting from Claudius Ptolemy's *Geography*, and on through the studies of Arabs[24] and the works of Gerhard Mercator, drawing precise nautical maps with a grid of latitudes and longitudes had always been important for every policy of exploration and conquest. As we know, the requirements of navigation had often influenced the evolution of the mathematical sciences. In order to determine the *qibla* [direction of the Mecca] and pray in accordance with the *Koran*, as in their trading across the Indian Ocean, the Arabs had elaborated relatively effective astronomical methods. They used catalogues of stars, measuring their height by means of manual methods or with an astrolabe, and they followed routes using a compass.[25]

"Grâce à la carte de Tupaia et à d'autres sources de renseignements Percy Smith a pu démonstrer que les Polynésiens et sourtout les Tahitiens avaient une maîtrise complète de l'Océan. Ils le traversaient dans tous les sens avec leurs pirogues, tandis qu'à la même époque, les Européens, dans des navires beaucoup plus importants, se contentaient de longer les côtes des continents de l'Est." ["Thanks to Tupaia's map and to other sources of information, Percy Smith was able to demonstrate that the Polynesians, and above all the inhabitants of Tahiti, had a complete control over the Ocean. They sailed across it in all directions in their pirogues, while the Europeans, in the same period, using much more important ships, were content to hug the coasts of the Eastern continents."][26]

The Ma'hoi were masters in the art of constructing *vaá* [pirogues] of all kinds, above all those that are their characteristic, *vaá tipae ama* [canoe with outrigger], *vaá tipae ati* [united canoes] and to famous *pahi* [canoes with two hulls]. They used them to "tere na te moana" ["sail over the deep abyss, the ocean"]. *Taàta orero* [person who speaks] was the name given at Tahiti to men who knew how to navigate,

[23] See above, Part II, Sect. 10.2.

[24] Kennedy 2002.

[25] Grosset-Grange 2002.

[26] Henry 2004, p. 287.

assuming the elevated rank among chiefs and priests. Tupaia had been a deposed *taàta orero* and was thus ready to reveal his secrets to the English.[27]

The abilities of ma'hoi navigators must have seemed so impossible that some have tried to verify them in practice with the help of the last survivors who still remembered them. In the second half of the twentieth century, Tevake took Westerners on board pirogues among various islands of the Pacific.[28] Cook stated that the *pahi* of Tahiti were much faster than his ship,[29] because they had two hulls. Without any instrument, the *taàta orero* maintained their route with the *aveià* [course of the stars].[30] At the Gilbert Islands, the sky was represented by the roof of the large hut for meetings. Teeta had learnt to navigate there. "A steering star has properly the bearing of an island at a slight but definite altitude, which is opposite the first or second beam of the meeting-house (*maneaba*). The star at this slight height (of about 15°) marks the correct bearing of the island. When it rises or sinks too far, another on the same bearing is used."[31]

For all these expert navigators, the stars, with their rising and falling on the horizon, represented a real compass, allowing them to pilot their pirogues northward, southward, eastward or westward, wherever they wanted to go. Without writing it down, or representing it on nautical maps, the coupling of stars with islands, and with the direction of the prow, was learnt by heart by reciting it.[32] This is why the navigator was called *orero*. At latitude 17° South, the Pole Star was not visible at Tahiti; and yet their songs call it Ana-nia [the upper cavern]. Thus they also navigated by using it in the north of the Ocean.[33]

The ma'hoi cultures exploited every element that was useful in order to navigate: the winds, the currents, the waves, the shadows of the sun at midday, the flight of birds. It was even related: "I have heard from several sources, that the most sensitive balance was a man's testicles, and that when at night or when the horizon was obscured, or inside the cabin, this was the method used to find the focus of the swells off an island."[34] Those who appreciate similarities will notice that also in this different culture, navigating was a question for males.

In the eighteenth century, also the Spaniard Andia y Varela had verified the precision of the Tahitians in navigating. "Since the wind is apt to vary in direction more than the swell does, they [the Tahitians] have their pennants [made] of feathers and palmetto bark, to watch its changes by and trim sail, always taking their cue for a knowledge of the course from the indication the sea affords them."[35]

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[27]Lewis 1994, pp. 56 and 373.

[28]Lewis 1994.

[29]Lewis 1994, p. 70.

[30]Lewis 1994, p. 104 and passim.

[31]Lewis 1994, p. 97.

[32]Lewis 1994, p. 382.

[33]Henry 2004 , p. 371. Lewis 1994, p. 111.

[34]Lewis 1994, p. 127.

[35]Lewis 1994, pp. 84 and 133.

Though aided by more sophisticated instruments, still today, those who sail the seas know that they have to bear in mind many different phenomena which contribute to the route, but also disturb it. The same was true for the natives of the Pacific, who invented their own systems to take into consideration the currents, prevalently westerly and south-westerly, generated by the easterly and north-easterly winds. Then they observed the swells and the wake of pirogues, to judge the leeway, that is to say, the movement of the boat not in the prow-stern direction, but slightly crosswise. They calculated their speed by observing the time taken to pass the foam on the swells.

But how did they measure what we are accustomed to calling 'time'? They used *mahana* [Sun] to indicate the weather, *tau* for the seasons and ages, and *taime* was the time that inexorably passes: the word was clearly learnt and borrowed from the English. The Ma'hoi did not count the years that passed[36]; the various moments of their days, called *ru'i* [nights], were marked by their events. Tides, for example, are constant all the year round: the morning low tide at 6 a.m., high tide at midday, low tide again at 6 p.m. and high tide at midnight.[37]

They were capable of moving amid all these various events, connecting up with them, and allowing themselves to be carried along by them. If they did not try to impose themselves on events by means of an external personal rhythm, what use could an abstract, absolute time be to them? And yet in their canoes with drums sounding, recurring rhythmic elements could help them to measure other phenomena, such as speed, or the winds, and so on. Did they sing, maybe, during their navigation? Did the *orero* recite their long litanies? Just as Galileo Galilei in Pisa Cathedral was to make use of music to study the pendulum, did the ma'hoi culture, perhaps, mark the rhythm of their navigation, and the course of life with songs and sounds? Aristotle had defined time as the measure of movement, and thus it was a derived, secondary notion which we know was rich in paradoxes in the Greek culture.[38] Was the primary notion of movement itself perhaps sufficient, then, for the Ma'hoi?

In any case, they showed the skill of navigating among innumerable faraway islands, even though Cook did not record how Tupaia succeeded in maintaining the route.[39] It has recently been admitted that ignorance likewise exists regarding the procedures followed, which were clearly capable, however, of obtaining effective results. May all this mask the unfathomable problem of how cultures guided by incommensurable values can communicate together?

At the Caroline Islands, they had invented a system for navigating called *etak*. They coupled stars with islands, but with the original idea that the star stood still, while the island changed its position during the voyage.[40] A Western navigator quite

[36]Henry 2004 , p. 196.

[37]Henry 2004 , pp. 333–342.

[38]See above, Part I, Chap. 2.

[39]Lewis 1994, p. 168.

[40]Lewis 1994, pp. 173–191. Ascher 1991, pp. 140–149.

rightly concluded that the system was incompatible with methods based on fixed maps: actually the representations of islands remain immobile on them. The Western way of abstracting what pertains to description from immediate reality ends up by fixing the elements at stake in a visible, static diagram known as a nautical map. On the contrary, the natives represented their islands as in movement, under fixed stars.[41] At the Marshall Islands, representing the islands on variously arranged sticks served, above all, to instruct apprentices and to aid the memory as regards how the great swells of the Pacific changed as they were encountered. From the swells thus, they succeeded in deducing the presence of an invisible island below the horizon.[42]

Map-makers place the reference system on the Earth, and thus navigators must chase the stars around the sky with sextants and clocks. At the Caroline Islands, abstractions were made in the head by referring to the starry sky: consequently, what moved had to be the Earth. In spite of Galileo Galilei and Kepler or Newton, the former had remained Ptolemaic, while at open sea. Whereas, pending the arrival of self-styled civilisation, our 'wild men' of the Pacific had placed their reference system in the fixed stars, and were thus Copernican (not to say relativistic)! If we could choose where to fix the reference points in order to understand and guide the movement of boats, not only was the Micronesian viewpoint not 'mistaken', but according to many testimonies, its efficacy was comparable to the Western one.

In the Ocean, winds, currents, waves, clouds, animals, storms, and corals are at work; whether large or small, boats make leeway, tossed here and there, and their speed continually changes; the movements of the Moon, the Sun, the planets and the other stars are not perfectly regular. Some contemporary philosopher might call it a "complex" system, and verify the limits or the defects of the 'simplified' Galileian-Keplerian-Newtonian-Eulerian model on it. In this case, it is legitimate to think that the manner of navigation followed by the natives of the Pacific equally was as successful as those of the various brigs commanded by Cook, Louis Bougainville (1729–1811) or Samuel Wallis (1728–1795). Isn't it simply Eurocentric arrogance to call the former 'primitive' and the latter 'scientific'?

In keeping with our style, I will not deal with the controversial problem of where the inhabitants of the Pacific came from, and how they spread.[43] And yet, if we chose to take into consideration the ma'hoi religious rites, and above all their music, a certain similarity exists with the cultures of India.[44] The hypothesis cannot be immediately discarded that the pre-Vedic Indians came from the Pacific Ocean, although it is more likely that the opposite is true.

Bernhard Rigo has recently underlined the cultural diversity of the Ma'hoi, stressing the characteristics of their language which is particularly polysemous and contextual. Among these, he noticed that the Western primacy maintained by the notion of "being" with the relative verb did not exist in Polynesia, because the word

[41] Lewis 1994, pp. 184–187.

[42] Lewis 1994, pp. 224–252.

[43] Lewis 1994, pp. 342–356.

[44] See above, Part I, Chap. 4.

was not present in their vocabulary. Furthermore, nouns and adjectives are confused with verbs, which do not have past, present or future tenses.

In contrast with Cartesian dualisms, heaven and earth were expressed as united by means of terms indicating their qualities: "atea papa" ["clear, transparent, spatious / rocky, stony"]. The ma'hoi language did not express the typical Western separation between space and time. "Ra'a" means both "there" [place] and "then" [time]. "Muri" means both "after" and "behind", "mua" both "before" and "in front of". Only the context can make the distinction.[45] The ma'hoi universe is made up of a continuous chain, without any interruptions, of living creatures generated from one another: gods, men, animals, vegetables. "Tupu te mau mea oraora o te tai, o te vai e o te fenua …" ["The things living in the sea, in sweet water and on land grew …"].[46]

Giancarlo Scoditti has underlined the important role played by music in the Melanesian culture of Kitawa. Through this, we can get a better idea of their concepts of time and space, and thus their distance from the Western sciences. From a precise study of their flutes, too, it would be possible to work back to the particular musical scales used on the island. But the relative studies have not been carried out yet. The impression of a pentatonic scale "typical of Chinese music" needs to be verified. This Italian ethnologist, who received an Anglo-Saxon training, though, has recorded how the natives of Kitawa distributed their notes into three tones: "kaiga-veka, kaiga-werai, kaiga-kekita" ["high tone, middle tone, low tone"]. He is in favour of extending its organisation along a "logarithmic spiral", like those engraved as decorations on canoes, inspired by the nautilus mollusc.[47] Without doubt, it would be more simple to measure the lengths of their flutes, but we are not given any findings or examples, whereas we possess various forms of drums. May there be a *taboo* on flutes?

The most singular aspect of this culture is the existence of *taboo* pieces of music, which must not be played: the "namunamu megwa" ["mute song, execution without sounds"]. "La voce del cantore oltre ad essere ascoltata viene, paradossalmente, vista: i suoni cantati con la voce concretano sullo spazio, rendendola quasi materica, l'immagine composta dal poeta. Ma ci sono anche immagini mentali che la voce non esegue e che il villaggio non ascolta: sono le immagini arcaiche, forse tra le più antiche della cultura nowau, racchiuse nella mente dei maghi (*tobwagau*) di Kitawa. Immagini classificate come pericolose, immagini che, se espresse con la voce, potrebbero materializzare il loro contenuto negativo, confermando che il suono attribuisce consistenza all'immagine pensata. Ancora una volta il tempo, il flusso dei suoni che realizza un'immagine, viene ridotto dal pensiero nowau a spazio, e assenza di voce è assenza di spazio, una sottrazione che inquieta un abitante di Kitawa. Il mago che recita nel silenzio della mente una formula poetica muta, o a-sonora, rende immateriali le sue immagini che, non essendo ascoltate

[45] Rigo 2004, pp. 15, 19–20, 24, 95–96 and passim.

[46] Rigo 2004, p. 86.

[47] Scoditti 2003, pp. 78–79, 21, 45, 84, 89.

dal villaggio, lo precipitano in uno stato di tensione determinato dall'assenza dello spazio in cui si concreta il suono. Spazio e suono sono le coordinate mentali più concrete che la cultura nowau abbia concepito." ["Besides being heard, the singer's voice is paradoxically seen: the songs sung with his voice become concrete in space, making the image composed by the poet almost material. But there are also mental images that the voice does not perform, and which the village does not listen to: these are the ancient images, perhaps among the most ancient of the nowau culture, contained in the minds of the (*tobwagau*) magicians of Kitawa. Images classified as dangerous, images which, if expressed with the voice, could materialise their negative contents, thus confirming that the sound attributes consistency to the image conceived. Once again, time, the flow of sounds that realises an image, is reduced by nowau thought to space, and the absence of voice means the absence of space, a subtraction that disturbs an inhabitant of Kitawa. The magician who recites in the silence of his mind a mute, or a-sonorous poetic formula, makes his images immaterial, and as they are not heard by the village, they precipitate it into a state of tension determined by the absence of space in which sound is made concrete. Space and sound are the most concrete mental coordinates that the nowau culture has conceived."][48]

In the forgotten little island of Melanesia, Scoditti seems to have rediscovered Wagner's *Parsifal*, where "Time becomes space". Anyway the flow of sounds is capable of reconstructing the *continuum* of the genealogies, from the earliest forefathers to the present time, by means of the funeral laments and and the intoned recitals of mythical accounts. Thus the language is modelled on music.[49]

Whether cultural or otherwise, the diversities of inhabitants of the islands of the Pacific have been widely studied among anthropologists. But while their techniques in order to navigate across the great ocean swells have always attracted the interest of Western studies,[50] very little research has so far been conducted to throw due light on their special scientific knowledge.[51]

[48] Scoditti 2003, pp. 73–76, 88, 51–52 and passim.

[49] Scoditti 2003, pp. 27 and 86.

[50] Finney 1976.

[51] Morrison 1994. Tera'ituatini Pambrun 1994.

Chapter 14
Come on, Apophis

Exactly the same thing happens in the case of historical truth as with all the other truths: mistakes are made, to a greater or a lesser extent.

Marguerite Yourcenar

Actually items of human knowledge are not divine truths, but only ways to arrive at truths. . . . But men in this country are very proud of all the items of knowledge that the human world has acquired up to this moment. In these items they see truths. And although they learn that the knowledge of yesterday is confuted by that of today, all the same they firmly believe in the knowledge of today; as if no tomorrow or no day after tomorrow existed.

Joseph Roth

14.1 Gott mit uns

In the eighteenth century, Johann Gottfried Herder (1744–1803) wrote: "The finest research on history and on the manifold characteristics of the human mind and heart would thus be a *philosophical comparison of languages*; because the mind and the character of a people is impressed in each of them." [. . .] ". . . linguistic differences may lead to differences, or even to incompatibility in spiritual processes."[1]

Homogenized and extinct in the global academic world, some interesting hints can be found in a lateral discipline known as "Ethnomathematics".[2] Unfortunately, it is sometimes limited to the problems of teaching, instead of trying to understand better the nature of cultural differences.

[1] Quoted in Chomsky 1969, pp. 70 and 120–121.

[2] Ascher & Ascher 1994; Ascher & Ascher 1997. Ascher 1991.

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As regards Africa, the scholar who would have pushed forward in this direction is the European who emigrated there, Paulus Gerdes. "We developed a complementary methodology that enables one to uncover in traditional, material culture some hidden moments of geometrical thinking. ... We looked to the geometrical forms and patterns of traditional objects ... we learned the usual production techniques ... The traditional form reflects accumulated experience and wisdom."[3]

Books about acoustics have often proceeded in the opposite direction to what I have related here. In them, music is often left aside, and reduced to the minimum; although it is admitted that "This is shown by its concern for music, out of which indeed the science of acoustics developed ...". From the diagram (Fig. 14.1), the proportions attributed to music by books about acoustics are clear, and these are undoubtedly those conceived today for our epoch. In this essay, on the contrary, I have shown that the relationship should be inverted, if we wish to remain faithful to historical evolution. Otherwise, history is corrupted, reducing it simply to *advertising* for the discipline. As offered by Bruce Lindsay, its history is not an authentic *history*, but rather a *story*, that is to say, story-telling. In actual fact, the title of his article is "The Story of Acoustics". The expression is to be taken in its double meaning. The characters of these stories are not similar to ours, and when their names seem to coincide, we are given an image of them appropriate to magnify the glories and the errors along the route towards the birth of acoustics, culminating in Lord Rayleigh's *The Theory of Sound* which was published in 1877.[4]

It is difficult to find music in this kind of literature, as there was none in the essays of Truesdell about Euler. Conditioned by the common-places of his environment, Lindsay completely overlooked a musician like Vincenzio Galilei. Sure enough he wrote: "... the 'end corrections' [for open pipes] were understood some half-century later [XIX century]." On the contrary, it has long been known that this "end effect" to tune music with pipes correctly was already known to the ancient Chinese, and to Vincenzio Galilei in Europe.[5]

The Anglo-Saxon cultural context sounded different from the German one. At the beginning, his Lordship lamented the recent premature death of the astronomer from Oxford, William Fishburn Donkin, who had already started writing a treatise similar to his own. However, it was to have had one characteristic that was different: it still contained a little bit of music! For Donkin revealed a practical knowledge of the art made of notes, and would have liked to reserve the second volume of his work to music. Unfortunately, this was never written, and only the first one was published posthumously.[6] When it came out in 1870, it was reviewed in an issue of *Nature* in the following words: "... the part that is published treats of subjects of so fundamental a nature, and so little dependent on what would have followed them

[3] Gerdes 1999, p. vii.

[4] Rayleigh 1945. Lindsay 1981.

[5] Rayleigh 1945, pp. 180–183 and 487–491. Lindsay 1945. Lindsay 1972, pp. 2, 13 and passim. Grattan-Guinness 1994. See above, Part I Chaps. 3 and 6.

[6] Donkin 1870.

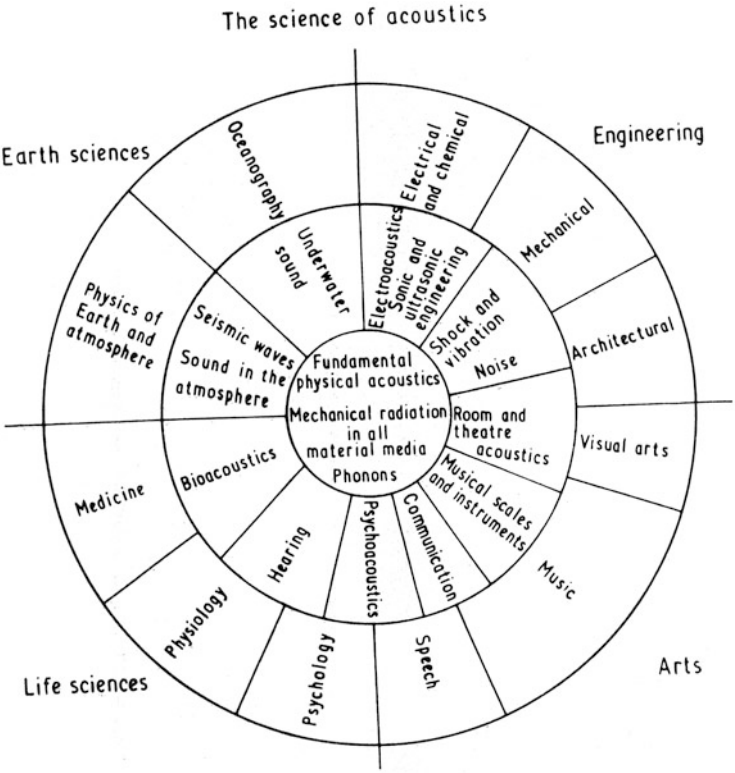

Fig. 14.1 How Lindsay conceived of the science of acoustics ("Acoustics" in Lindsay 1972, p. 2)

[music], that its intrinsic value is probably not much lessened by the absence of the remainder [music]."[7]

Rayleigh must have taken this observation as a suggestion, because he put very little music only in the Introduction and in the final chapter dedicated to the sensibility of the ear. Was perhaps he following Helmholtz in this? Like the German physicist and physiologist, the Englishman effectively appreciated the sounds generated by sirens, because in this way, it would be easier to measure the frequencies. The "…true intonation" would be the one based on ratios between whole numbers [like the scale of Zarlino]. For the English peer, the proportional ratio between the period $T \propto \frac{1}{\nu}$ of the sound and the length of the vibrating string, like that existing between T and the reciprocal of the root extracted from the tension, became the Mersenne's laws[8]: Vincenzio Galilei had disappeared.

[7]Foster 1870.

[8]Rayleigh 1945, v. I, pp. 11 and 180–184.

For chapter XXIII, the last one, Rayleigh relied above all on Helmholtz, complaining, however, that among scholars there were "... divergent views ... observations ... so discrepant that no satisfactory conclusion can be drawn at the present time." All chapter XXIII was added in 1896 for the second edition. "Although, as we shall see, some of the positions taken by the author [Helmholtz] have been relinquished, perhaps too hastily, by subsequent writers, the importance of the observations and reasonings contained in it, ..., ensure its long remaining the starting point of all discussions relating to sound sensations."[9]

However, it should be noted that, like Seebeck,[10] Rayleigh too registered the limits of the theory sustained by Ohm regarding the intensity of harmonics. Our English peer provided many experimental details about the differential sounds discovered by Tartini, reopening the controversy whether they were only due to the physiology of the ear, or were present also in the air, and could therefore be physically measured by means of resonators. In the end, he was in favour of the solution offered by Helmholtz, that, depending on the instruments, some, but not all, were only effects of the ear. The question "... seems to be more easily explicable upon the basis of Helmholtz's theory. Upon the whole, this theory seems to afford the best explanation of the facts thus far considered, but it presupposes a more ready departure from superposition of vibrations within the ear than would have been expected."[11]

The most important problems of music were either put aside, or traced back to the experimental acoustic effects that were most congenial for him. "It is not consistent with the plan of this work to enter at length into questions of temperament and just intonation." Helmholtz's theory about consonances and dissonances was based on beats, as we know,[12] and thus Rayleigh dedicated his attention to them, too, in order to verify the conditions. It seems that it was taken for granted that the physical phenomenon was sufficient to justify them, although this Englishman conscientiously recorded some varying opinions about how it was produced in the various cases. The comments made by various scholars were often critical of Helmholtz. However, there is no doubt that for Lord Rayleigh too, for example, the interval of a fifth between notes had the "... true ratio 3 : 2 ...".[13]

Acoustics was born losing all memory of, not to say the contact with, music, but now it began to benefit from electromagnetism and the invention of electromechanical instruments like the telephone and the phonograph.[14] Helmholtz had been less technical and less of a mathematician, whereas Rayleigh had written a real treatise of experimental physical acoustics, with mathematical calculations and many practical details.

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[9] Rayleigh 1945, v. II, pp. viii, 432 and passim.

[10] See above, Part II, Sect. 12.2.

[11] Rayleigh 1945, v. II, pp. 443–446, 462.

[12] See above, Part II, Sect. 12.2.

[13] Rayleigh 1945, v. II, pp. 455, 454 and passim.

[14] Rayleigh 1945, pp. 366–367, 440–448, 473–474 and passim.

When writing a historical introduction to recent reprints of *The Theory of Sound*, Lindsay too realised that something would be missing in current histories of sciences. But for him, the defect would be that "... the history of acoustics has been largely a neglected subject." Instead, what is usually neglected is music, which we know was largely present for centuries in the interests of many natural philosophers. Acoustics, as a physical discipline, was born only in the nineteenth century. Even though he was completely ignoring the question of which notes to tune on musical instruments, our physicist from the United States went so far as to complain that they "... lay rather aside from scientific progress in acoustics, ...". He even got to the point of maintaining that his acoustics would influence them in future. Isn't it a comical situation? Seeing how many experiments had already been conducted in this field during the past. As an experimental physicist, he was confusing the effects with the causes.[15] Although deformed, the stories that Lindsay tells remain illuminating up to a certain point, because they confirm the exit of music from the central scenes of sciences.

Within the field of computer science disciplines, with an abundant use of computer programmes, a certain interest has recently re-emerged to bring music to the surface from the history of the mathematical sciences. Is ancient Pythagoreanism returning in another guise? Anyway, in this new context, Jean Dhombres has tried to justify, by pleading the habits of academics long specialised in mathematical physics, Lagrange's declared indifference to music. But in the enthusiasm of his argument, unfortunately, our French historian forgot that the mathematician from Turin denied the harmonics of notes, and followed the theory of music proposed by Euler.[16] For the period preceding Vincenzio Galilei, unfortunately, the good summary compiled by Manuel Pedro Ferreira avoided insisting on Aristoxenus, as the antagonist of the Pythagorean-Platonic philosophies in musical practice.[17]

Benedetto Scimemi has related that in the eighteenth century, theoreticians and makers of instruments succeeded in coming close to the equable temperament. For this, we find the Mesolabium used by Zarlino (and by Descartes), Giuseppe Tartini's calculations, or how to arrange the frets on the neck of a lute. The Italian mathematician concluded: "It is difficult to decide whether, in the course of history, music gained more from mathematics or the contrary. What we certainly must not do, at any rate, is to place the two disciplines in two separate and incommunicable worlds. History teaches us that the boundaries of art, craftsmanship and science are often shaded and movable."[18]

Nowadays, no science historian has paid so much attention to music as Eberhard Knobloch. He has sometimes concentrated on combinatorial analysis, as inspired by notes, and applicable to them.[19] This German scholar has dealt with Athanasius

[15] Rayleigh 1945, pp. xi and xvii.

[16] Dhombres 2002. See above, Part II, Sect. 12.1.

[17] Ferreira 2002. See above, Part I, Chap. 2.

[18] See above, Part II, Sect. 9.2. Scimemi 2002, p. 62.

[19] Knobloch 1979. Knobloch 1989. Knobloch 2002. See above, Part II, Chaps. 9 and 10.

Kircher, whom I have overlooked. As a partial excuse for this, I may now add that Kircher "...completely depended on Marin Mersenne's two great monographs on music theory, ...".[20] When writing about Kepler or Euler, he was careful not to omit their musical interests and papers.[21] Considering the wide range of his studies, it is to be hoped that Knobloch will offer us an overall work reflecting them.

If we were to accept the recurring competition, and compile tables of who would have arrived first, or had done better, in reality the book that I am terminating reveals that natural philosophers, mathematicians and physicists have drawn the most benefit from music. But unfortunately, we have also discovered how reluctant they, and science historians, have been to admit it. Whereas, on the contrary, the freedom of musicians did not benefit from their relative theories, and found difficulty in breaking out of the (useless?) numerical cages with the force of their ears, guided by the pleasure derived from music.

The patient reader may have noticed that, starting from Plato, the mathematical sciences often served, in Europe, to discriminate kinds of music considered for various reasons unsuitable for different contexts. I will finish this work by rendering explicit what I have allowed to emerge here and there during the story. The decision to move away from musicians and their music impoverished both natural philosophers, first of all, and then mathematicians and physicists. This influenced and facilitated the development of their research in those main directions which are known to everybody, but which continue to deserve to be criticised for their limitations and their (negative?) effects on our life.

However, our history does not come to an end here. Musicians, mathematicians and physicists will continue to create and to face among themselves. Who knows, for better or worse, what may be reserved by a future that is always open to any kind of change or catastrophe (in the sense, or not, of René Thom).

Thanks

My thanks are due, also in Swift's sense, to all the people who had, despite themselves, helped me, sometimes with other intentions. Oscar João Abdounur, Fabio Acerbi, Rocco Altieri, Matteo Aria, Mohammad Bagheri, Michele Barontini, Giacomo Benedetti, Remo Bodei, Umberto Bottazzini, Massimo Cacciari, Karine Chemla, Marco Ciaurro, Marcello Cini, Alberto Conte, Joseph Dauben, Elisabetta Donini, Umberto Eco, Paolo Fabbri, Giuseppina Fenaroli, Paul Forman, Paolo Gallarati, Giorgio Gallo, Giovanni Gargani, Antonio Garibaldi, Veronica Gavagna, Livia Giacardi, Franco Giannessi, Carlo Ginsburg, Giulio Giorello, Enrico Giusti, Paolo Gozza, Giovanni Gronchi, John Heilbron, Maurizio Jacono, Eberhard Knobloch, Liu Dun, Geoffrey Lloyd, Jean-Claude Martzloff, Paolo Marzocchi,

[20] Knobloch 2005, p. 336.

[21] Knobloch 2002a, Knobloch 2007, Knobloch 1987.

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> For truth to be complete, it must include the beginning, the middle and the end of a thing. Otherwise, it is partly false. For this reason, the Hebrew word *hemet* [truth] is spelt using the first letter, the middle letter and the last letter of the alphabet.
>
> Rabbi Na'hman of Breslev

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Index

Page references referring to volume 1 are in *italics*.

Abaris ou les Boréades, 423
Abel, *15*
Abel, Niels, 502
Abelard, Pierre, 308, 396
Abert, Hermann, 330
Achmet, 281
Ad Tridentinae Synodi Patres, *296*
Ad Vitellionem Paralipomena, 51
Adler, Max, 489
Adler, Oscar, 488, 490, 491
Aesthetic Measure, 501
Agamemnon, *22*
Agnesi, Maria Gaetana, 437
Agon, Carlos, *xiv*
Agustine of Hippo, 306
Airs, waters, places, *92*
Aiton, Eric J., 261
Alberti, Leon Battista, *276*, *279*, *286*
Alembert, Jean le Rond d', 1, *5*, *11*, *202*, 291, 301, 335, 336, 352, 353, 358, 360, 362, 363, 366, 368–383, 387, 388, 391–397, 399–404, 406, 411, 414, 415, 418–424, 431–434, 449, 451, 453, 455, 469, 480
Aleni, Guido, 33
Alexander the Great, *39*, *60*, *64*, *91*, *162*
Alf laila wa laila, *244*, *249*
al-Fusul fi 'l-hisab al-hindi, *231*
Algarotti, Francesco, 409
Algebra, *286*
Algorithmi de numero indorum, *203*
Allard, André, *272*
Almagest, *53*, 53, *205*, *217*, *225*, *250*, *257*
Al-Muadalat, *230*
Ammann, Peter J., 64, 107, 137
An Essay Concerning Human Understanding, 246
Anaxagoras, *85*
Anaximander, *85*
Angelucci, Eugenia, 407
Anselm of Canterbury, 173
Antiquae Musicae Auctores Septem, 201
Aphrodite, *25*, *63*, *67*, *73*, *77*, *80*, *93*
Apollo, *17*, *69*, *89*
Apollonius, *54*, *60*, *64*, 142, 245
Apomasaris Apotelesmata, 281
Applications of Mathematics to the Sciences of Nature: Critical Moments and Aspects The, *xiii*
Aquinas Saint Thomas, *39*, 145, *279*, *285*, 305
Arago, Pierre-François, 437
Archadelt, Jacques, 59
Archibald, R.C., 502
Archimedes, 11, 26, 34, *54–60*, *64*, *81*, *88–90*, 136, 142, *166*, 194, *228*, 245, 246, *259*, 261, *281, 282*, *292*, 294
Archytas, *16*, *18, 19*, *21*, *26*, *36*, *47*, *59*, *64*, *162*, *264, 265*
Aria, Matteo, 514, 517, 518
Ariosti, Attilio, 236
Ariosto Ludovico, 23
Aristarchus, *55*, *94*, *162*
Aristotle, 3, 18, *21*, 22, 24–26, *34*, *38–40*, 50, 51, *55*, *58*, *64*, 68, *83*, *85*, *87–89*, *91, 92*, *95*, 99, 110, 116, 120, 132–134, *156*, *162, 163*, 177, 179, 189, *211*,

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213, *218, 219*, *221*, *223, 224*, *234*, *246*, *253, 254*, *258*, 265, *269*, *279*, *290*, *299*, *305*, *308*, *326*, 451, 452, 522
Aristoxeni Musici Antiquissimi, 189
Aristoxenus, *4, 5*, 5, 6, *11*, *13*, 17, *18, 19*, 20–22, *35–38*, *40*, *42*, *45, 46*, 47, 48, *53*, 62, *64*, *94, 95*, 100, 105, 114, 128–130, *160*, *162*, 164, 165, 184–186, 189, 191, 192, *201*, 201, *202*, 202–204, 208, *213–215*, *224*, 233, 234, *235*, 235, *236, 237*, 237, 238, *242*, *245*, *248, 249*, *262–264*, *267–269*, *271*, *281*, *294*, *297*, *299*, 299, *300*, 300, *301*, 301, *303*, *309*, *311–316*, *319*, *323–326*, 341, 343, 350, *365*, *372*, 385, 393, 419, 421, 452, 505, 531
Arithmetic, *61, 62*, *226*, *229*
Arithmetica, 142
Arithmetica infinitorum, 187, 188
Arithmetica integra, *296*
Arithmétique, 7
Arius, *257*, *271*, *297*
Arlecchino, Imperatore della Luna, 264
Armi ed intenzioni di guerra, *xiv*
Armide et Renaud, 375, 376, 384, 385, 401
Armonikon, 37, *41*, *52, 53*, 53, 62, 63, 183, 201, *215*, *279*, *310*
Arnauld, Antoine, 144, 177, 306
Arnold, Vladimir I., 289
Arrighi, Gino, *259*
Ars magna, sive de regulis algebraicis, *286*
Arteaga Esteban de, 435
ArthaŚastra, *192*
Artusi, Giovanni Maria, 200
Aryabhata I, *181–183*, *187*, *203*
Aryabhatya, *195*, *203*
Asbab huduth al-huruf, *212*
Ascher, Marcia, 513, 514, 527
Ascher, Robert, 513, 514, 527
Aśoka, *192*
Aṣtadhyayi, *173*, *190*
Aston, Francis, 310
Astronomia nova, 38, 57, 80, 99
Atharva Veda, *170*
Atombau und Spektrallinien, 490
Attarzade, Muhammad ibn Husayn, *248*
Aubrey, John, 285
Augustine of Hippo, *90*, 145, 150, 156
Augustus, Octavian, *91*
Authier, Michel, *59*, *90*, *93*
Ave verum corpus, 449
Baas, Bernard, 237, 260, 261, 264
Babur mughal, *205*
Bach, Carl Philipp Emanuel, 404
Bach, Johann Christian, 404
Bach, Johann Sebastian, 10, *38*, 60, 237, 298, *304*, 322, 324, 331, 423, 425, 427, 428, 437, 440, 442, 453, 461, 477–479, 485, 487, 505–507
Bacon, Francis, 3, 119, 120, *145*, 189–191, 221, 238, *240*, 245, 284, 285, 297, 306, 317, 318
Bag, A.K., *173*, *177*
Bailhache, Patrice, 121, 229, 233, 234, 237–239, 241, 264, 419, 433, 504, 505
Bald, R.C., 78
Balslev, Anindita Niyogi, *189*
Balzac, Honoré de, 491
Ban, Jan Albert, 162, 178, 180, 211
Banchieri, Adriano, *304*
Banville, John, 75
Barbieri, Patrizio, 409
Barbour, J. Murray, 295–298
Bardi, Giovanni, *309*
Barker, Andrew, *53*, *95*, *336*
Barone, Francesco, 270
Barontini, Michele, *xiv*, *211*, *230*, *246, 247*, *335*
Barresi, Pietro, *361*
Barrow, Isaac, 270, 273–275
Bartók, Béla, 494
Basra Halil ibn Ahmad, *254*
Basso, Alberto, 331, 426, 427
Baumgardt, Carola, 66, 72, 73, 75, 107, 492
Bausani, Alessandro, *213*, *219–222*
Bayle, Pierre, 231, 242, 317
Beaugrand, Jean, 165, 177, 208
Beauval, H. Basnage de, 206
Beeckman, Isaac, *5*, 10, 158, 160, 161, 163, 164, 169, 170, 177–179, 181–183, 189, 192, 198, 209, 210, 221
Beethoven, Ludwig van, 437, 454, 455, 467, 482, 485, 487, 500–502, 507, 508
Bellarmino, Roberto, 26, 79
Bellissima, Fabio, *26–28*
Ben Miled, Marouane, *62*, *230*
Benedetti, Giacomo, *xiv*, *169*, *173*
Benedetti, Giovan Battista, 46, 113, 114, 123, 192, *254*, 297, 298, *299*, *303*, *305–308*, *310*, *312*
Benedetto, Croce, 503
Benoit, Paul, *224*, *244, 245*, *257*, *259*
Berg, Alban, 40, 492

Berkeley, George, 293–295
Bernoulli, Daniel, 291, 336, 337, 358–360, 362–365, 367, 378, 418, 420, 421, 432, 433, 440, 445
Bernoulli, Eduard, 357
Bernoulli, Jacob, 216, 291
Bernoulli, Johann I, 216, 291, 336, 337, 339, 345, 351
Bernoulli, Johann II, 336
Betti, Renato, 361, 368
Bhagavad Gita, *169*, *181*, *195*, 463–465
Bharata, *185*, *189*, *196–199*, *201*
Bhaskara I, *183*, *195*
Bhaskara II, *182*, *184*, *186, 187*, *196*, *206*, *225, 226*
Bible, *15*, 26, 29, 30, 35, 51, 56, 63, 69, 74, 84, *89*, *98*, 134, 145, 147, 150, 154, *155, 156*, 156, 173, 231, 253, *258*, 262, 280, 281, *284*, 289, 305, *324*
Bijaganita, *182*, *184*
Biot, Jean-Baptiste, 439
Birchensha, John, 191
Birkhoff, George David, 501
Biruni, Al-, *203–205*, *218*, *232*, *234*, *251*
Bismarck, Otto von, 458
Blankenburg, Quirinus van, 199
Bloch, Ernest, 487
Boccadoro, Brenno, *327*
Bodde, Derk, *135, 136*
Bodin, Jean, 50, 51
Boehme, Jakob, 261
Boësset, Antoine, 180, 211
Boethius, Severinus, 7, 21, 45, 114, 123, 165, *217*, *259*, *262–271*, *276*, *279*, *286*, *292, 293*, *296*, 299, *303*, *307*, *310*, *314*, *316*, *322*, *325*, *352*, *356*, *359–361*, *365*, *372*, *375*
Bohlman, Philip V., *224*
Bohr, Niels, *145*
Boltzmann, Ludwig, 466–469, 481
Bolyai, János, 502
Bombelli, Raffaele, *286*
Bonaparte, Napoleone, 436
Bononcini, Giovanni, 384
Book of the benefit about the knowledge of melodies, 138
Borren, Charles van den, 60
Borsacchini, Luigi, *326*
Borsi, Franco, *276, 277*
Bos, Henk G.M., 10, 213, 218, 220
Bottazzini, Umberto, 352, 366, 433, 444
Bougainville, Louis, 523
Boulliau, Ismael, 165, 208
Bourbaki, Nicolas, *35*
Boyer, Carl, *10*, *15*, *33*, *40*, *53–55*, *60–63*, 144, 187, 188, 191, *203*, *210*, 213, *227*, *259*, 263, *275*, 295, *300*, 361, 366, 418, 433, 470
Boyle, Robert, 285
Brahe, Tycho, 38, 56, 61, 62, 64, 70, 72, 80, 83, 86, 88, 102, 228
Brahma Sphuta Siddhanta, *187*, *195*, *203*
Brahmagupta, *181*, *185–187*, *195*, *203, 204*, *232*
Brahmasutra, *170*
Brahms, Johannes, 455, 478, 480, 481, 485, 505
Brandenburg Concertos, 427
Brecht, Bertold, 32
Brhatsamhita, *206*
Briggs, G. Morton, 414
Briggs, Henry, 197
Brouncker, William, 271
Brouwer, Jan, *161*, *191*
Brown, Howard Mayer, *325*
Bruce, Edmund, 96, 97
Bruno, Giordano, 77, 118, 313, 320
Brush, Stephen G., 466
Bryennius, Manuele, 141
Bull, John, 298
Bürgi, Jost, 92, 197
Burnett, Charles, 114
Busoni, Ferruccio, 487
Butler, David, 483
Buxtehude, Dietrich, 321
Byrd, William, 321

Caccini, Giulio, 127
Cadmus et Hermione, 320
Cai Yong, *154*
Cai Yuanding, *136*
Calderon de la Barca, 106
Calvin, Jean, 389
Caneva, Kenneth L., 462
Cantor, Georg, *32, 33*, *184*, *258*
Cantor, Moritz, *29*, *225*
Caparrini, Sandro, 361, 368
Cardano, Girolamo, 11, 56, 66, *286*, *291*, *297–301*, *303*, *324*, *326, 327*
Cardi, Ludovico, 23
Cardona, George, *185*, *190*, *195*
Careri, Enrico, 407
Carissimi, Giacomo, 321
Carnot, Lazare, 437
Carnot, Sadi, 311
Carter, Michael G., *253*

Caruso, Enrico, 500
Casari, Mario, *204*, *206*
Casini, Paolo, 270, 290
Caspar, Max, 56, 64, 65, 69–77, 80–84, 91–93, 95, 104
Cassini, Giandomenico, 416
Castelli, Benedetto, 35
Castelnuovo, Guido, 503
Castor et Pollux, 334, 423
Catel, Charles Simon, 435
Catullus, 235
Cavalieri, Bonaventura, 190, 245, 274
Cavalli, Pietro Francesco, 196, 406
Centrone, Bruno, *15*, *19*
Cerrai, Paola, *xiii*
Cervantes, Miguel, *257*
Cesi, Federico, 20
Chailley, Jacques, 328, 332
Chandahsutra, *183*
Charbonnières, Jacques Champion de, 195
Charlemagne emperor, 308
Charlonye, Gabriel de la, 164
Chateaubriand, Viscount, 146
Châtelet, Gabrielle-Émile de, 295
Chattopadhyaya, Debiprasad, *173*
Cheddadi, Abdesselam, *251*
Chemillier, Marc, 512
Chemla, Karine, *120, 121*, *142*, *145*, *150*, *158*
Chen Cheng-Yih (Joseph), 16, *104–106*, *110, 111*, *137*
Chen Congzhou, *166*
Cheng Dawei, *xiii*, *5*, 12, 15, *106–111*, *329*
Chern Shiingshen, 465
Cherubini, Luigi, 439
Chladni, Ernst, 438, 446, 471
Chomsky, Noam, 319, 527
Chongxu zhenjing, *127*
Choron, Alexandre-Étienne, 435
Choubachy, Chérif, *252*
Christensen, Thomas, *327*, 367, 393, 420–422, 432
Christine of Lorraine, 21, 35
Christmannus, Jacob, 88
Chua, Daniel K.L., *326*
Chunqiu fanlu, *134*
Chunqiu [Springs Autumns], *98*, *133*, *139*, *146*, *151*, *159*, *161*, *164*, *167, 168*
Ciccotti, Giovanni, *158*
Cicero, Marcus Tullius, *56*, *59*, 64, *73*, 150, 204, 205, *260, 261*, *285, 286*, *292, 293*, 316, *366*
Cini, Marcello, *158*
Clark, Suzannah, *327*
Clarke, Samuel, 259, 270, 314
Clavis Apocalyptica, 281
Clavius, Christophorus, 27, 86, 135, *296*, 302
Closs, M.P., 513
Cocchi, Gioachino, 384
Codex Atlanticus, *288*
Coelho, Victor, 301
Cohen, David E., 337
Cohen, H. Floris, 122, 164, 178, 182, 183, 206, 221, 301
Colbèrt, Jean Baptiste, 212, 220
Collection, *63*, *91*
Colombo, Cristoforo, 461
Comba, Antonella, *192*, *205*
Combarieu, Jules, 482
Commentaries on Mars, 55, 88
Commentarii in Somnium Scipionis, *257*
Compasso geometrico militare, 34
Compendium Musicae, 158, 162, 163, 169, 179, 180, 192, 193, 271, 302, 328
Comte, Auguste, 436, 437
Concerto for pianoforte, 428
Concerto nach italienischem Gusto, 427
Condillac, Etíenne, 428
Condorcet, Marie-Jean Nicolas, 437
Confessions, 395, 396
Confucius Kong Fuzi, 12, 30, *97–99*, *102*, *119, 120*, *133*, *138, 139*, *141*, 146, *154*, *162*, *164, 165*, *168*, 251, *258*, 417, 418, 450
Conics, *60, 61*
Constantine emperor, *257*, 308
Conti, Lino, *xiii*
Cook, James, 519, 521
Cooke, R., *19*
Copernicus, Nicolas, 11, 22, 30, 31, 38, 63, 70, 77–81, 118, *257*, *292*
Corelli, Arcangelo, 321, 384
Correspondance, 161
Così fan tutte, 428
Cosmotheoros, 211, 212
Costruzione dell'immagine scientifica del mondo La, *xiii*
Couperin, François II, 321, 323
Courant, Maurice, 297
Courlander, Harold, 513
Cousin, Louis, 199
Cowl, Carl, *337*
Crombie, Alistair Cameron, 107, 113, 119, 177, 299
Cromwell, Oliver, 188, 321
Crozet, Pascal, *229–231*, *243*
Cullen, Christopher, *120*
Cuse, Nicolas de, 313
Cyclus Harmonicus, 206

Dadai liji, *129*
Dafne, 127
Dagens, Bruno, *190*
Dalalat al-khairin, *234*
Damm al-malahi, *223*
Danesh Nama, *216*, *219*, *232*
Daniélou, Alain, *199*, *201*
Dante Alighieri, *286*
Daodejing [Classic of the way and of the virtue], 3, *97*, *124*, *127*, *139*, *141*, *147, 148*, *154*, *167*
Dardanus, 400
Darwin, Charles, *1*, *39*
Das Rheingold, 456
Das wohltemperierte Clavier, 298, 324, 331, 427, 487
Datta, B., *181–187*, *194, 195*
De Architectura, *91*, *276*
De arte cabalistica, *280*
De Beghinselen der Weeghcons, 9
De divina proportione, *283, 284*, *286, 287*
De Institutione Arithmetica, *259*, *361*
De institutione musica, *259*, *292*, *359*, *361*
De Lassus, Roland, *304*
De libris propriis, *299*
De Maria, Michelangelo, *158*
De Monte, Philippe, 101
De Morgan, Augustus, 502
De musica, *297*, *299*
De natura deorum, *285*
De occulta philosophia, *279*
De Potentia restitutiva, 285
De proportionibus, *298*
De prospettiva pingendi, *281, 282*
De quinque corporibus regularibus, *281*, *283*
De re aedificatoria, *276*
De Republica, *260, 261*
De rerum natura [On the nature of things], *64*, *74*, *82*
De rivolutionibus orbium celestium, 30
De Rore, Cyprien, 59, 298, *304–306*, *316*
De stella nova, 54
De Ursis, Sabatino, 86, 87
De vita libri tres, *277*, *279*
De vita propria, *298*
Debussy, Claude, 327, 493
Dedekind, Richard, *32, 33*
D'Elia, Pasquale, 31, 87, 91
Della natura de principii et regole musicali, *297*
Delsedime, Piero, 418
Democritus, *40*, *54*, *65*, *85*, *90*, 113, 132
Démonstration du principe de l'harmonie, 372
Der biblische Weg, 486
Desargues, Girard, 134, 140, 141, 143, 177
Descartes, René, *1*, 1, 3, *5*, *9*, 10, *11*, 11, *29*, *63*, 64, 84, 119, *120*, 140–144, 147, 151, 153, 157–164, 166–183, *185*, 187, 188, 192–194, 197, 199, 211, 212, *217*, 218, 219, 221, 223, *230*, 234, 243–245, 249, 252, 256–258, 261, 262, 268, 269, 271–276, 278, *281*, 284, 286, *291*, 293–295, 301–304, 310, 316–321, 324, 328–331, 333, 357, 368, 380, 393, 409, 418, 469, 506, 512, 531
Deschamps, Théodore, 162
Desprez, Josquin, *304*, 453
Dhombres, Jean, 531
Di Benedetto, Renato, 407
Dialogo della musica antica et della moderna, 17, 20, 40, 59, 98, 104, 128, *309*, *321–325*
Dialogo sui massimi sistemi, 18, 24
Dias, Manuel, 29
Dickreiter, Michael, 46, 49, 97, 98, 100–104
Dictionnaire de musique, 387, 388, 391, 392, 394, 395
Diderot, Denis, 335, 368, 375, 393, 394, 403–405, 411, 413, 422, 423
Didymus, *50*, 123, 203, *306*, *316*, 374
Die Kunst der Fuge, 426
Die Lehre von den Tonempfindungen, 442, 444
Die Musik der Araber, 453
Die Natur der Harmonik und Metrik, 454
Die Zauberflöte, 453
Dijksterhuis, E.J., 9, 220
Dimostrazioni Harmoniche, 204
Din Nasir, Al-, *233*
Din Safi, Al-, *219*, *224*, *230*
Dini, Pietro, 31, 35
Diogenes Laertius, *29*
Diogenes of Sinope, *17*, *95*
Dionysius of Siracusa, *19*
Diophantus, *54*, *61, 62*, 142, *186*, *227*, *229*
Dioptrics, 168
Diotima, *25*
Discorsi e dimostrazioni matematiche intorno a due nuove scienze, 18, 24, 166
Discorso intorno all'opere di Messer Gioseffo Zarlino, *309*, *320*, *322*, *325*
Discours de la cause de la Pesanteur, 219
Discours de la méthode, 64, 157, 158, 166, 168, 169, 171, 174, 175, 333
Dissertatio de arte combinatoria, *234*, 239, 254
Diversarum Speculationum Mathematicarum et Physicarum Liber, *305*

Doctrine of the Mean The, *99*
Don Giovanni, 428, 453
Don Quijote da la Mancha, *257*
Donini, Elisabetta, *158*, 308
Donkin, William Fishburn, 528
Donne, John, 77, 78, 80
Dostrovsky, Sigalia, 273, 296, 297, 299, 300, 438, 439, 447
Drake, Stillman, 21, 140, 191, 192, 299
Du Bois-Reymond, Emile, 482, 491
Du Caurroy, François Eustache, 127
Du Fu, *168*
Du Shiran, 224
Dudink, Ad, 29
Dufay, Guillaume, 300
Duishu tanyuan, 463
Dukas, Helene, 487
Dumont, Henri, 195, 196
Durante, Francesco, 392
Dürer, Albrecht, *285*, *289*
Dvarakanatha, *174*
Dvorak, Antonin, 500

Eco, Umberto, 318
Einige canonische Veraenderungen über das Weynacht-Lieder, 426
Einstein, Albert, *xiii*, *xiv*, *1*, *6*, *64, 65*, 66, *126*, *161*, 177, 230, 289, 444, 465, 481, 507
Einstein, Alfred, 330, 428
Eitz, Carl, 477
El Héroe, 157
Elémens de musique théorique et pratique suivant les principes de M. Rameau, 352, 371
Elementa Harmonica, *36–38*, *248*
Elementa Musicalia, *361*
Elements, *26, 27*, 27, *28, 29*, 29, *31*, *33*, 33, *34, 35*, *58, 59*, *62, 63*, *83*, 85, *89*, *92*, *111*, *142*, *174*, *176*, *183*, *188, 189*, *191*, *195*, *205*, *217*, *225, 226*, *229–231*, *233, 234*, *245*, *284*, 462
Eléments de Mathématique, *35*
Eléments ou principes de musique, 199
Eloise, 308, 396
Elvin, Marc, *143*
Empedocles, *85, 86*
Encyclopédie, *5*, 335, 375, 377, 379, 384, 387, 388, 391–393, 396, 402, 404, 405, 413, 414, 425
Engelfriet, Peter M., *121*
Engels, Friederich, 310
Enriques, Federigo, *33*, 503
Ephemerides, 54
Epicurus, *64, 65*, *71*, *83*, *86*, 113, *275*
Epigonius, 202
Epitome Astronomiae Copernicanae, 88, 95
Erasistratus, *87*
Erasmus from Rotterdam, 3, 5, 324
Eratosthenes, *58*, *90*, *94*
Erlanger, Rodolphe d', *211*, *214, 215*, *221*, *235*, *239*, *243*, *336, 337*
Esquisse d'un tableau historique des progrès de l'esprit humain, 437
Essais, 147
Essay On the Origin of Languages Where People Speak of Melody and Musical Imitation, 388
Ethics, *21*
Euclid, *1*, *4, 5*, 9, *11*, 11, *15*, *21*, *25*, 25, *26, 27*, 27, *28*, 28, *29*, 29, *30–33*, 33, *34*, 34, *35*, 35, *36–38*, 38, 41, 45, *55–60*, 61, *62–64*, *81*, *83, 84*, 85, *88, 89*, 90, 91, *92*, 92, *95*, 102, 103, *111*, 114, *116, 117*, 119, 133, *143*, *153, 154*, *156*, *162*, *164, 165*, 165, 168, *169*, 170, 171, *174–178*, *183*, 184, 187–189, 203, *205*, *209*, *214, 215*, *217*, 217, *220*, *225, 226*, *228–234*, 234, *237*, 238, *243*, *245*, 245, *246–251*, *254*, *259*, *261*, 261, *262*, 262, *272*, *275*, *282*, *284, 285*, *287*, *291, 292*, *295*, 295, *296*, 302, 306, 379, 380, 399, 419, 449, 462, 469, 505
Eudoxus, *33*, *54, 55*, *57*, *59*, 397
Euler, Leonhard, 1, *2*, *5*, *11*, *201*, 291, 327, 336, 337, 339–363, 365–369, *370*, 370, 371, 373, 374, 377, 379, 385, 394, 402–404, 409, 415, 418–421, 424, 431–434, 436–438, 443–445, 449–451, 455, 457, 484, 504, 528, 531, 532
Euridice, 127
Eutyche, *258*
Eximeno, Antonio, 424
Extrait d'une réponse de M. Rameau à M. Euler, 349

Fabbri, Natacha, 302, 303
Fabricius, David, 81
Fang Zhongtong, *125*
Fano, Fabio, *309*
Fano, Gino, *61*
Farabi, Al-, 191, *217*, *219*, *221–224*, *234–247*, *249–254*, *336–339*
Faris, Ibn, *253*

Farmer, Henry George, *224*, *248, 249*
Fasch, Karl Friederich, 357
Faust, 491
Fazari, Al-, *203*
Fechner, Gustav Theodor, 483
Fenollosa, Ernest, 1, *143*
Fermat, Pierre de, 140–143, 153, 177, 183, 187, *230*
Ferrari, Ludovico, *286*
Ferreira, Manuel Pedro, 531
Ferro, Scipione del, *286*
Fêtes, de l'Hymen et de l'Amour, 377
Feyerabend, Paul, 23, 32, 36
Ficino, Marsilio, 103, 122, *277–280*, *283*, 283, *298*, *327*
Finney, Ben R., 525
Flamant, Jacques, *261, 262*
Flamsteed, John, 287
Fleckenstein, J.O., 336
Fletcher, Neville H., *104*, *322*
Fludd, Robert, 64, 99, 107, 118, 137, 138, 257, 504
Fokker, Adriaan D., 10
Fontaine, Jean de la, 406, 430
Fontenelle, Bernard le Bovier, 158, 264, 313, 401, 404
Forman, Paul, *82*, 491
Foscolo, Ugo, 412
Foster, C., 529
Fourier, Joseph, *6*, 367, 421, 431, 433–437, 439, 444–446, 450, 455, 459, 460, 462
Fowler, David, *21*, *34*
Fox Keller, Evelyn, *143*
Fraiese, Attilio, *33*
Franceschi, Francesco De, *303*
Frescobaldi, Girolamo, 298, 321
Freudenthal, Hans, 443, 444
Fritz Kurt von, *19*, *40*
Fubini, Enrico, 381, 388, 403, 405, 422–425
Fünf Klavierstücke, 498
Funkenstein, Amos, 304–307, 317
Fux, Johann Joseph, 435

Gabir ibn Hayyan Geber, *245*
Gabrieli, Andrea, 100, 101, *304*, 321
Gabrieli, Giovanni, *304*, 321, 453
Gaffurio, Franchino, 127, *316*, *323*
Galen, *34*, *85*, *87*, *89–91*, *93, 94*, *156*, *163*, *166*, *205*
Galilei, Galileo, *xiii*, *1*, 1, *5*, *11*, 18–26, 29–33, 35–38, 40, 41, 46, 52, *58*, 63, *66*, 75, 79, 82, 84, 85, 87, 91, *92*, 92, 96, 97, 103, 105, 117, 118, 120, 130, 134, 140, 166–168, 170, 171, 174, 175, 181, 191, 194, 196, 198, 200, 209, 212, 216, *217*, 221, 223, 243, *254*, 256–258, 268, 275–277, 280, *281*, 285, 293, 299, 301, 303, 305, *308*, 310, 312, 313, *314*, *319*, 320, *322, 323*, 358, 400, 412, 445, 522, 523
Galilei, Vincenzio, 1, *5*, 6, 17, 18, 20, 40, 45, 47, 49, 59, 62, 96–98, 100–102, *103*, 103–105, 114, 122, 126–128, 130, 140, 159, 181, 183, 186, 189, 191, 192, 201, 204, 205, 207, 208, *212*, 237, 238, 268, 298, *299*, 299–301, *303*, 303, *309–327*, 350, 358, 388, 393, 412, 419, 439, 452, 453, 528, 529, 531
Gallé, 208
Galuppi, Baldassarre, 384
Gandhi, Mahatma, *169*
Ganita Sarasamgraha, *195*
Gassendi, Pierre, 177
Gauguin, Paul, 514
Gaultiers Jacques, 195
Gauss, Carl, 224
Gebruyck of ongebruyck van 't Orgel in de Kerken der Vereenighde Nederlanden, 192
Geistliche Gesänge, 474
Geminiani, Francesco, 378, 407
Generalbaßschule, 346
Génération harmonique, 332, 385
Genesis, *155*
Geography, *54*, 520
Géométrie, 163, 166, 168–171, 174, 273, 302
Geometry, *63*
Gerardo from Cremona, *211*, *259*
Gerdes, Paulus, 511–513, 528
Germain, Sophie, 438
Gernet, Jacques, 29, *145*, *148*, *155, 156*, *158*, 255
Geron, *162*
Gesualdo, Carlo, 298, *304*
Geymonat, Ludovico, 40, 103
Ghayat al-Hakim, *278*
Ghazzali, Al-, *219*, *247*, *250, 251*
Gingerich, Owen, 78, 94, 170
Ginsburg, Carlo, *280*
Giorgi, Francesco, *277*
Gitalamkara, *185*, *197*, *199*
Glareanus, Heinrich, *316*
Gluck, Christoph Willibald, 425
Godowsky, Leopold, 487
Goethe, Wolfgang, *v*, 491

Gogavino, Antonio, *41*, 189
Goldbach, Christian, 230, 232, 233, 235, 347
Golden verses, *280*
Gospels, *258*
Goudimel, Claude, 390
Gouguyi, 28, 85
Gouk, Penelope, 159, 187, 190, 238, 270–273, 283–286
Gowzgani, *216, 217*, *233*
Gozza, Paolo, 21, 106, 191, 300, *327*, 505
Gracián Baltasar, 157, 158
Gradus ad Parnassum, 435
Graham, Angus Charles, *124, 125*, *127–129*, *131–134*, *138*, *141, 142*, *144*, *146–148*, *150–155*, *157, 158*, *160*, *162*, *326*
Granet, Marcel, *101*, *106*, *111*, *121*, *124, 125*, *152*, *157*
Grassmann, Günther, 481
Grassmann, Hermann, 502
Grasso Caprioli, Lionella, 407
Grattan-Guinness, Ivor, 352, 366, 433, 434, 528
Great Learning, *99*
Green, Burdette, 483
Grimm, Friedrich, 395
Grossato, Elisa, 407
Grosset-Grange, Henri, 520
Grossi, Ludovico da Viadana, 205
Guanzi, *127*, *132*, *165*
Guicciardini, Lodovico, 200
Guido D'Arezzo, *10*, 46, 66, 161, 234, *259*, *269–271*, *292*, *297*, 299, *309*, *316*, *342–344*, *346*, *348*, *351*, *362, 363*, *365*, *371*, *374*, *376*, 393, 434
Guldenmann, Katharina, 75, 82
Guldin, Paul, 69
Guo Po, *136*
Guo Shuchun, *120*
Guo Xiang, *124*
Guoyu, *134*
Gutenberg, Johannes, 418

Haase, Rudolf, 239
Haeckel, Ernst, 482
Haggag ibn MatarAl-, *225, 226*
Hahn, Otto, 478
Haidao suanjing, *117*
Halleaux, Robert, 67
Halley, Edmund, 275, 278
Halliday, David, *322*
Hamlet, 3
Hammer, Franz, 91
Hammond, Frederick, 32
Han Lin, 34
Handbuch der physiologischen Optik, 457
Händel, Georg Friedrich, 235, 236, 321, 346, 437, 453
Hankins, Thomas L., 414, 418
Hansen, Chad, *139*, *141*, *146–148*, *152, 153*, *156*
Hanshu [Book of the Han], *99*, *103*, *105*, *107*
Hanslick, Eduard, 445, 452, 484, 485
Harbsmeier, Christoph, *139*, *142*, *144*, *146*, *150*, *152*, *157, 158*, *160*
Harmonices Mundi Libri Quinque, 37, 40, 57, 67, 68, 70–74, 77, 82, 94–96, 98–100, 102–104, 107, 117, 171, 232, 240, 270, 275, 303, 403, 450, 491
Harmonie Universelle, 107, 110, 114, 117, 119, 128, 163, 165, 166, 177, 180, 193, *225*
Harmonielehre, 493, 494
Harnach, Adolf von, 482
Harriot, Thomas, 183
Harrison, John, 289, 309
Hassler, Hans Leo, 101
Hassler, Jacob, 101
Hauer, Matthias, 498
Hauptmann, Moritz, 454
Hayashi Takao, *178*, *185*, *196*
Haydn Franz Joseph, 501
Heath, Thomas, *20, 21*, *29, 30*, *32*, *177*
Hegel, Georg Wilhelm, 463, 465
Heilbron, John, 470, 476
Heisenberg, Werner, *145*
Helmholtz, Hermann von, *6*, 239, 357, 418, 419, 439–459, 462, 467, 469, 470, 472, 473, 476–484, 498, 500, 503–505, 508, 529, 530
Hemony brothers, 198
Henfling, Conrad, *103*, 234, 235, 237–240
Henry, Teuira, 514, 515, 518, 519, 521, 522
Heraclitus, *11*, *16, 17*, *24*, *162*, *166*
Herbart, Friederich, 441
Herder, Johann Gottfried, 527
Hermann, Jacob, 337
Heron, *54*, *61*, *64*, *91*, *166*, *210*
Herophilus, *87*, *93*
Herschel, William, 502
Herwart von Hohenburg, Georg, 73, 96, 98
Hessen, Boris, 309–311
Heydon, Christoph, 67
Hiebert, Erwin, 502

Hilbert, David, *27*, *32, 33*, *35*, *58, 59*, 173, *326*, 503
Hindemith, Paul, 75, 494
Hipparchus, 88, *187*
Hippasus, *15*
Hippocrates, *163*
Hippolyte et Aricie, 334, 375
Hisab al-wasaya, *227*
Histoire des Mathématiques, 412
Histoire Universelle du grand Royaume de la Chine, 213
History of Mathematics, 405
History of the Mathematical Sciences II, *xiv*
Hitler, Adolf, 188, 488
Hitzler, Daniel, 101
Hobbes, Thomas, 177, 188, 305
Hofmann, Joseph E., 255, 261, 263
Hofstadter, Douglas R., 506
Hogendijk, J.P., *231*
Holbach, Paul Thiry, 413
Holbein, Hans, 215
Holder, William, 295
Holton, Gerald, 23, 40, 99
Homer, *22*, *24*, *61*, *81*, *84*, *95*
Hooke, Robert, *5*, 186, 190, 265, 273, 280, 283–288, 318
Hooykaas, R., 161, 189
Horace, 264
Horner, William George, *122*
Horologium Oscillatorium, 220
Hou Hanshu, *135*
Houzel, Christian, *245*
Hu Danian, *161*
Huainanzi, *128*, *139*, *162*, *165*
Huangdi neijing lingshu, *159*
Huangdi neijing suwen, *128*
Huashu, *130*
Hudde, Jan, 242
Hudjwiri, Al-, *244*
Hulin, Michel, 464
Humai, *335–339*
Humboldt, Wilhelm von, 463–465
Husserl, Edmund, 484, 499, 507
Huygens, Christiaan, 1, *5*, 8, 10, 11, 21, 144, 180, 192–195, 197, 199, 200, 203, 205–209, 211–213, 215, 219–221, 223, 224, 227–230, 233, 234, 236–239, 244–247, 256, 265, 275, 285, 286, 289, 294, 296, 323
Huygens, Constantijn, *5*, 8, 10, 192–195, 198, 209, 211
Hypatia, *54*, *63*
Hypparchum, 89
Ignatius of Loyola, 77
Ihsanoglu, Ekmeleddin, *224*, *249*
Ikhsa al-ulum, *234*
Ikhwan al-Safa, *219*, *252*
Il cimento dell'armonia e dell'invenzione, 322
Il sogno di Scipione, 427
Iliad, 24, *81*
In praise of folly, 3, 5
Introduction to arithmetic, *62*
Introduction To Astrology As a Hidden Science, 488
Isaac, Henricus, *281*, *291*, *304*
Istituzioni analitiche ad uso della gioventù italiana, 437
Istituzioni armoniche, *298*, *303*, *307*
Itard, Jean, 433

Jacobi, Carl Gustav Jacob, 415, 502
Jakobsleiter, 491, 494, 498
James, William, 314
Jami, Catherine, *152*, 224
Jansen, Cornelius, 144–146, 155, 157
Jartoux, Pierre, 224
Jayadeva, *199*
Jayasimha, Sawai, *205*
Jean des Murs, 109
Jihe yuanben, 27, 34, 462
Jing Fang, *139*
Jingwu quanbian, 33
Jiuzhang suanshu, *117–122*, *125*, *142*, *145*, *149, 150*, *230*
Joachim, Joseph, 454, 480, 481, 505
Jocasta, *86*, *92*
John the Evangelist, *145*, *155*
Jolivet, Jean, *250*
Jommelli, Niccolò, 384, 392
Jona-Lasinio, Giovanni, *158*
Jordan, William, 22
Joseph, George Gheverghese, *181*, *184–187*, *203*, *205*, *227*
Jullien, François, *130*, *140–143*, *146–148*, *150, 151*, *154, 155*, *159–161*, *165–167*
Jung, Carl Gustav, 494
Justinian, *63*, *204*

Kahir Al-, *249*
Kalbeck, Max, 481
Kalpasutra, *183*
Kangro, Hans, 470, 479
Kangxi, 248
Kant, Emmanuel, 313, 439, 457, 479

Karagi, Al-, *229*, *231*
Kashf al-mahdjub, *244*
Kassler, Jamie Croy, 273, 300
Kauffman Stuart, *xiii*
Kautilya, *192*
Kelvin, William Thomson, 444
Kennedy, Edward S., 520
Kepler, Johannes, *xiii*, *1*, 1, *5*, *11*, *21*, 37–41, 43–85, 87, 89, 91–109, 112, 114, 117–119, 126, 127, 133, 137, 140, 156, 158, 159, 168, 170, 171, 175, 182, 205, 212, *217*, 219, 221, 223, 227–232, 240, 250, 256, 257, 262, 269, 270, 272, *274*, 275–278, *281*, 289, 293, 300, 303, 320, 323, 362, 363, 368, 398, 403, 409, 439, 450, 480, 490–494, 500, 504, 506, 523, 532
Keynes, John Maynard, 291
Khayyam, Umar al-, *xiv*, *5*, *209*, *228–230*, *244*, *246, 247*, *255*, *335–337*
Khusraw, Amir, *204*
Khwarizmi Al-, *181*, *203*, *226–229*, *244*
Kiesewetter Raphael, *224*, 453
Kindi, Al-, *224*, *234*
King, David A., *232*
Kircher, Athanasius, 64, 102, 204, 283, 450, 504, 532
Kish, George, 309
Kitab al Išarat wa'l Tanbihat, *218*
Kitab al-Gabr wa-l-muqabala, *226*
Kitab al-Hind, *203*
Kitab al-Inam fi marifat al-angam, *225*
Kitab al-iqa, *248*
Kitab al-Manazil al-sab, *230*
Kitab al-Musiqi al-Kabir, *221*, *234*, *237*, *241*, *243*, *336–338*
Kitab al-Qanun fi 'l-tibb, *211*
Kitab al-ru'us, *248*
Kitab al-Šifa, *211*, *217*, *219*, *232*, *260*, 299, *335–339*
Kitab fi ma yahtagu ilay-hi al-kuttab wa-l-ummal min ilm al-hisab, *231*
Klein, Felix, 444
Kleinert, Andreas, 366
Kline, Morris, 188, 189, 355, 361, 366, 433, 444, 470
Knobloch, Eberhard, 502, 504, 531
Koestler, Arthur, 69, 75, 84, 99, 312
Kol Nidre, 493
Koran, 156, *204*, *210*, *217*, *220, 221*, *227*, 231, *232*, *245–247*, *249–252*, *258*, 520
Kowaleswskaia, Sofja, 502
Koyré, Alexander, 99, 261, 273, 278, 280, 289, 290, 295, 315, 318
Kramer, Edna E., 438
Kraus, Karl, 507
Kreiling, Frederick, 255
Kreisler, Fritz, 487
Kuhn, Thomas S., 301
Kumarila, *189*
Kutub al Qudama, *339*

La recherche de la vérité, 317
La serva padrona, 381, 382, 384, 390, 395, 422
Lagrange, Giuseppe Luigi, 291, 352, 367, 369, 431–434, 436–438, 531
Lambert, Michel, 195
Laozi, *162*
Laplace, Pierre Simon de, 243, 291, 437
Lasker, Emanuel, 493
Laue, Max von, 476
Lavoisier, Antoine Laurent de, 436
Laws, *87*
Le devin du village, 382, 384, 391, 425
Le leggi del contrappunto, 410
Leeuwenhoek, Antonie van, 258
Leibniz, Gottfried Wilhelm, *5*, *11*, 50, 72, *80*, 113, *124*, 127, 142, 144, *145*, 153, 169, *185*, *196*, 210, 215–219, 223, 224, 227–233, *234*, 234–248, *249*, 249–253, *254*, 254–265, 268, 270, 274, 276–278, 280, *281*, 283, 284, 286, 288, 290, 291, 293–297, *303*, 304, 305, 307, 313, 315–321, 323, 324, 337, 340, 346–348, 353, 356, 357, 368, 380, 404, 413, 415, 419, 421, 426, 433, 439, 444, 491, 504, 511
Lenin, Ulianov Vladimir, 40
Lenoble, Robert, 107, 115, 116, 118, 119, 124, 137, 139, 140, 143, 144, 177, 182, 194
Leo, Leonardo, 384, 392
Leonardo da Pisa, 44, *181*, *207*, 232, 234, *259*, *271*, *273–275*, *282*, *286*
Leonardo da Vinci, *xiii*, 26, 54, *161*, *276*, 278, *280*, *287–289*, 289, *290, 291*, *297*, *301*
Leopardi, Giacomo, 430, 431, 508
Les Indes galantes, 375, 401
Lester, Joel, 337
Lettre sur la musique françoise, 382, 383
Lettres à une princesse d'Allemagne, 350

Levis, John Hazedel, *106*, *139*
Lévi-Strauss, Claude, *3*, 393, 512
Lewis, David, 519, 521–523
Li Shanlan, 462, 463
Li Shizhen, *132*
Li Yan, 224
Li Zhizao, 29
Liang, Shuming, *148*
Liber Abbaci, *259*, *272–275*
Liber de ludo aleae, *300*
Liebig, Justus von, 458
Lieder, 500
Liezi, *127*, *132*
Life and Letters, 492
Life is a dream, 106
Liji [Memories of Rites], *98*, *134*
Lilavati, *196*, *206*
Lindsay, Bruce, 528, 531
Lissajous Jules, 447, 458, 462
Little prophet, 395
Liu An, *162*, *165*
Liu Hui, *117–120*, *122*, *145*, *150*
Liu Tianjun, *98*
Liu Zhi, *131*
Ljapunov Aleksandr, 25
Lloyd, Geoffrey, *84–93*, *98*, *127*, *129*, *133*, *140*, *143*, *145*, *147*, *149*, *153–156*, *158–163*, *165, 166*, *168*
Lobkowitz, Juan Caramuel, *327*
Locatelli, Pietro Antonio, 407
Locke, John, 231, 233, 246, 315
Lombardi, Anna Maria, 75, 103
Longobardi, Nicola, 249, 253
Loriti, Heinrich Glareanus, 98
Loulié, Etienne, 199
Lounsbury, Floyd G., 513
Lovejoy, Arthur Oncken, 313
Lü Buwei, *161*, *164*, 413
Lu Gwei-Djen, *136*
Lucia di Lamermoor, 500
Lucretius, Titus Carus, *4*, *64–82*, *161*, *279*
Lulli, Giovan Battista, 195, 196, 221, 320, 321, 375, 376, 380, 384–386, 401, 405, 406, 425, 437
Lülü chengshu, *122*
Lumpkin, Beatrice, 511
Lundberg, Robert, 23
Lunheng, *129*
Lunyu [Dialogues], 30, *98*, 251
Luppi, Andrea, 236, 239, 254, 255, 260, 261
Lüshi chunqiu, *128*
Luther, Martin, 56, *280*, *296*, 306, *326*
Lüxue xinshuo, 17
Mac Laurin, Colin, 269
Mace, Thomas, 271
Mach, Ernst, 479, 482
Machault, Guillaume de, 300
Machiavelli, Nicolò, 79
Macrobius, Ambrosius Theodosius, *19*, *257*, *260–262*, 266, 267
Magia naturale nel Rinascimento La, *279*
Magini, Giovanni Antonio, 85, 86
Magiste Syntaxis Mathematica, *225*
Magriti Al-, *278*
Mahabharata, *189*, *193*, *195*, *197*
Mahavira, *187*, *195*
Mahler, Gustav, 467
Mahmud sultan, *204*, *218*
Mahoney, Michael S., 171
Maimonides, Moses, *234*, 305
Major, John S., *106*
Malamoud, Charles, *170*, *181*, *194*
Malebranche, Nicolas, 317
Mambella, Guido, *327*
Mamiani, Maurizio, 280, 282, 283
Mamone, Capria Marco, *xiii*
Mamun, Al-, *224*, *226*, 281
Mann, Thomas, 499, 507
Mansur, Al-, *203*
Manzoni, Alessandro, 503
Maqala fi l-gabr wa l-muqabala, *228*
Marcellus, *59*, *90*, *93*, 136
Marconi, Guglielmo, 467
Marenzio, Luca, 298, *304*
Marmontel, Jean-François, 412
Mars, *63*, *80*, *93*
Martini, Giovanni Battista, 335, 336, 435
Martzloff Jean-Claude, 35, *117*, *120*, *122, 123*, *142*, *152*, 224, 287, 463
Marx, Karl, 311
Masala fi'l-musiqi, *224*
Massera, Giuseppe, *279*, *290*, *303*, *309*
Mästlin, Michael, 73, 74, 96, 98
Matematiche del Tao Le, *xiii*
Matematizzazione della biologia La, *xiii*
Mathematics and Computation in Music, *xiv*
Mathiesen, Thomas J., *95*
Mattheson, Johann, 346
Maupertuis, Pierre Louis, 502
Maurolico, Francesco, *xiii, xiv*, *5*, 5, 17, 18, 46, 56, 113, 119, 123, 138, 159, 161, 178, 179, 181, *238*, 284, *291–297*, *299*, *303, 304*, *307*, *324*, *326*, 329–331, *341*, *348*, *350*, *352–354*, *356*, *360*, *362, 363*, *369*, *371, 372*, *374*

Mawsili, Al-, *209*
Mayer, Julius Robert von, 311, 458
Mazzacurati, Carlo, *19*
Mazzini, Giovanni, 139
McGuire, J.E., 268–270, 300, 301
Mechanicam, 340
Mechanik des Ohres, 441
Mede, Joseph, 281
Medici, Maria de, 127
Mei, Girolamo, 127, *312*
Mei, Juecheng, 224
Mei, Wending, 34, 35, *141*, 224
Meibomius, Marcus, 196, 201, 202
Melissus, *166*
Menaechmus, 163
Mencius, *99*, *138*, *141*
Menelaus, *187*
Meng Kang, *100, 101*
Mengoli, Pietro, 190, 191, 197, 223, 394
Mengqi bitan, *131*, *135*
Mengzi, *138*
Meno, *21*, *173*, *225*
Mental Radio, 489
Mercator, Kremer Gerhard, 11, 97, 197, 309
Mercator, Nicolaus, 197, 295
Merola, Tarquinio, 298
Mersenne, Marin, *5*, *9*, 10, 102, 107–145, 157, 159, 161–166, 170, 171, 173–179, 181–183, 186, 187, 193, 194, 200, 204, 206–210, 212, *213*, 213, 215, 221, 223, *225*, 233, 237, 271, 273, 285, 296, 299–302, *303*, 303, 304, 316, 317, 336, 358, 388, 394, 439, 480, 504, 529, 532
Metaphysics, *40*
Metastasio, Pietro, 392, 427
Meteors, 168
Method The, *57, 58*, *60*
Methodus incrementorum, 369
Methodus inveniendi lineas curvas maximi minimive proprietates gaudentes, 361
Micheau, Françoise, *224*, *244, 245*, *257*, *259*
Mikami, Yoshio, 224
Mila, Massimo, 236, 323, 330
Milton, John, 78, 264
Ming, *4*
Ming Antu, 224
Minkowski, Christopher, *195*
Minnaert, M.G.J., 9, 12
Mises, Richard von, 490
Misura del cerchio, 34
Mittelstrass, Jürgen, 261
Molière, 195, 390, 405
Monde ou Traité de la Lumière Le, 169, 170, 175, 219
Monge, Gaspar, 437, 502
Montaigne, Michel de, 144, 147, 150, 151, 154, 157
Montesquieu, Charles Louis de, 417, 430
Monteverdi, Claudio, 59, 196, 321, 453
Montucla, Jean, 398, 412
Moray, R., 212
More, Thomas, 77
Morgan, Auguste de, *35*
Morone, Giovanni, *297*
Morrison, John, 525
Morse, Edgar W., 458
Moscheo, Rosario, 123
Moses und Aron, 486, 494, 498
Mozart, Wolfgang Amadeus, 330, 412, 423, 427–429, 437, 449, 453, 455, 467, 487
Mozi, *141*, *146*, *160*, *162*, *165*
Muris, Johannes de, 300
Musica, *x*, *xiv*, *5*, *292, 293*, *296*, *303*, *341*
Musicae Traditiones, 178
Musicalische Temperatur, 205
Musick's Monument, 271
Musil, Robert, 431
Musique Arabe La, *211*, *336*
Musique ou Traité du Son La, 169
Musurgia Universalis, 64, 102, 204, 283
Mysterium cosmographicum, 38, 56, 67, 70, 72

Na'hman of Breslev, 533
Napier, John, 197
Napolitani, Pier Daniele, *56*, *58–60*, *64*
Napolitano, Ernesto, 506
Nasr, Seyyed Hossein, *210*, *219*, *250–252*, *255*, *278*
Nateli, Abu Abdallah, *217*
Natya Śastra, *199, 200*, *202*
Naumann, Ernst, 454
Needham, Joseph, 12, 13, *97–101*, *103*, *105, 106*, *108*, *111, 112*, *117*, *121–123*, *125*, *127–139*, *141–146*, *148–150*, *152*, *154*, *157, 158*, *160, 161*, *165*, 224, 311, *326*
Nernst, Walther, 502
Nestorius, *258*
Nettesheim, Agrippa von, *279*
Neugebauer, Otto, *3*, *54*, *210*
New Atlantis, 189
Newton, Isaac, *xiii*, *1*, 1, *5*, *11*, 50, 52, *58*, *80*, 83, 84, 99, 102, 142, 144, 170, 176, 188, 210, 216, 218, 219, 223,

224, 227–230, 246, 256, 258–261, 264–266, 268–280, *281*, 281–284, 286, 287, 289, 290, *291*, 291–297, 300, 301, 305–318, 320, 321, 324, 325, 337, 352, 356, 357, 362, 365, 369, 380, 388, 398, 406, 410, 412, 414–416, 418, 420, 441–446, 457, 458, 465, 469, 479, 480, 492, 523
Nicolodi, Fiamma, 407
Nicolson, Marjorie, 78
Nicomachus of Gerasa, *62*
Nietzsche, Friedrich, *274*, 501
Ninth Symphony, 467, 502
Noble, David F., 76, *259*, 307, 316
Noble, William, 190
North, Francis, 190, 273
Nouvelles réflexions sur le principe sonore, 348
Nouvelles suites de Pièces de Clavecin, 423
Novissima Sinica, 247
Nuh siphir, *204*
Nuvole in silenzio (A. Schoenberg svelato), *xiii*

O'Keefe Vincent, 504
Ockham, William of, 305
Odyssey, *95*
Ohm, Georg Simon, 445, 447, 458–462, 469, 530
Oldenburg, Henry, 190, 270, 272, 278
On animals, *278*
On the construction of the artillery, *91*
Opera Mathematica, *xiv*
Opere Le, 20
Optical Lectures, 272
Opticks, 272, 294
Orcalli, Angelo, 439
Oresme, Nicolas, 300
Origen, *90*, 308
Orlando furioso, 24
Ormandy, Eugen, 488
Osiander, Andreas, 118, *297*
Ovidio, Publio Nasone, *86*

Pacioli, Luca, *xiii*, 11, *21*, *276*, *280*, *282–287*, *290*, *295*, *300*, *324*, 331
Packham, Ron, *65*
Palestrina, Giovanni Pierluigi, 59, *304*, 449, 452, 453
Palisca, Claude V., 21, 301, *303*, 303, *312*, *320*, *322, 323*, *325, 326*
Palladio, Andrea, *276*
Panca-Siddhantika, *183*
Paola, Francesco da, 108, 134
Pânini, *173*, *182, 183*, *188*, *190*
Paolo dell'Abaco, *276*
Pappus, *54*, *62, 63*, *91*, 142, 168, 171, 410
Paracelsus, 64
Paradise Lost, 264
Parmenides, *16*, *83, 84*, *86*, *88, 89*, *162*, *166*
Parsifal, 467, 525
Pascal, Blaise, *5*, 32, 107, *123*, 140, 143–157, 183, 187, 188, 240, 248, 255, 304, 436, 462, 511
Pascal, Etienne, 140, 143
Passio secundum Matthaeum, 427, 478
Passions, 425
Pathodia Sacra et profana, 193
Pauli, Wolfgang, 64, 83, 489, 494
Paumgartner, Bernhard, 330
Pell, John, *186*, 190
Pensées, 107, 147, 154, 157
Perez, David, 384
Pergolesi, Giovanni Battista, 335, 381, 384, 387, 391, 392, 395, 400, 405, 406, 422, 423, 425, 437
Peri, Giacomo [Jacopo], 127
Perinu, Roberto, *195–197*, *200, 201*, *203*
Persico, Enrico, 369, 444
Pesic, Peter, *297*, *300*, *303*, *326*
Peters, Arno, *54*
Petrarca, Francesco, 60
Philidor, François-André, 404
Philo of Byzantium, *91*, *166*
Philolaus, *372*
Philosophia Mosaica, 257
Philosophiae Naturalis Principia Mathematica, 265, 266, 270, 272, 275, 276, 279, 282, 287, 290, 291, 295, 309, 310
Philosophical Essay of Music A, 273
Piano sonata op. 90, 454
Picasso, Pablo, 502
Picatrix latinus, *278*
Piccinni, Nicola, 425
Pico della Mirandola, Giovanni, 54
Piéces de clavecin, 323
Piero della Francesca, *xiii*, *276*, *280–287*, *289, 290*
Pigot, Thomas, 190
Pingala, *183*, *185*
Pingree, David, *195, 196*, *206*
Pitagorici, *15, 16*, *19*, *40*
Planck, Max, 1, *6*, 466, 470–483, 494, 499, 505
Platée, 381, 382, 400
Plato, *4*, *11*, 11, *13*, *15, 16*, *18–24*, 24, *25*, 25, *26*, 27, *30, 31*, *33*, *35*, 36, *37, 38*, 38,

39, 49–51, *55*, *57–59*, *61–63*, 63, *64*, 66, *70*, 72, *83–88*, *90–95*, 102, 116, 137, *145*, 150, *156*, *162, 163*, *166*, *173*, *192*, 195, *202*, *211*, *214*, *219*, *221*, *224*, *226*, *234*, 238, 239, *241*, *249*, *258*, *260–262*, *269*, *275–279*, *282–285*, *290*, *300*, *302*, 302, *309*, 314, *324*, *327*, 389, 505, 532
Plato's Testament for the Education of Young Men, *249*
Plautus, 177
Pliny, 133
Pliny the Elder, *91*
Plotinus, *279*
Plutarch, *29*, *59*, 133, *319, 320*
Poincaré, Henry, 25, 469, 482
Politeia [Republic], *21*, *61*, *64*, *92*, *94*, *166*
Politics, 50
Polo, Marco, *153*
Pompadour, 383
Pope, Alexander, 313, 325
Porpora, Nicola, 384
Poseidon, *15*
Pound, Ezra, 1, *143*, 509
Practica geometriae, *274*
Pratricha d'astorlogia, *276*
Prey, T.W., 507
Principes d'acoustique, 394
Principes de composition de l'école d'Italie, 435
Proclus, *19, 20*, *29*, 51, 55, *63*, 74, *91*, 92, 103, *260*, *262*, 266
Projet de paix perpétuelle, 417
Proni, Gaspar de, 439
Proposal to Perform Music in Perfect and Mathematical Proportions, 186, 271
Proserpine, 405
Provinciales Les, 145, 155
Prunières, Henri, 406
Ptolemies the kings, *60*, *64*, *90, 91*, *94*, *162*, *166*
Ptolemy, Claudius, *3, 4*, 7, *11*, 22, *27*, *34*, 37, *41–44*, 44, *45*, 45, *46*, 46, *47*, 47, *48, 49*, 49, *50, 51*, 51, *52, 53*, 53, 62–64, 70, 71, 79, 80, *83*, 86, *88*, *90*, *94*, 96, 105, 128, 137, *156*, *162*, 165, 184–186, *187*, 187–189, 200, 201, 203, *205*, *215*, *217*, *220, 221*, *225*, *240*, *244*, *248, 249*, *257*, *259*, *261, 262*, *268, 269*, *279*, *282*, *297–299*, *302*, 302, *304–306*, *310, 311*, *314*, *316*, *325*, 341, *365*, 412, 498, 520
Purana, *186*
Purcell, Henry, 321
Pygmalion, 333, 350, 375, 428
Pyrrho of Elis, 148, 150, 151
Pythagoras, 7, *10–15*, *21*, *25*, *28*, 28, *29*, 29, *34*, *38*, 38, *40*, 43, *52*, *56*, *63*, 78, *81*, *85*, 85, *86*, *88*, 90, 91, *92*, 99, 102, 105, 106, 122, *158*, *162*, *171*, *174*, 174, *178*, 186, 189, *194*, *203*, *219–221*, *224, 225*, *233*, 239, 250, *263*, 267, 269, *271, 272*, *278*, *280*, 284, 285, *287*, 301, *319*, *324*, 333, 341, *342*, 345, 349, 350, *367, 368*, 371, 397, 398, 403, 416, 423, 428, 430, 445, 450, 469, 480, 498, 512

Qian Hanshu [Books of the early Han], *100*, *103*, *134*, *138*
Qing, Manchu, *4*
Qinshi emperor, *99*, *119*, *148*, *161*, *164, 165*, 247, 418
Quattro libri dell'architettura I, *276*
Questiones ... in Genesim, 107
Quinault, Philippe, 196
Qurra, Tabit ibn, 189, *224–226*, *228*, *233*
Qusta ibn Luqa, *229*

Rabelais, François, *275*
Racine, Jean, 405
Radelet de Grave, Patricia, 291
Raffaello Sanzio, *21*, *33*
Ramanujan, Srinivasa, 465
Ramayana, *193*
Rameau, Jean-Philippe, *5*, 10, *38*, 60, 196, 321, 327–337, 340, 348–353, 357, 358, 360, 367, 370–381, 384–388, 392, 393, 395–407, 409, 412, 418–426, 428, 432, 437, 439, 451–455
Rannat al-awtar, *248*
Rasa'il Ikhwan al-Safa, *219*
Rasch, Rudolf A., 10, 196, 211, 221, 298
Rashed, Marwan, *254*
Rashed, Roshdi, *225*, *227–230*, *232–234*, *246, 247*, *249*, *254*
Rashid, Al-, *204*
Rattansi, Pyo, 268–270, 300, 301
Ravel, Maurice, 494
Ravetz, Jerome R., 433, 434
Rayleigh, John lord, 468, 528–530
Redondi, Pietro, 32, 36
Regiomontanus, Mueller Johannes, *59, 60*
Regole di contrappunto pratico, 435
Rehding, Alexander, *327*
Rekhaganita, *205*

Rembrandt, 215
Republic, 49, 50, 72, 116, 239, *249*, *261*, *324*, *366*, 389
Requiem, 453
Resnick, Robert, *322*
Reuchlin, Johannes, *280*
Revelation, 492
Riccati, Giordano, 409, 410
Riccati, Jacopo, 409
Ricci, Matteo, 27–29, 33–35, 85, 86, *143*, 248, 249, 253, 417
Riccioli, Giovan Battista, 94
Richardson, Samuel, 396
Riemann, Bernhard, 1, *6*, 439–447, 456–458, 466, 469, 470, 495, 500
Riemann, Hugo, 483–485, 495
Rig Veda, *170*
Righini, Pietro, *224*, *248*
Rigo, Bernhard, 523, 524
Risala fi aqsam al-ulum al-aqliyya, *232*
Rise of Music in the Ancient World. East and West The, *335*
Ritmica, *36*
Rivoluzioni del teatro musicale italiano, 435
Robaiyyat [Quatrains], *209*, *244*, *247*, *255*
Roberval, Gilles Personne, 140, 163, 177
Robinson, Kenneth Sirdwood, 12, 13, 15, *98*, *103–106*, *108*, *111*, *129–139*, *141–143*, *145*, *148*, *154*, *161*, *165*
Roland, de Lassus, 10, 59, 98, 100
Roman, Paul, 466
Rosen, Edward, 78
Rosenfeld, B.A., *228*, *230*, *246, 247*
Rossing, Thomas D., *104*
Rossini, Gioacchino, 437
Rousseau, Jean-Jacques, 335, 368, 375–379, 381–397, 400, 401, 405, 406, 411–413, 417, 422, 423, 425, 449
Rudolff, Christoph, 170
Ruffini, Paolo, *122*
Russo, Lucio, *34*

Sabaino, Daniele, *300*, *327*
Sabattini, Mario, *98, 99*, *165*, 248, 416
Sachs, Curt, *242*, *335*
Sacred Books of the East, *193*
Saggiatore Il, 19, 23
Saggio di una difesa della divina rivelazione, 367
Sah, Muhammad, *205*
Sala, Nicola, 435
Saliba, George Asad, *245*
Salinas, Francisco, 127, 165, 200, 203, 206–208
Salmon, Thomas, 186, 187, 271
Sama Veda, *170*, *195*, *197, 198*
Sambursky, S., *39*, *85–88*, *90*, *92, 93*, *95*
Samgita Ratnakara, *198*, *200*, *206*
Samratsiddhanta, *205*
Sani, Saverio, *193*
Santangelo, Paolo, *98, 99*, *165*, 248, 416
Sanyal, A.N., *201*
Sapir, Edward, *2*
Sappho, *325*
Śarngadeva, *199*, *206*
Śatapatha Brahmana, *170–172*, *177*, *192, 193*
Satyanarayana, Nookala Chinna, *199*
Saul from Tarsus, 324
Saussure, Ferdinand de, *190*
Sauveur, Joseph, 237, 296, 297, 332, 343, 358, 359, 388, 404, 420, 421
Savart, Félix, 438
Savonarola, *279*
Saydawi, Al-, 138, *225*
Sayili, Aydin, *225, 226*
Scaliger, Julius, 88
Scarlatti, Domenico, 298, 323, 401
Schall von Bell, Adam, 33, 213, 247, 417
Schering, Ernst, 443
Schipper, Kristofer, *111*
Schlegel, August, 464
Schnabel, Arthur, 487
Schoeneberg, Bruno, 466
Schönberg, Arnold, *xiii*, *6*, 240, *304*, 483, 485–499, 507
Schopenhauer, Arthur, *25*, 468, 501
Schostakovitch, Dimitri, 494
Schott, Gaspar, 238
Schreck Johannes, Terrentius, 33, 84, 90, 92
Schroedinger, Erwin, *82*
Schubert, Franz, 467, 478, 487, 500
Schumann, Robert, 454, 478
Schuppener, Georg, 91
Schütz Heinrich, 321, 475
Schütze, Ingo, *300*
Schweitzer, Albert, 479
Scienza e i vortici del dubbio La, *xiii*
Scimemi, Benedetto, 531
Scoditti, Giancarlo, 524, 525
Scoto, Duns, 305
Scriba, Christoph J. , 183, 187, 189
Sebokt, Severus, *203*
Sectio Canonis [Division of the monochord], *26*, *28*, *83*, *95*, *215*, *237*, *247*
Section of a ratio, *60*

Seebeck, Ludwig Wilhelm August, 458–461, 530
Seidenberg, Alexander, *34*, *170*, *175*, *179*
Semedo, Alvarez, 213
Sen, S.N., *173*, *177*
Seneca, 150
Seraphita, 491
Serenade, 498
Serravezza, Antonio, 480, 481, 484
Serres, Michel, *80, 81*, 229, 240, 259–262, *291*, 436, 437
Settle, Thomas B., 21, 23
Shadwell, Thomas, 287
Shakespeare, William, 3, 5
Sharh el Mushkil min Kitab al Musiqa, *247*
Sharh ma ashala min musadarat Kitab Uqlidis, *229*, *246*
Shea, William R., 119, 163, 168, 170, 172, 175–178, 181, 193, 194
Shen Gua, *131*, *135*
Shi Kuang, *139*
Shigu, *101, 102*
Shiji [Memories of a Historian], *99*
Shijing [Classic for Odes], *98*, *132, 133*
Shiloah, Amnon, *216*, *220*, *223–225*, *234*, *242*, *246*, *248–250*
Shuduyan, *125*
Shujing [Classic for Books], *98*, *151*
Shuli benyuan, *126*
Shuli jingyun, *126*
Sicanicarum rerum compendium, *296*
Siculus, Diodorus, *368*, 423
Siddhanta Śiromani, *187*, *196*
Sidereus nuncius, 22
Siemens, Werner von, 458
Sima Qian, *99*, *137*
Sima Tan, *99*
Sima Zheng, *134*, *137*
Simonini, Alessandra, 505
Simplicius, *163*, 243
Sina Ibn Avicenne, *210–219*, *221, 222*, *224*, *229*, *232–234*, *245, 246*, *250*, *254*, *260*, *278*, *290*, 299, *336–339*
Sinaat al-gabr, *229*
Sinclair, Upton, 489
Singh, A.N., *181–187*, *194*
Singh, S.L., *xiv*
Siu Man-Keung, *152*
Sivin, Nathan, *82*, *98*, *127*, *129*, *133*, *140*, *145*, *147*, *149*, *151*, *153–156*, *158–163*, *165, 166*, *168*
Six livres de la république Les, 50
Sleepwalkers, 312
Smith, David E., *29, 30*
Smith, Percy, 520
Snell, Willebrord, 119, 143, 168
Snellius, Willebrord, 11
Socrates, *19*, *21–23*, *25*, *92–94*, *251*
Solon, *87*
Sommerfeld, Arnold, 490
Somnium Scipionis, 64, *260*, *366*
Somnium seu opus posthumum de astronomia lunari, 64, 212
Sonata Pastoral, 500
Sonatas, 323
Sonnets , 5
Sophocles, *86*
Sorabji, Richard, *254*
Sorge, Georg, 448
Speculazioni di musica, 190
Speiser, Andreas, 500
Speiser, David, 366
Spengler, Oswald, 499
Speziale, Fabrizio, *204, 205*
Spinoza, Baruch, 252, 259, 304, 487
Śrautasutra, *183*
Staal, Frits, *179*, *188*, *190*
Stabat mater, 449, 452
Stapulensis, Faber, *292, 293*, *352*, *361*
Steffani, Agostino, 236
Stèndhal, 436
Stengers, Isabelle, 32
Stephenson, Bruce, 64, 94, 96, 97
Stevin, Simon, *xiii*, *5*, 5–12, 16, 17, *161*, 163, 164, 182, 186, 198, 206, 207, 221, 238, 301, *327*
Stifel, Michael, 11, *296*, *326*
Stomachion, *59, 60*
Storia della Croce, *281*
Storia della musica, 435
Straub, Hans, 362
Strauss, Richard, 487, 500
Strawinsky, Igor, 494, 502
Strena seu de Nive Sexangula, 67
Struik, Dirk J., 11, *230*
Stuckenschmidt, Hans Heinz, 487
Stumpf, Carl, 483, 494
Suanfa tongzong [Compendium of calculating rules], *5*, *329*
Suario, Giuseppe, 247
Subbarayappa, *205*
Suhrawardi, Al-, *219*
Śulvasutra, *173, 174*, *176–184*, *187*
Summa de arithmetica, geometria, proportioni et proportionalità, *283, 284*, *286, 287*

Summa theologica, 145
Sundararaja, *174*
Sun, Xiaochun, *98*, *101*, *104*, *107*, *112*, *116*, *122*, *128–132*, *134*, *136, 137*, *144*, *147*, *154*, *162*
Sun, Yuanhua, 33
Sunzi, *137*, *165, 166*
Sunzi bingfa, *136*
Surya Siddhanta, *187*
Swift, Jonathan, 3, 265
Sylva Sylvarum, 189
Sylvester, James, 508
Symposium [The banquet], *24*
Szczesniak, Boleslaw, 91

Tabulae Rudolphinae, 88, 89
Tacitus, 150, 382
Tagliagambe, Silvano, 244
Taittirya Brahmana, *170*
Taittirya Samhita, *171*
Taixuan, *129*
Tamburlaine, *205*
Tangherlini, Silvia, 51, 63, 83, 103
Tannery, Paul, *27*, *95*, 197, 297, 503
Tartaglia, Niccolò, 11, *92*, *286*, *301*
Tartini, Giuseppe, 372, 379, 392, 395, 397, 401, 407–411, 420, 432, 448, 451, 530, 531
Tasso, Torquato, 23, 264, 320
Taton, René, 141, 144
Taylor, Brook, 363, 369, 445
Teeta, 521
Telemann, Georg Philipp, 321
Tentamen de motuum coelestium causis, 290
Tentamen novae theoriae musicae, 340, 341, 349, 351, 358, 450
Tera'ituatini Pambrun, Jean-Marc, 525
Terracini, Alessandro, *61*
Terradellas, Domingo, 384
Tertius interveniens, 52
Testi & contesti [Texts & Contexts], *82*
Tetrabiblios [The Four Books], *53*, 53, *310*
Tevake, 521
Thales, *87*
The battle of the books, 3
The Great Chain of Being, 313
The Interpretation of Nature and the Psyche, 494
The Red and the Black, 436
The Social Contract, 391
The Testament of Astrology, 493
Theodoricus, *260*
Theon, *54*, *63*
Theory of Sound The, 528, 531
Thèses mathématiques, 7
Thibaut, George, *170*, *173*, *175*, *177*, *179*
Thom, René, 25, *40*, *80*, *161*, *167*, 440, 441, 532
Thomas, D., *322*
Thomson, George, *16, 17*
Tianxue chuhan, 29
Timaeus, *19–21*, 51, 266
Timotheus, *318*
Tiresias, *86*
Toccata and Fugue, 427
Tonietti, Tito M., *4*, 6, 12, 13, 15, 16, 25, *27*, 27, 28, 34, *35*, 35, *39, 40*, *59*, *81*, 85, *97*, *103–106*, *112*, *114*, *117*, *119*, *121*, 123, *125, 126*, *130*, *140*, *142, 143*, *151, 152*, *154–159*, *161*, 161, *167*, 173, *184*, *191*, *196*, *201, 202*, *230*, 230, *234*, 240, *246, 247*, 256, 261, *266*, 266, 268, *269*, 270, 271, *274*, *287*, *290*, 290, *291*, *293*, *296, 297*, 298, 303, *304*, *326*, *335, 336*, *352*, *360*, 441, 444, 462, 466, 470, 481, 485, 486, 488, 489, 494, 497–499, 503, 506
Torella, Raffaele, *189, 190*, *192*, *197*
Torricelli, Evangelista, 143, 144, 227
Traité d'Harmonie, 435
Traité de l'harmonie réduite à ses principes naturels, 327, 454
Traité de l'homme, 159
Trattato d'abaco, *281*
Trattato di musica secondo la vera scienza dell'armonia, 407, 409
Treatise of Algebra, both Historical and Practical, 188
Treatise of Angular Sections, 189
Treatise of Harmony, 295
Treatise on Algebra, *229*
Trio Dumky, 500
Triomphe de l'Amour, 405
Trios, 500
Truesdell, Clifford, 358–363, 365–369, 371, 418, 422, 434, 528
Tupaia, 519–522
Turin, Alain, 188
Turner, Frederick, *143*
Turner, R. Steven, 457, 458

Upaniṣad, *170*, *187*, *193, 194*
Uqlidisi, Al-, *231*
Ustuqusat, Al-, *225*, *234*

Vacca, Giovanni, *33*, *92*
Vahabzadeh, Bijan, *228*, *230*, *246, 247*
Valla, Lorenzo, *307, 308*, *316*
Vallance, John, *84*, *87*
Vande Spiegheling der Singconst, 8, 164, 182, 198
Varahamihira, *183*, *206*
Varela, Andia y, 521
Variationen für Orchester, 498
Varignon, Pierre, 242
Vasari, Giorgio, *283*
Vayer, Fr. de la Mothe le, 225
Vecchi, Orazio, 100, 101
Veda, 231, *258*
Vedanga, *183*, *186*
Vedanta, *170*, *190*
Vegetti, Mario, *34*, *82*, *84–93*, *95*
Vendrix, Philippe, *327*
Veracini, Francesco Maria, 407
Verbiest, Ferdinand, 247, 287
Verité des sciences, 119
Vernant, Jean-Pierre, *140*
Vicentino, Nicola, 1, 203, 207, 208, *301–303*, *326, 327*
Viète, François, 142, 177, 188, 234, 243, 245, 273
Vijaganita, *206*
Villiers, Christophe de, 165, 166
Vinci, Leonardo, 384
Virtuoso The, 287, 291
Vitruvius, Marcus Pollio, *84*, *91*, 136, *166*, *276*, *286*, 480
Vitry, Philippe de, 300
Vivaldi, Antonio, 321, 322, 330, 487
Viviani, Vincenzo, 20, 130, 227
Vogler, Georg Joseph, 357
Vogler, Martin, 357
Voltaire, 291, 295, 415–418, 422, 429
Vom Musikalisch-Schönen, 484

Wackher, Matthias, 66
Waerden Bartel L. van der, *172*, *179*, *186*, *210*
Wafa, Abu 'l, *230*
Wagner, Richard, 456, 467, 478, 485, 487, 525
Walker, Daniel Pickering, 100, 183, 244, 254, 255, 261, *277*, *279*, 299, 301, *303*, *320*, *322, 323*, *325*
Wallenstein, Albrecht von, 52, 76, 84, 90
Wallis, John, *5*, *13*, *41*, *46*, 102, 158, 183–190, 200, 201, 203, 221, 273, 285, 296, 393
Wallis, Samuel, 523
Wang Bo, *139*
Wang Chong, *129*
Wang, Hsin-huei, 565
Wang Ling, 12, 13, *98*, *100, 101*, *103*, *105, 106*, *108*, *111, 112*, *121–123*, *125*, *127–139*, *143*, *145, 146*, *148*, *152*, *161*
Wardhaugh, Benjamin, 186, 187, 190, 191, 197, 272, 286, 290, 363
Warrain, Francis, 98
Weber, Ernst Heinrich , 483
Weber, Heinrich, 466
Weber, Max, 498
Webern, Anton, 505
Werckmeister, Andreas, 102, 103, 205, 324
Westfall, Richard S., 265, 270, 273, 278, 283, 287, 289, 310
Weyl, Hermann, *21*, 173, 230, *326*, 465, 499, 507
White, David G., *193*
Whitman, Walt, *169*
Whorf, Benjamin Lee, *2*
Wigner, Eugene P., 293
Willaert, Adrien, 59, 100, *304, 305*
William of Moerbeke, *59*
Winnington-Ingram, R.P., *35*, *38*
Wittgenstein Ludwig, *139*, *161*
Wolf, Hugo, 500
Wolff, Christian, 426
Wotton, sir Henry, 80
Wren, Christopher, 285
Wrońsky, Józef Maria, 439
Wu, king, *137*
Wu Wenjun, *152*
Wujing, 231
Wudi, *165*
Wylie, Alexander, 462, 463
Wymeersch Brigitte van, *230*, *327*

Xenakis, Iannis, 506
Xenophanes, *89*
Xerse, 196
Xifa shenji, 33
Xin Dufang, *135*
Xu Chunfang, *117*
Xu Guangqi, 28, 30, 33, 34, 85
Xunzi, *133*, *165*

Yadav, B.S., *xiv*
Yajur Veda, *170*
Yang, C.N., 465
Yang Xiong, *129*
Yantraraja, *206*

Yijing [Classic for Changes], *98*, *100*, *102*, *104, 105*, *107*, *112*, *116*, *119*, *121, 122*, *125*, *129*, *131*, *144, 145*, *148*, *150*, *165*, 249, 250, 253–255, 288, 319, 413, 418, 439
Young, Thomas, 458
Youschkevitch, A.P., *228*, *230*, *246, 247*, 348, 350, 357, 362
Yueji, *106*, *134*
Yuejing [Classic for Music], *97, 98*
Yueling, *135*

Zanoncelli, Luisa, *26*
Zarlino, Gioseffo, 6, 7, 9, 44–48, 104, 105, 112, 127, 128, 159, 161, 165, 186, 187, 198–200, 203, 206–208, 284, *298, 299*, *302*, 302, *303–307*, *309–314*, *316*, *320*, *322–325*, *327*, 328, 329, 331, 335, 350, 408, 409, 445, 476, 498, 505, 529, 531
Zarqali, Al-, 88
Zaslavsky, Claudia, 511
Zeng Guofan, 35
Zeno of Elea, *16*, *18*, *39*, *56*, 120, *125*, *235*, *254*, 263
Zeuthen, Hieronymus, *29*, *33, 34*, *187*
Zhang Dianzhou, 466
Zhang Heng, *129*, *145*
Zhang Zai, *130*
Zhao Shuang, *111, 112*, *116*, *119, 120*, *159*
Zhengmeng, *130*
Zhou Mi, *132*
Zhou Xu, *164*
Zhoubi suanjing, *30*, 85, *111*, *116–119*, *121*, *125*, *141*, *143*, *148*, *150*, *153*, *159*, *225*, *284*, 416
Zhouli, *134*
Zhu Xi, *124, 125*, *131*
Zhu Zaiyu, *xiii*, *5*, 5, 12–14, 16, 17, 27, 102, *137*, 208, 452
Zhuangzi, *98*, *120*, *124*, *127*, *139*, *141*, *146*, *148*, *150*, *154*, *160*, *165*, *167, 168*
Zhuzi quanshu, *124*, *131*
Zig al-Sindhind, *244*
Zigeunerlieder, 478
Ziguxi zhai suanxue, 462
Zipoli, Domenico, 298
Zoroaster, 249
Zoroastre, 375
Zu Chongzhi, *123*
Zuozhuan, *133*, *136*